



An Interpretation of the Gray's Elegy Argument

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ABSTRACT

In this essay, I first argue that the Gray's Elegy Argument—the dense passage in Bertrand Russell's 'On Denoting'—can be interpreted as a single, coherent argument against the notion that a definite description corresponds to what I call a multifaceted object—an object having multiple facets or sides. I then look into some manuscripts Russell wrote in 1904 and in 1905. I show that he had envisaged the notion of a multifaceted object and used it for two different purposes before he discovered various objections to it, which he turned into the Gray's Elegy Argument.

1. Introduction

My aim in this paper is to offer an interpretation of the *Gray's Elegy Argument*—the dense passage in Bertrand Russell's 'On Denoting'. I will develop a line of thought that authors such as Noonan, Makin, Levine, Salmon and Simons have presented in their interpretations of the passage (Noonan 1996; Makin 2000; Levine 2004, 2005; Salmon 2005; Simons 2005). They attribute to him the idea that if we speak about the *denotation* of a definite description—the object that the description singles out—*whenever* we use the description, then we cannot speak about the *meaning* of the description in virtue of which the object is singled out. With this idea, we can indeed make sense of much of the passage. Nevertheless, to make sense of the whole of it, we need something more, for the idea does not explain why Russell in the passage concludes that the meaning of a definite description is *identical* to the denotation. To explicate how he draws this conclusion, I attribute to him the idea that a definite description stands for an object of a peculiar

kind that contributes different *facets* or *sides* to complexes in which it occurs, depending on different ways it occurs there. I will argue that by ascribing this idea to him, we can interpret the whole passage as a single, coherent argument. The reader may find this idea far-fetched. So I will also attempt to show that he indeed endorsed it, albeit tentatively, in his attempts to resolve the set-theoretic paradox (among others) and to give an account of the nature of complex objects. It will also be seen that when he discovered some of the arguments that constitute the Gray's Elegy Argument, he was examining the idea in question.

In my view, the authors mentioned above have indicated a plausible way of understanding what Russell calls the 'difficulty in speaking of meanings'. In 'On Denoting', he illustrates this difficulty using such descriptions as 'the first line of Gray's Elegy'. Suppose this expression is correlated with a *denotation* and a *meaning*. Suppose further that whenever we use the expression, we speak about the denotation. These two suppositions constitute what may be called the *notion of denoting*. As those authors argue, the notion makes it impossible to speak about the meaning of a definite description in any straightforward manner. One may try to do so using such expressions as 'the meaning of 'the first line of Gray's Elegy'', referring to the description.¹ But, they point out, Russell is not content with any linguistic way of specifying the meaning. One may then try to use such an expression as 'the meaning of the first line of Gray's Elegy', using the description. But this whole expression only enables us to speak about the meaning of the first line of the poem—not the meaning of the description but the meaning of the denotation of it—precisely as the expression 'the square of the even prime' only allows us to speak about the square of the denotation of 'the even prime', that is, the square of 2. If we speak about the denotation of a given description whenever we use it, then we are left with no straightforward way to speak about the meaning of the description.

This difficulty is only a part of the Gray's Elegy Argument, however. We are yet to see how it leads to 'an inextricable tangle', which Russell indicates at the end of the passage. He apparently concludes that if we endorse the notion of denoting, we need to think the two propositions "Scott was the author of *Waverley*" and "Scott was Scott" are *identical* (as

¹In what follows, I will use single quotation marks either to quote or refer to an expression (even if they are nested), reserving double quotation marks for technical purposes.

complex objects), while the fact that George IV wanted to know if the former proposition, but not the latter, was true requires that they should be *different*. But the notion of denoting does not explain why we have to identify those two propositions with each other. On the contrary, if the relation of denoting is a binary relation holding between two separate entities, the two propositions must be different from each other. For the former contains Sir Walter Scott—the denotation of the description—as a constituent, while the latter, the meaning of the description. It is then natural to regard, as Makin does, the ‘inextricable tangle’ as a charge ‘independent’ of the rest of the passage (Makin 2000, 32). To be fair, those who read the passage in different ways are not in a better position. Many of them see the ‘inextricable tangle’ as being an ‘additional argument’ (Turnau 1991, 65) or even as being ‘either unnecessary or fundamentally misconceived’ (Rosenkrantz 2017, 11). It has thus been customary in the literature to interpret the passage as composed of two separate arguments, not as a single argument, or to focus on the ‘difficulty in speaking of meanings’, dismissing the ‘inextricable tangle’ as being confused if not incoherent.²

Yet we can understand the whole argument as a single, coherent argument by attributing to Russell the idea that a definite description corresponds to an object having the denotation of the description as a *side*. I propose using the term ‘multifaceted object’ to speak of an object that has different *facets* or *sides*. The idea is that when a multifaceted object occurs in a complex, it contributes different sides as a constituent to the complex depending on the different ways in which it occurs there. To endorse the notion of a multifaceted object is to draw a distinction between *occurring in* a complex and *being a constituent of* it—when a multifaceted object occurs in a complex, only one of its sides becomes a constituent of the complex.³ Those objects which occur in a proposition are concerned with the identity of the proposition (taken as a complex object), while the constituents of the proposition are concerned with the truth of it. In this view, when we say something about an entity, that is, when we assert a proposition concerning the entity, whether it is true or

²Wahl and Levine interpret the whole passage as offering a single argument (Wahl 1993; Levine 2004, 2005). I compare their interpretations with mine in Section 2.

