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Wittgenstein and Frege on Negation and Denial

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ABSTRACT

Frege maintains that there are not two distinct acts, assertion and denial; rather, denying p is one and the same as asserting not- p . Wittgenstein appears not to recognise this identity in Frege, attributing to him the contrary view that a proposition may have one of two verbs, “is true” or “is false”. This paper explains Wittgenstein’s attribution as a consequence of Frege’s treatment of content as theoretically prior to the act of judgment. Where content is prior to judgment, the denial of p —what is rejected in asserting p —is distinct from the assertion of not- p . Wittgenstein’s own embrace of Frege’s identity is then considered, an embrace explained in part by his repudiation of Frege’s theoretical order.

1. Introduction

“To each thought”, Frege writes, “there corresponds an opposite, so that rejecting one of them coincides with accepting the other” (PW 198). There are not two distinct acts, assertion and denial. Rather, a thought p opposes its negation not- p , so that denying p is one and the same as asserting not- p .

The Tractarian Wittgenstein takes the same view. He appears, however, not to recognise its presence in Frege. Indeed, Wittgenstein ascribes to Frege the contrary position that a proposition may have one of two verbs, “is true” or “is false” (TLP 4.063). This reading is surprising not only because there are passages in which Frege explicitly rejects “two ways of judging” (CP 385), but also because Frege nowhere theorises with any second “verb”. Wittgenstein’s attribution is nonetheless quite correct, in that it follows from a foundational commitment in Frege that denial is indeed distinct from assertion. Throughout his career, Frege

maintains a “separation of the act from the subject matter of judgment” (CP 149): propositional content is theoretically independent of, and so theoretically prior to, the acts of judgment and assertion. And given what Frege wants to mean by denial, and what Wittgenstein does mean by it, it is a consequence of this priority that the contents p and not- p are not opposed in such a way that denying p is the same as asserting not- p .

Section 2 of this paper briefly outlines Frege’s separation of judgment from content. Wittgenstein, it is held, both recognises this Fregean separation and rejects it. A Tractarian content is not theoretically prior to assertion but is, rather, something *to be judged*. In Section 3 an explanation is offered of Wittgenstein’s attribution to Frege of an act of denial distinct from assertion. The denial of p is that act whose correctness is the incorrectness of the assertion of p , and given Frege’s separation this cannot be the assertion of another content, not- p . Section 4 outlines the grounds of Wittgenstein’s own commitment to the identity of denying p and judging not- p . Finally, Section 5 expands the Tractarian account, considering how, with judgment internal to content, to judge that p is to reject the judgment that not- p .

2. Judgment and Content

Frege distinguishes in his concept script between what indicates a propositional content and what effects its assertion:

A judgment will always be expressed by means of the sign “|-” which stands to the left of the sign, or the combination of signs, indicating the content of the judgment. (BS §2)

The judgment that p is expressed by a formula “|- p ”. Within this formula, the symbol “ p ” indicates a propositional content but does not present this content as judged. To do this, to present the content as judged, the “ p ” is prefixed by the sign “|-”. This distinction between what expresses a content and what effects its assertion reflects, I shall maintain, a certain order in Frege’s theorising. Specifically, it reflects that Fregean content is theoretically independent of, and so theoretically prior to, the acts of judgment and assertion.

This ordering is not something Frege dwells on, but it is visible in various aspects of his work. One such aspect is his parallel treatment

of assertion and questioning. Comparing assertoric and interrogative sentences, Frege writes:

An interrogative sentence and an assertoric one contain the same thought; but the assertoric sentence contains something else as well, namely assertion. The interrogative sentence contains something more too, namely a request. (CP 355)

Frege’s parallel remarks here suggest that the thought contained in both the interrogative sentence and the assertoric sentence has no more in itself to do with the act of asserting than it does with the act of questioning. (Frege might it seems have augmented his script with a sign “?-” such that where “|- p ” asserts that p , “?- p ” asks if p .) And so they may suggest that the thought has *nothing* in itself to do with the act of assertion. As he writes elsewhere:

When something is judged to be the case, we can always cull out the thought that is recognised as true; the act of judgment forms no part of this. (PW 251)

A propositional content is something which may be judged. Equally, it is something which may be asked, hoped, guessed etc. And there is indeed a parity here, in that these acts are all external to the content itself; they “form no part” of it. Frege has of course a particular interest in the act of judgment: his script is a script for the prosecution of inference, and so for the representation of judgment. But this interest notwithstanding, judgment is on a level with hoping and asking as subsequent in the order of explanation to content.

This is Dummett’s reading of Frege. For Frege, Dummett writes, “the sense of the sentence is . . . given in advance of . . . the activity of asserting” (1959, 143). A substantial defence of this reading would need to do a good deal more than collate suggestive remarks. It would for example examine Frege’s “replacement of the concepts *subject* and *predicate* by *argument* and *function*” (BS 7); or it would examine the priority of Fregean grasping over Fregean judgment (e.g., PW 267); or it would consider how the normative laws of judgment are grounded for Frege on prior, descriptive laws of truth (e.g., CP 351). It is not my ambition here, however, to enter such a defence. More important for our purposes are two attributions to Wittgenstein, attributions which will be supported by the work of the paper as a whole. The first of these is that

Wittgenstein shares in Dummett's reading of Frege: Fregean content is constitutively prior to judgment and assertion. Second, Wittgenstein shares Dummett's rejection of this attributed view.

