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Review: After Gödel. Platonism and Rationalism in Mathematics and Logic by Richard Tieszen

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This volume is aimed at developing and defending a theory of knowledge that would best explain our relation to the abstract objects of mathematics and logic. This theoretical stance is developed in relation to Gödel's philosophical conception, but does not pretend to be faithful either to Gödel's intentions or to his writings. The philosophical approach developed in the book is called *consti*tuted Platonism and is presented as a form of rationalism that combines some aspects of what Gödel called the rightward tendency (the belief in the cognitive role of reason) with some aspects of the leftward tendency (the impossibility of having cognitive access to noumena). This combination of left and right is exactly what Gödel appreciated in Husserl's work in his 1961 essay on the modern development of the foundations of mathematics in the light of philosophy. The volume is conceived as a phenomenological essay about phenomenology in mathematics and logic: constituted Platonism appears as a Gödelian strategy as well as a consistent development of a Husserlian perspective applied to mathematics and logic, but also as an alternative to Quine's holism and an answer to Carnap's conception. The first and the last chapters, which set the stage and draw some conclusions on the role of reason in science, are perhaps the most informative and interesting parts of the book, because Gödel's ideas are discussed in relation to different authors (especially Kant, Husserl, Carnap and Quine). Gödel's distinction between a leftward and a rightward tendency in philosophy of mathematics and logic drives the agenda, and is useful in understanding both Gödel's refusal of any form of reductionism and the phenomenological way out from the Kantian duality of phenomena and noumena.

Constituted Platonism is based on the idea that there are two notions of independence from the mind (pp. 101—102): some objects are strongly independent from the mind because they are independent of any possible experience, whereas other objects are independent from the mind in the sense that they appear to us as invariant in different experiences and our knowledge of them undergoes certain constraints (grammatical, formal, meaningtheoretic, structural, and so on). Constituted Platonism claims that mathematical and logical objects are mind independent in the second sense, because their objectivity is constituted by consciousness, and in particular by the overlapping horizons of different meaning-bestowing monads (p. 96). The author does not pretend that Gödel actually considered all mathematical objects as independent from the mind in the second sense. For example, he concedes to Mark van Atten that Gödel might have taken natural numbers to be ideas in the mind of God and thus independent from any human mind (p. 17). Setting aside certain aspects of Gödel's conception, in this and in several other cases, the author aims to develop a defensible form of Platonism in mathematics and logic. Strong mind-independence is taken to be incompatible with human knowledge, and thus subject to definitive criticism concerning cognitive accessibility to abstract objects. So, Tieszen chooses weak mind-independence, which allows us to understand the experience of abstract objects by analogy with sensory experience: abstract objects are known by a rational intuition that is not immediate but allows illusion, errors and knowledge revision. This explains how our knowledge of abstract objects might be incomplete, but objective. This is for example the case of set theory, where concepts are exact but our intuition of them is partly indeterminate, and a clarification of our intuition of the concepts is needed in order to solve problems. Set-theoretical paradoxes are not the symptom of a cognitive impasse, but rather an example of correctable illusions, because "in the presence of the contradiction (paradox), we can shift to a new perspective on sets that does not appear to be contradictory" (p. 162).

The author's intention to develop a coherent philosophical conception, without limiting himself to a literal reconstruction of Gödel's own thinking is certainly a very interesting and difficult task, which the volume achieves in a convincing and successful way. The exaltation of reason is obtained through the emphasis on intentionality, and in particular, on the differentiation of several intentional acts involving proof, justification, ideation, exactness, axiomatization, formalization, and the search for necessities (p. 141). Abstract objects are the invariant objects that constitute the correlative contents of such acts. Knowledge of abstract objects is possible because "we are not only directed by intentions toward the invariants that we conceive in mathematics, but [...] some of these intentions can be fulfilled, partially fulfilled, or frustrated" (p. 157). Husserl's phenomenological approach to consciousness and intentionality, as well as Gödel's emphasis on the capacities of reason, are used to develop a coherent theoretical approach to the philosophy of mathematics and logic. Yet, the choice to depart from some of Gödel's ideas, and maybe also from some of Husserl's ideas, raises some questions concerning the methodology of the research.