Wittgenstein doesn't consistently use Frege's language of judgment and assertion, preferring instead a mixed bag which includes besides judgment and assertion a terminology of thinking, picturing, saying and representing. These Tractarian acts all share in common with Fregean judgment and assertion, however, that they are correct or incorrect depending on how things are. Wittgenstein writes of picturing:

The picture agrees with reality or not; it is right or wrong, true or false. (TLP 2.21)

Like a Fregean assertion " $\vdash p$ ", a Tractarian depiction " p " is either right or wrong: it is right if p and wrong if not- p . Crucially, moreover, it is in relation to such an act that content is explained in the *Tractatus*. For both Frege and Wittgenstein, a content is fundamentally a *content*, something to be thought and expressed. Where in Frege, however, such content expression is equally present in both assertion and question and is explanatorily prior to both acts, a Tractarian sense is fundamentally the content of a *picture*, and so fundamentally the content of something correct or incorrect depending on how things are.

3. Frege's Two Verbs

3.1.

The divergence between Frege and Wittgenstein on the relation between judgment and content is at once a divergence on the relation between judgment and truth. Truth and content are theoretically coeval in both Frege and Wittgenstein: truth is explained with a notion of propositional content, and a propositional content is precisely a condition of truth (see, e.g., GG §32 and TLP 4.431). Frege's position that content is prior to judgment and assertion is thus a position at once that truth is prior to assertion. And Wittgenstein's contrary view that content is fundamentally the content of a saying is at once a view that truth is fundamentally correctness in saying.

Wittgenstein's opposition to Frege on this matter is manifest at various points in his book. One key such point is section 4.063, the

section in which he sets out his (in)famous "analogy to illustrate the concept of truth". In this analogy, Wittgenstein compares points on a piece of paper which are either black or white to sentences " p " which are either true or false. The analogy breaks down, Wittgenstein then writes, in that whilst a point on the paper may be recognised independently of a determination of when it is called black and when white, there is nothing meant by " p ", and so no contentful " p " here to recognise, separately from a determination of when " p " is called true—separately, that is, from a determination of its sense:

The point at which the simile breaks down is this: we can indicate a point on the paper, without knowing what black and white are; but to a proposition without a sense corresponds nothing at all, for it signifies no thing (truth-value) whose properties are called "false" or "true"; the verb of the proposition is not "is true" or "is false"—as Frege thought—but that which is true must already contain the verb. (TLP 4.063)

Every proposition must already have a sense; assertion cannot give it a sense, for what it asserts is the sense itself. (TLP 4.064)

At first glance, Wittgenstein's correction of Frege may seem to be as follows. A proposition has no meaning separately from a determination of its truth condition—separately, that is, from the determination of its sense. Frege held however that prior to having a truth condition, a proposition refers to a truth-value, and that it is only through a subsequent act of assertion that the proposition comes to have its sense. The symbol " p " of the Begriffsschrift formula " $\vdash p$ " refers to a truth value, but it does not in itself have a truth condition: for that, we need to apply Frege's assertion sign " \vdash ".

At this first glance, Wittgenstein has Frege badly wrong. It is a very poor misreading of Frege to suggest that whilst an unasserted symbol " p " of his script refers to a truth value, such a symbol does not yet express a Fregean thought. Wittgenstein does not, however, blunder in this way.¹ What section 4.063 displays is not incompetence in reading Frege, but an important difference between Fregean thought and Tractarian

¹Proops disagrees, holding that Wittgenstein misreads Frege's claim that "in a mere equation there is as yet no assertion: ' $2 + 3 = 5$ ' only designates a truth value, without its being said which of the two it is" (GG §5). "Frege's point", Proops writes, is "that ' $2 + 3 = 5$ ' designates a truth value in contrast to expressing a judgment. He is not claiming that ' $2 + 3 = 5$ ' designates a truth value in contrast to expressing a sense. I think it likely, however, that Wittgenstein read this passage in the second of these ways" (1997, 132).

sense. Unlike a Fregean thought, a Tractarian sense is something *to be judged*: a sense is, essentially, a correctness condition for judgment and assertion. What has a sense, it follows, is something correct or incorrect depending on how things are. And in Frege's script, what is correct or incorrect depending on how things are is not the propositional symbol " p " which expresses only understanding, but the formula " $\vdash p$ " which expresses a judgment. In this way, Frege posits propositions—symbols with propositional content—constitutively prior to Tractarian sense, and maintains that the expression of such sense arises only with a subsequent act of assertion.

Removing this puzzling appearance of a blunder in reading Frege still leaves us, however, with a second such appearance, one which will hold us for rather longer. This concerns Wittgenstein's ascription to Frege of two "verbs of the proposition". The terminology here of a "verb" occurs nowhere else in the *Tractatus*.² It derives, however, from Russell's *Principles of Mathematics* where it indicates a distinguished propositional element which contains, or gives, the assertion (1992, secs. 51–52). The ascription to Frege of two verbs for his propositions thus strongly recalls Frege's own description in *Begriffsschrift* of his script as having a "single predicate for all judgments, namely, 'is a fact'" (BS §3). Explaining his new formal script, Frege compares it to a language in which "the proposition 'Archimedes was killed at the capture of Syracuse' is expressed in the following way: 'The violent death of Archimedes at the capture of Syracuse is a fact'" (BS §3). Such a language, he continues:

... would have only a single predicate for all judgments, namely, 'is a fact' ... Our *Begriffsschrift* is a language of this sort and the symbol ' \vdash ' is its common predicate for all judgments. (BS §3)

Frege's script has a single "predicate" by which a propositional content is asserted, a single "verb", namely "is true" or "is a fact", whose application to a symbol with propositional content constitutes the expression of a judgment.³ In *Tractatus* 4.063, however, Frege's single predicate "is a fact", his sign " \vdash ", has become not one verb but two: "is

²Section 4.063 is copied verbatim from the 1913 *Notes on Logic* and could in various ways have used a rewrite.

³This "predicate" is of course not really a *predicate*—at least, not as conceived in the Aristotelian tradition against which Frege is reacting. (So *Begriffsschrift* 3 begins with the avowal that "a distinction between subject and predicate finds no place in my representation of a judgment" (BS §3).) Frege's presentation of his assertion sign as his

true" and also "is false". Given a propositional symbol " p " of Frege's script, Wittgenstein apparently thinks, there are two ways of proceeding to Tractarian sense: one can assert as true, or alternatively one can deny as false.

How could this attribution be correct? Frege's script has no denial sign correlate to his assertion sign: as we are told, the language has a *single* predicate. There are of course formulae of the form " $\vdash p$ ", but Frege could hardly be more explicit—including in *Begriffsschrift* section four—that the vertical negation stroke figuring in this formula does not serve to indicate a second assertoric act but contributes instead to the content asserted. "Negation does not belong to the act of judging", Frege later writes, "but is a constituent of a thought" (PW 253). The formula " $\vdash \neg p$ " expresses not a negative way of judging that p , but a regular, truth-oriented judgment that not- p . What is more, there are passages in which Frege explicitly considers and rejects the idea of an act of denial distinct from assertion:

To each thought there corresponds an opposite, so that rejecting one of them coincides with accepting the other. To make a judgment is to make a choice between opposite thoughts. Accepting one of them and rejecting the other is one act. So there is no need of a special sign for rejecting a thought. We only need a special sign for negation as such. (PW 198)

The content of any truth is 'a content of possible judgment', but so too is the opposite content. This opposition or conflict is such that we automatically reject one limb as false when we accept the other as true, and conversely. The rejection of the one and the acceptance of the other are one and the same. (PW 8, see also 185, 149)

Not only does Frege's script have no denial sign, Frege furthermore explicitly disavows the idea of an act of denial distinct from assertion. There is, indeed, such a thing as denial, but this is not a distinct, second act (there are not "two ways of judging" (CP 385)); rather, to deny p is to assert not- p . So again, what are we to make of Wittgenstein's ascription to Frege of two verbs?

The first and obvious thing to say here is that Wittgenstein is not giving a reading of Frege in the sense of an attempt sympathetically to

single predicate for all judgments is rather an explanation of the sign as the sole locus in his script of assertoric force.

elaborate Frege's thinking on its own terms—no more indeed than he does when he says that a Fregean proposition gains a sense through assertion. That was never going to be Wittgenstein's game. Rather, as I shall suggest, Wittgenstein has singled out what he sees as fundamental to Frege's theorising, namely that judgment is external to content, and from that basis he has reconstructed a wider "Fregean" view. This view is then attributed to Frege, even though it may bear little relation to what Frege himself says, or would readily recognise. The logic of Wittgenstein's reconstruction and attribution is not, however, alien to Frege. On the contrary, it is visible in the justification Frege himself gives for rejecting a separate act of denial.

In the passages just cited, Frege reasons as follows. There is an act of rejecting or denying that p opposite to that of accepting or judging that p . In addition to his assertion sign, someone might therefore think, we need also a sign for rejection. This inference overlooks, however, that contents themselves oppose each other: the thought that p opposes, and is opposed by, the thought that not- p . And so what may seem to call for distinct, opposing acts is accommodated instead by a single act taking two opposing contents: denying p may be identified with asserting not- p . In the *Tractatus*, I shall suggest, Wittgenstein turns this reasoning against Frege. Frege's separation of judgment from content is at once a separation of judgment from negation. And this entails that the Fregean contents p and not- p do not in fact oppose each other in a manner which permits the identification of denying that p with judging that not- p . Despite his explicit rejection of a second verb, Frege is thus committed to just such a thing, to an act of denial separate from assertion.

Let's take this in two steps. First, let's consider what the opposition is to be between judging p and denying p , and so what is meant here by denying p . And then second, let's consider how Frege's separation means that denying p cannot be identified with judging not- p .

3.2.

Frege writes that there is an opposition or conflict amongst contents, and that this opposition obviates the need for a separate act of denial. He is less explicit how the idea initially arises of such a separate act. The implicit thought is however clear enough: an act of denial is needed to complement, as its opposite, the act of assertion. Asserting that p is one

half of an opposition, the other half of which is denying that p . What, then, is this opposition of acts?

A first stab at answering this question would make a simple appeal to *correctness*. Judging that p opposes rejecting that p in that the former is correct only if the latter is incorrect. These acts oppose or conflict in the sense that the one is right if, and only if, the other is wrong. This first stab isn't entirely off beam, for the conflict must indeed be understood in terms of correctness. Nonetheless, the suggestion is clearly too weak. If we are to make sense of Frege's reasoning, two related conditions need to be met. First, the idea of an act opposite to judging p must make the proposal look reasonable of a second way of judging, a proposal which is then rebutted by identifying denying p with judging not- p . And second, this identification must depend upon, and so tell us something significant about, the nature of negation: the contents p and not- p are opposed to each other. Neither of these conditions holds, however, if the opposition between judging p and denying p consists merely in the material equivalence of the judgment's correctness with the denial's incorrectness. If the idea of denying p is that of an act correct if, and only if, judging p is incorrect, then judging not- p is trivially adequate for the role—its adequacy dependent only on the fact that the content p is true (and so correct to judge) if, and only if, the content not- p is false (and so incorrect to judge). There will be no suggestion of a separate act of denial, and no significant commitment in play concerning negation.⁴