The volume is not based on a merely historical approach, because, as the author himself declares, the version of platonic rationalism advocated in the book does not "reflect all the elements of Gödel's own view", yet one of the aims of the volume is that of informing the reader about the content of "Gödel's relevant writings or the supplementary materials" (p. vii). And this latter aim is well achieved by a thorough and detailed presentation of Gödel's

remarks on Plato, Leibniz, Kant and Husserl in the first chapter. Yet, there are not as many comments on Plato as expected, and those on Leibniz are related only to his project of a universal characteristic without any references to monadology or to his metaphysical writings. The author acknowledges that "there are suggestions in some of Gödel's writings that he was prepared to go Leibniz's way on this matter", but adds that "for a variety of reasons, I am not going to pursue this line of inquiry" (p. 11). This choice is not further elucidated, but one should consider that Gödel did not make fully explicit in his published writings or in the considered supplementary materials how deep the influence of Leibniz's philosophy was on his own work. So, one will have to wait for the imminent publication of the transcription of Gödel's manuscripts on philosophy (the so-called MaxPhil) by Gabriella Crocco's research group, in order to better evaluate the role of Leibniz's ideas on Gödel's own thinking.

A second methodological question concerns the possibility of following and betraying Gödel at the same time: on the one hand the author wants to start from Gödel's own texts, and on the other hand he develops a coherent and well-grounded philosophical conception that might not correspond entirely to Gödel's philosophy. This is the reason for the choice of the title "After Gödel", which is very appropriate, because the author follows Gödel up to a certain point that seems defensible. The survey on Gödel's remarks on philosophy in the first part is very interesting and adherent to Gödel's texts. Richard Tieszen declares from the outset that he will steer clear of Gödel's remarks on religion, angels, demons, ghosts and the like, because he aims to develop a form of rationalism about mathematics and logic only (p. viii, 204). This disciplinary boundary is of course a useful restriction, if one wants to build a theory about mathematical and logical objects. Yet, it has the effect of steering clear of Gödel's interdisciplinary approach

too, based on the integration of metaphysics, logic, mathematics, history, theology and natural sciences.

A third methodological question concerns the reference to mathematical practice in the arguments used to defend constituted Platonism, for example when the author claims that mathematical objects are not conceived as moments of real entities. The author rarely gives specific examples, and even when some mathematical examples are mentioned (e.g. from number theory or set theory), they are separated from their historical and cultural context. The interest for mathematical practice is not only compatible with, but also crucial to a phenomenological approach based on the investigation of appearances, but there are many different mathematical practices and styles, some of which might be compatible with empiricism, such as the tradition of using rational numbers rather than real numbers in measurement theory. So, either the author mentions some specific mathematical practices that are based on Platonism, or mathematical practice is not a sufficient argument in favor of Platonism against certain forms of strong empiricism. For example it is not enough to say that our practice shows that mathematics cannot be "a set of empty hypotheses, conjectures, expectations, or problems", or that it cannot lack rational intuition.

Such critical remarks notwithstanding, the book is a very interesting and stimulating text that achieves a rare and difficult task: it connects in a new and fruitful way two traditions that have been separated and opposed for poor reasons and for too long: the analytical and the continental tradition. On the one hand, the author makes Husserl (and some of Husserl's language) understandable to logicians and philosophers of mathematics; on the other hand it invites Husserl's scholars to take into serious account the correlations to Gödel's technical results, including the incompleteness theorems and the remarks on Turing's machines. The accuracy of the quotations and the richness of textual references, together with the comparisons with other authors of the analytical

tradition such as Quine and Carnap, do not only improve the clarity of the text, but will certainly stimulate a revival of studies of Gödel's philosophy. One of the main merits of the volume, from an historical perspective, is exactly that of telling us what Gödel's philosophy cannot be, destroying a widespread but oversimplified understanding of his Platonism, and inviting the reader to complete the framework by a thorough investigation of Gödel's unpublished materials.

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