Looking for a stronger notion of opposition, the ready thought will be to tighten the correctness connection. Consider here winning and losing in competitive games. Jack is playing Jill at chess, and so Jack will win the game if, and only if, Jill loses. More than this, though, for Jack to win his game against Jill is for Jill to lose her game against Jack. There are not two separate events here, or indeed two separate conceptions of a single event; rather, the thought of Jack winning is the thought of Jill losing. And so too we may conceive of judgement and denial. Judging p is opposed to denying p not merely in that the one is correct if, and only if, the other is incorrect, but in that the correctness of the judgment is one and the same as the incorrectness of the denial. To judge that p is to take a stand, or position, essentially against an opposite, the denial of

⁴One might also wonder how, if this is the opposition between judgment and denial, there is to be a *single* act for Frege of denying p : judging that it is incorrect to judge that p is correct if, and only if, judging that p is incorrect.

p , such that for the judgment to succeed (be correct) is for the denial to fail (be incorrect).

Frege says that to judge is to choose (PW 185, 198). In judging, Frege maintains, the subject chooses between a thought and its negation—that is, between judging a thought and judging its negation. Frege helps himself at these points to the identity in current question between judging not- p and denying p . But holding back on that identification, recognising judgment as a stand against an opposite provides for its recognition as a choice. In judging, the subject understands her act to be correct: judging p , she understands herself to be correct so to judge. Insofar as her act is a stand against an opposite, however, the understanding of her judgment as correct is at once an understanding of its opposite, the corresponding denial, as incorrect. The correctness of the judgment is in conception the same as the incorrectness of the denial. In judging that p , it follows, the subject understands the denial that p to be incorrect: she rejects the denial that p . Judgment is thus indeed as Frege claims a choice. The subject is asked, “Is it raining?”. Her answer “Yes” is correct if, and only if, the answer “No” is incorrect. More than this, though, her answer “Yes” represents a choice between answering “Yes” and answering “No”, for her answer “Yes” is a rejection of the answer “No”.

3.3.

The notion of opposition between judgment and denial with which Frege’s reasoning sets off in the passages cited, I am suggesting, is that of a pair of internally opposed stands such that for the judgment that p to be correct is for the denial that p to be incorrect. From here, Frege identifies denying p with judging not- p . This identification is supported with a claim of an opposition at the level of content. The contents that p and that not- p oppose each other, Frege says, and “this opposition or conflict is such that . . . the rejection of the one and the acceptance of the other are one and the same” (PW 8). Now Frege may indeed be able to claim a significant notion of oppositeness between his contents that p and that not- p . These are internally opposite truth conditions, and we’ll say more of this anon. With judgment separate from content, however, judgment will be separate also from negation, an operation at the level

of content. And this means that Frege cannot identify denying p with judging not- p .

To help us see this, consider the following analogy. A maths teacher shows a student a series of cards, each of which bears a complex name of a real number, for example “ $(54/4) - 12$ ”. The student’s task is to work out whether the number on the card is positive. She is to say “Yes” if the number is positive, and “No” if it isn’t. In saying “Yes” to a certain card, we may take it with Frege, the student opposes saying “No” to that card; her calling “Yes” is a rejection of the call “No”. But the student does not, in saying “Yes” to a certain card *similarly* oppose any call on any *different* card. For example, if the card is marked “ $(54/4) - 12$ ”, then in saying “Yes” to the card, the student doesn’t oppose saying “Yes” to a card marked “ $-((54/4) - 12)$ ”. Or again she does not, in saying “Yes” to “ $(54/4) - 12$ ” oppose saying “No” to “ $((54/4) - 12) + 1$ ”. Of course, calling “Yes” on a card “ N ” is correct only if “Yes” on “ $-(N)$ ” is incorrect. And calling “Yes” on “ N ” is correct only if “No” on “ $(N) + 1$ ” is incorrect. But this falls short of the opposition we would have in view, the opposition which exists between saying “Yes” on a card and saying “No” on that same card. If the student says both “Yes” and “No” to a certain card, she would appear—incoherently—to have taken a stand against herself. If she says “Yes” to both “ N ” and “ $-(N)$ ”, or “Yes” to “ N ” and “No” to “ $(N) + 1$ ” then she has made a mistake in her sums—but that is all.

This example parallels how matters are for Frege with asserting and denying propositional contents. To assert p is to oppose denying p . But the assertion of p is not *so* opposed to the assertion or denial of any *other* content. In particular, the subject does not, in asserting p , oppose asserting not- p . And the reason for this is the same as it is in the maths example. The subject saying Yes to “ N ” does not therein reject saying Yes to “ $-(N)$ ”, for even if the subject’s understanding of the card “ N ” includes a recognition that it has a negation “ $-(N)$ ”, still the operation by which “ $-(N)$ ” results from “ N ” has nothing in itself to do with the acts of “Yes” and “No”. Multiplying by minus 1 is not in itself a “Yes”-to-“No” operation, a “Yes”-to-“No” move between numbers. And so recognising “ N ” and “ $-(N)$ ” as connected in this way does not—not *in itself*—mean recognising one’s “Yes” to “ N ” as requiring a “No” to “ $-(N)$ ”, and so as ruling out a “Yes” to “ $-(N)$ ”. There is thus no opposition between

“Yes”-ing “N” and “Yes”-ing “-(N)” of the kind that exists between “Yes”-ing “N” and “No”-ing “N”, no incoherence beyond the fact that a mistake has been made. Similarly, Frege’s separation of judgment from content means that negation has nothing in itself to do with judgment. Negating is not in itself an assert-to-deny operation, an assert-to-deny move between contents. Recognising the content p as having a negation not- p does not therefore—not *in itself*—mean recognising one’s assertion of p as requiring a denial of not- p , and so as ruling out the assertion of not- p . There is therefore no opposition between asserting p and asserting not- p of the kind that exists between asserting p and denying p . Rather, the contents p and not- p are from the perspective of judgment simply different contents. The subject does not in judging the one, have any judgmental regard for the other.

The Fregean subject’s assertion that p is correct if and only if the assertion that not- p is incorrect, but that is all: her act is not a stand against the assertion that not- p . Insofar as the assertion that p it is not a stand against the assertion that not- p , however, no identification can be made between asserting that not- p and denying that p , for denying that p is precisely that act against which asserting that p is a stand. Having separated judgment from content Frege is thus committed—as Wittgenstein sees in TLP 4.063—to a separate act of rejection, a second, falsity-oriented way of judging, a second verb for the proposition.

3.4.

What then are Frege’s options? Well, assuming that he is not going to retract his separation of judgment from content first presented in *Begriffsschrift* and embodied in his ongoing use of his assertion sign, and assuming also that he is not going to embrace Wittgenstein’s attribution of a second verb, Frege must backtrack on the conception of judgment and denial that set the ball rolling. To judge that p is not to take a stand against an opposite, the denial that p . A judgment is not internally opposed to a corresponding act of denial. Rather, to judge a certain thought is simply to “think-true” the thought. Of course, we could introduce here a second act of denial, an act of “thinking-false” a thought. But in the absence of any idea of internally opposite stands, this introduction would seem superfluous, for the correctness condition of “thinking-false p ” would be the same as that of “thinking-true not- p ”.

Contents have negations, and so for any act of “thinking-false”, we already have an act of “thinking-true” correct in the same circumstance. There is, it may therefore seem, no theoretical value to be had in dealing with a separate act of denial.

And so indeed Frege argues in ‘Negation’ against a distinct act of denial not, as we have seen him do elsewhere, by appeal to an opposition at the level of content, but by suggesting that it is an unnecessary “nuisance” (CP 383) to distinguish two different ways of judging. For reasons of “economy of logical primitives” (CP 384), we should not theorise with an act of denying p distinct from that of asserting not- p .

4. Negation and Denial in the *Tractatus*

4.1.

Frege’s separation of judgment from content means that his acts of judging p and judging not- p are not strongly opposed, as we might say. The correctness of the one entails the incorrectness of the other, but the subject does not, in judging p , reject the judgment that not- p . If Frege wants an act strongly opposite to judging p , an act against which judging p is a stand, then this act of denying p will necessarily be distinct from judging not- p . What I want now to do is to outline how, with judgment internal to content, Wittgenstein endorses the position that the judgment that p is a stand against the judgment that not- p . As he puts it more generally: “Every proposition that contradicts another denies it” (TLP 5.241).

There are two moves to be made here. The first sets judgment to one side and locates in Wittgenstein a view deriving in part from Frege’s *Begriffsschrift* that the propositions “ p ” and “ $\sim p$ ” form an internal pair: it is basic to the sense of “ $\sim p$ ” that it contradicts “ p ”, and basic to the sense of “ p ” that it contradicts “ $\sim p$ ”. This ascription is relatively uncontroversial, and it will be briskly made. The second move reintroduces judgment, arguing that where judgment is internal to content, this internal connection between senses amounts to a strong opposition between judgments.

So let’s again begin with Frege. In *Begriffsschrift*, Frege explains his conditional assertion “ $p \supset q$ ” with reference to four possibilities: (1)

the contents p and q are both true, (2) p is true and q false, (3) p is false and q true, and (4) p and q are both false. The assertion " $p \supset q$ " is then defined as expressing "the judgment that the third of these possibilities does not obtain, but one of the other three does" (BS §5).⁵ Two sections later, Frege introduces his negation sign, explaining his formula " $\sim p$ " as "express[ing] the circumstance that the content does not obtain" (BS §7). In these introductions, Frege does not supply an element of meaning for his negation or implication sign. He does not specify an item contributed by " \supset " to the meaning of the proposition " $p \supset q$ ", something which would constitute an explanatory ground for the way in which this proposition's truth value depends upon those of " p " and " q ". Rather, implication is explained directly in terms of the truth and falsity of the conditional proposition for the different cases of truth and falsity of its antecedent and consequent. There is no element of sense introduced by " \sim "; rather, the negation sign is explained by saying that " $\sim p$ " is true if, and only if, " p " is false.⁶

Wittgenstein was impressed by this *Begriffsschrift* position, writing in 1913 that:

All that is essential about molecular functions is their T-F schema (i.e., the statement of the cases when they are true and the cases when they are false). (NB 98/100)

The endorsement of Frege continues into the *Tractatus*,⁷ where Wittgenstein presents a molecular proposition as a table specifying the proposition's truth or falsity for the various possible cases of truth and falsity of its atomic components (TLP 4.43–4.46). As in *Begriffsschrift*, the Tractarian logical particles do not introduce grounds within a sense for

⁵Here in *Begriffsschrift*, Frege gives the cases not in terms of what is true and false, but in terms of what is affirmed and denied. As Sullivan (2004, 676–77) however notes, it is uncertain what weight to put on this. Negation is introduced with reference only to what does and does not obtain (BS §7); and writing a year or two later Frege explains his conditional of *Begriffsschrift* §5 in terms of true (*richtig*) and false (*falsch*) (PW 11).

⁶It may have been noticed that I have not, in discussing Frege, considered at any point his later assimilation of propositions to names. By that assimilation, the proposition " $\sim p$ " has the form Fa , and " $p \supset q$ " the form aRb . There is a property of negation and a relation of implication. Wittgenstein dismisses this view, writing that "it is self-evident that \vee , \supset , etc. are not relations in the sense of right and left" (TLP 5.42). It does a disservice to Frege, however, to dwell on the later treatment of propositions as names. Little is learnt about Wittgenstein by considering that he opposes it.

⁷In section 4.431 Wittgenstein refers with approval to Frege's explanation of "the signs of his conceptual notation".

connections of truth value; rather, they represent those connections themselves.

Frege and Wittgenstein maintain furthermore that there *are* no such grounds. There is no ground within the sense of " $\sim p$ " for the connection in truth value it bears to " p ". There is no explanation *from* what " $\sim p$ " means *to* its being true in the case of " p "'s falsity and false in the case of " p "'s truth. So much is entailed not by Frege and Wittgenstein's treatment of the logical particles alone, but also by their notion of a proposition as the *expression* of its sense. A *Begriffsschrift* or Tractarian proposition doesn't "single out" its sense; it doesn't identify it in some particular way; rather, the proposition *gives* its sense.⁸ In particular, where the proposition " $\sim p$ " represents its sense as that which obtains if, and only if, " p " is false, this representation is not the indication of a content by the signalling of some property it (uniquely) bears. Rather, the representation tells what the sense is. There is, it follows, no sense of " $\sim p$ " separate from, and so explanatorily prior to, the truth-value connection made explicit in its representation as the *Begriffsschrift-Tractatus* negation of " p ".

One further comment is necessary to complete the sketch. It is consistent with what has just been said that a certain priority obtain of atomic content over non-atomic. First there is the elementary proposition " p "; subsequently, " $\sim p$ " is defined as that which is true if, and only if, p is false. First there are " p " and " q "; subsequently, " $p \supset q$ " is defined in the manner of Frege in terms of the truth and falsity of these prior atoms. There are I think deep, systematic reasons to reject such a constructive position.⁹ Sticking to brisk exegesis, however, we note simply that this constructive view of sense is certainly not Wittgenstein's. "All logical operations", Wittgenstein writes, "are already contained in the elementary proposition" (TLP 5.47). The elementary proposition is not something prior to negation, implication etc., and so something prior to the molecules in which it participates. Rather, the logical operations are already implicated in the elementary proposition. So in the *Notes to Moore* we find:

⁸The notion of expression is central within Frege and Wittgenstein's thought. For Wittgenstein see e.g., TLP 4.431; for Frege see in particular PW 12.

⁹One can sense straightaway a problem of uniformity: how are the defined molecules to be of the same kind—the kind *sense*, or *proposition*—as their prior atoms?

Logical functions all presuppose one another. Just as we can see $\sim p$ has no sense, if p has none; so we can also say p has none if $\sim p$ has none. The case is quite different with φa , and a ; since here a has a meaning independently of φa , though φa presupposes it. (NB 118)

φa presupposes a prior a , Wittgenstein writes. But it is not like this with negation: $\sim p$ is implicated in p just as much as p is in $\sim p$. An elementary proposition " p " is not constitutively prior to its non-elementary negation ' $\sim p$ '; rather, the elementary proposition presupposes its negation just as much as it is presupposed by it.

It is basic to the sense of " $\sim p$ " that it is true if and only if " p " is false, and likewise basic to the sense of " p " that it is true if and only if " $\sim p$ " is false. In this way, a content—any content—is fundamentally one half of a pair of opposites such that the truth of the one is the same as the falsity of the other. The thought that p is intrinsically the thought that p rather than not- p , it is the possibility that p rather than not.^{10,11}

4.2.

Considerably more work would be needed properly to explain Wittgenstein (and Frege's) notion of expression and to set out how truth-functionality is internal for Wittgenstein to the elementary proposition.

¹⁰So Wittgenstein writes in 1913: "we say 'A believes that p is true', and in order to make the direction of p even more explicit, we might say 'A believes that " p " is true and "not- p " is false'" (NB 97). The truth of " p " is the truth of " p " and the falsity of " $\sim p$ ".

¹¹Insofar as this position is also Frege's, it might be thought to provide him with a defence against the "objection" in Section 3.3 above that where judgment is external to content, the judgment that p is not a stand against the judgment that not- p . Judgement for Frege is the recognition of the truth of a thought. And the current ascription to Wittgenstein—and possibly also to Frege—is that the truth of a thought is one and the same as the falsity of its negation. A defence might then go as follows. To judge that p is to recognise the truth of the thought p . The truth of p is the falsity of not- p . So, to judge that p is to recognise the falsity of not- p : it is to reject not- p . There is a slide here, however, between recognising-true the thought that p and recognising that the thought that p is true. The thought that the thought p is true is indeed the thought that not- p is false. So if "recognise the truth of the thought p " means "recognise that the thought p is true", then recognising the truth of p will mean recognising the falsity of not- p . But recognising that not- p is false can't be cast here as rejecting not- p : it is simply recognising that it is false that not- p —i.e., that not-not- p . For a rejection of not- p we would need to start with "recognising-true the thought that p " and identify that with an act of "recognising-false the thought that not- p ". But no identification of the truth of p with the falsity of not- p will deliver that.

I set aside also the question whether in *Begriffsschrift* an atomic content "presupposes" its negation as it does for Wittgenstein, or whether, for Frege, atomic content is prior to molecular. Let's continue instead with a return to judgment.

In our maths game, the act of saying "Yes" to the card " N " is correct only if saying "Yes" to " $\sim(N)$ " is incorrect. This entailment is no mystery but is explained by the fact that if a number is positive then its negation is non-positive (indeed negative). Similarly, the Fregean judgment that p is correct only if the Fregean judgment that not- p is incorrect, and this is explained by the fact that if a content is true then its negation is false. In both cases, we explain the "weak opposition" between the two acts with reference to something prior, a connection at the level of content. In Fregean terms, the normative laws of judgment are grounded on the prior, descriptive laws of truth.¹² For Wittgenstein too, of course, judging that p is correct only if judging that not- p is incorrect. Here though, no explanation of this is possible by reference to a prior matter of content for there is no such matter: the relation of contradiction is not for Wittgenstein theoretically independent of the act of judgment. So, how else is the weak opposition to be understood?

To think of judgment as internal to content—to think of a content as something to be judged—is to think of a content as a way of judging. A content is not an independently given object of an act of judgment, but a manner of so acting. (Frege complains of "usual practice" that it fails to recognise "a distinction between judgment and content of possible judgment" (PW 11), but if Wittgenstein is right, Frege's distinction should not be recognised.) The judgeable content that p is a certain manner of judging, a certain type of judgment: it is the claim, or judgment, that p . To think of judgment as internal to content is moreover to think of judgment as internal to truth, and so it is to think of truth as fundamentally correctness in judgment. Where judgment is internal to content, and therewith also to truth, it is not only internal to judgment that it aims at truth, but internal to truth that it is correctness in judgment—and internal to falsity that it is incorrectness in judgment. Within this Tractarian frame, relations of entailment between contents are not prior to and explanatory of a judgment's inferential profile: they do not supply a prior ground for the correctness and incorrectness of

¹²See for example the start of "The Thought" (CP 351).

judging on the basis of a judgment that p . Rather, to say that if the content that p is true then so too is the content that q —to say that p entails q —is to say nothing other than that if the judgment that p is correct then so too is the judgment that q . Or again, to say that the content that p is true only if the content that not- p is false—to say that p contradicts not- p —is to say nothing less than if the judgment that p is right, then the judgment that not- p is wrong. And here, as we can see, Frege’s weak opposition between judging that p and judging that not- p becomes strong. An inconsistency between two contents is as such for Wittgenstein an opposition between two judgments. As we saw in the last section, however, a content’s inconsistency with its negation is internal to the content. The judgment that p is thus opposed *in and of itself* to the judgment that not- p .

It is basic to “ $\sim p$ ” that it is true just in case “ p ” is false. The truth of “ $\sim p$ ” is the falsity of “ p ”. Similarly, the truth of “ p ” is the falsity of “ $\sim p$ ”: the two propositions form in this way an internally opposite pair. So far, the early Frege may well agree. For the *Tractatus*, however, the truth of a content does not merely imply the correctness of its judgment: it *is* that correctness. So for the *Tractatus*, the correctness of judging p is one and the same as the incorrectness of judging not- p . As we have put it, the judgment of a content is strongly opposed to the judgment of its negation: these two are stands against each other. In this way, Wittgenstein maintains the identity Frege wants but cannot have between the denial of p —what is rejected in asserting p , that against which asserting p is a stand—and the assertion of not- p .

5. Judging and Endorsing

At the end of Section 3.2 judgment was cast as a species of choosing: to judge that p , we said, is to reject the denial that p . This depended on an identity between the correctness of judging p and the incorrectness of denying p , an identity belonging to the notion of denying p as entertained by Frege and Wittgenstein. It also depended, however, on a second thought that in judging that p the subject understands herself to be correct so to act. In judging that p , the subject takes herself to be correct, and so takes the judgment that p to be correct; the correctness of judging that p is the incorrectness of denying that p ; so, in judging

that p the subject takes the denial that p to be incorrect. This second thought has been termed the “self-consciousness of judgment” and has been discussed at length by various authors (see in particular Rödl 2007, 2017; and Kimhi 2018). Its application to Frege and Wittgenstein might, however, be considered anachronistic. This charge would I think be misguided (see Johnston 2021), but rather than take that point on, let’s close by seeing that we have the resources even without an appeal to self-consciousness to recognise a sense in which for Wittgenstein (though not for Frege) to judge is to endorse, and so to reject, and so to choose.

To think of a content as something to be judged is to think of a content as a way of judging. Compare here a dance. There are such things as dances: the Viennese Waltz, for example, and the Macarena. These dances may be the objects of various attitudes, or acts. One might revere the Viennese Waltz, say, and detest the Macarena. One might prefer the Salsa to the Rumba. The possibility a dance bears of being detested or preferred is however external to the dance itself. The Macarena is a dance, and as such may be loved, but this liability is not constitutive of the dance’s basic nature. One does not recognise what the Macarena fundamentally is by understanding it as something one might love. What is not so external to the Macarena, on the other hand, is that it may be *danced*. A dance—any dance—is fundamentally something *to be danced*. Elaborating this point, we might speak of *manners of dancing*. Where the Macarena is danced, one’s dancing does not have an act-object structure. There is not within one’s dancing of the Macarena an entity danced discernible separately from one’s dancing of it. Rather, the dance one dances is the form one’s dancing takes, the manner of one’s dancing. To dance a certain dance is not to act with a certain object, as one does when one adores the Waltz or eats a cake; rather, it is to dance in a certain manner. In dancing, one *exemplifies* the dance one there dances. And as a dance is something to be danced, so a Tractarian sense is something to be judged. Where for Frege, asserting a content is like eating a cake—the content is a prior object for the act—for Wittgenstein asserting (saying, representing) a content is akin to dancing a dance: the content is not a prior object for the act but a manner of asserting exemplified by the act.

Some care is needed here for by a manner of dancing we don’t mean that a dance is a mere similarity relation across episodes of dancing

(identified perhaps by independently given bodily movements). Rather, a dance is a *routine* for dancing. This is made plain by the observation that people very often dance without there being any dance they are there dancing. (Where dancers in a ballroom typically dance dances, dancers in a nightclub typically do not.) To dance a certain dance is not merely to dance in a certain manner in the sense of making certain movements in a certain order in (approximate) time to music. Rather, to dance a certain dance is to *follow* that dance. A dance—the Macarena or the Waltz—is a way of dancing such that so dancing means *following* the dance one there dances. To exemplify the dance is to follow what one there exemplifies, and to follow the dance is to exemplify what one there follows. As one might say, a dance is the following of itself. A similar structure is to be found with Tractarian sense. A sense is not a constitutively independent object of the act of judgment. Rather, a sense is something for judgment, something to be judged, a way of judging exemplified by instances of its judgment. We should not be misled here, however, into supposing that a sense is a mere similarity relation across token judgments—and then worry how to identify such relations. The position is rather that to judge a certain sense is to judge a certain manner of judging: it is to judge a certain judgment. But what does this mean, to judge a judgment? Well, what does it mean, to dance a certain dance, a certain manner of dancing? It means, I have said, to follow that dance. And to judge a certain judgment means to *endorse* that judgment. Where a content is something to be judged, to judge a content is to judge a judgment: that is, it is to endorse a judgment.

The Tractarian view of a content as something to be asserted might be put as the view that a content is a *claim*: a claim is something to be made, and the making of a claim is an assertion. And where a content is a claim, to assert the content is to *endorse* the claim. It is, that is to say, to endorse a certain act of claiming: precisely that act of claiming one thereby exemplifies.

This result replicates in a non-Fregean context Frege's position that "predicating [truth] is always included in predicating anything whatever" (PW 129). Or again, it constitutes an understanding of the identity between the propositions "*p*" and "It is true that *p*".¹³ What is more,

¹³Some philosophers endorsing an identity of content between "*p*" and "It is true that *p*" take the identity to reflect that truth is metaphysically insubstantial, that the word

it delivers a manner independent of judgment's self-consciousness in which to judge is to choose. To judge that *p*, we are saying, is to endorse the judgment that *p*: that is, it is to judge the judgment that *p* to be correct. For Wittgenstein, however, the correctness of the judgment that *p*—the truth of the content *p*—is one and the same as the incorrectness of the judgment that not-*p*—the falsity of the content not-*p*. So, to judge that *p* is to judge the judgment that not-*p* to be incorrect. It is to reject the judgment that not-*p*.

References

- Dummett, Michael. 1959. "Truth." *Proceedings of the Aristotelian Society* 59 (1): 141–62.
- Frege, Gottlob. 1972. *Conceptual Notation and Related Articles (BS)*. Edited by T. W. Bynum. Oxford: Clarendon Press.
- . 1984. *Collected Papers on Mathematics, Logic, and Philosophy (CP)*. Edited by Brian McGuinness. Translated by Max Black and V. H. Dudman. Oxford: Blackwell.
- . 1991. *Posthumous Writings (PW)*. Edited by Hans Hermes, Friedrich Kambartel, and Friedrich Kaulback. Translated by Peter Long and Roger White. Oxford: Wiley.
- . 2013. *Basic Laws of Arithmetic (GG)*. Edited by Philip Ebert and Marcus Rossberg. Oxford: Oxford University Press.
- Johnston, Colin. 2021. "Frege, the Self-Consciousness of Judgement, and the Indefinability of Truth." *British Journal for the History of Philosophy* 29 (6): 1124–43.
- Kimhi, Irad. 2018. *Thinking and Being*. Cambridge, MA: Harvard University Press.
- Proops, Ian. 1997. "The Early Wittgenstein on Logical Assertion." *Philosophical Topics* 25 (2): 121–44.
- Rödl, Sebastian. 2007. *Self-Consciousness*. Cambridge, MA: Harvard University Press.
- . 2017. "Self-Consciousness, Negation, and Disagreement." *Proceedings of the Aristotelian Society* 117 (3): 215–30.
- Russell, Bertrand. 1992. *The Principles of Mathematics*. London: Routledge.
- "true" is merely a linguistic device of generalization or disquotatation, or something like that. Such a perspective couldn't be further from the *Tractatus*, or indeed for that matter from Frege.

Sullivan, Peter. 2004. "Frege's Logic." In *Handbook of the History and Philosophy of Logic Vol. 3*, edited by D. Gabbay and J. Woods, 659–750. Amsterdam: North Holland.

Wittgenstein, Ludwig. 1922. *Tractatus Logico-Philosophicus (TLP)*. Translated by C. K. Ogden. London: Routledge and Kegan Paul.

———. 1979. *Notebooks 1914–1916 (NB)*. Oxford: Blackwell.