³This is not Russell’s own terminology though it is largely in accordance with his use of such expressions as ‘occurrence’ and ‘constituent’ in ‘On Denoting’ and the relevant manuscripts.

not depends on the constituents of the proposition including the entity itself, while what we assert—the identity of the proposition—depends on those objects which occur in it.

Russell first considers a natural way of applying this idea to definite descriptions: a definite description corresponds to a multifaceted object having meaning and denotation as its two sides. When we use a definite description plainly, the corresponding multifaceted object makes its denotation a constituent of a proposition, and we can thereby speak about the denotation. When we use a definite description with inverted commas, the corresponding object makes its meaning a constituent of a proposition, and then we can speak about the meaning. But he repudiates this approach because of the ‘difficulty in speaking of meanings’. If we are to use such expressions as ‘the meaning of ...’ and ‘the denotation of ...’ so as to speak generally about the meaning and denotation of any multifaceted object, we need to speak about multifaceted objects themselves. Otherwise, we end up speaking about the meaning and denotation of the denotation of a given description. Nevertheless, the idea that a definite description corresponds to a multifaceted object does not explain how we can talk about the object itself. One may try to overcome this difficulty by claiming that we can use inverted commas to speak about multifaceted objects. This is indeed how Russell is led to the idea that the meaning of a definite description *is* a multifaceted object and it has the denotation as its sole side. Yet, how can we make sense of the notion of a multifaceted object having only one facet? This notion may not involve contradiction, but it does lead to the ‘inextricable tangle’. In every proposition in which the meaning occurs, not the meaning but the denotation is a constituent of the proposition. Thus, the distinction between a multifaceted object and its sole side, or the distinction between *occurring in* and *being a constituent of*, becomes virtually non-existent. Then the two propositions “Scott was the author of *Waverley*” and “Scott was Scott” are *identical* because they have the same constituents and they can no longer be differentiated on the grounds that a multifaceted object occurs in the former but not in the latter.

In what follows, I will first expand on this interpretation in Section 2 before I turn to some of the manuscripts Russell wrote in 1904 and in 1905 to offer textual evidence for my attribution of the notion of a multifaceted object to Russell in Section 3. It is true that he does not employ any

specific term for the notion of a multifaceted object in 'On Denoting'. But, as I will argue in Section 2, some sentences in 'On Denoting' may indeed be read as claiming that this notion is required to 'preserve' the connection between meaning and denotation. In addition, as I will argue in Section 3, there are three points we can make for the attribution of the notion to him. First, the *zigzag theory*—his attempt to resolve the set-theoretic paradox by distinguishing *legitimate* propositional functions from *illegitimate* ones—led him to interpret a second-order variable as an object having two different sides. Second, in the 1905 manuscript named 'On Fundamentals', he develops the view of complexes as having two sides so that he can account for what may be called the *dualistic* nature of complexes—a complex is *one* object composed of *many* objects. Third, in 'On Fundamentals', he finds various arguments against the idea that definite descriptions correspond to 'one complex with the two aspects of meaning and denotation'. Those arguments appear in the part that contains some passages re-used almost verbatim in 'On Denoting'. He turned those arguments into the Gray's Elegy Argument. He was thus examining the notion of a multifaceted object when he discovered the argument, and this suggests that the argument was indeed directed against the idea that definite descriptions correspond to multifaceted objects. In what follows I refer to 'On Denoting' and 'On Fundamentals' by 'OD' and 'OF' respectively.

2. The Interpretation

In this section, I shall present my interpretation of the Gray's Elegy Argument (or for short the GEA).⁴ The passage consists of eight paragraphs, to which it has been customary to refer as '(A)', '(B)', ..., and '(H)' (Blackburn and Code 1978). As I cannot quote the whole passage, I will have to assume the reader's access to a copy of OD.⁵

⁴By 'GEA', I refer either to the passage itself or to the argument presented there.

⁵All the references to OD will be made to the fourth volume of *Collected Papers of Bertrand Russell* (Russell 1994), where inverted commas are used in the same way as in the original article published in *Mind*. All the italics in quotations are emphasised in the source unless otherwise stated.

2.1. The target of the argument

In this section, I comment on paragraphs (A), (B) and (C). I argue that the GEA is not simply targeted at the notion of denoting but in conjunction with the view that the relation of denoting holds *within* a multifaceted object.

(A) Russell begins the GEA with the following remark: ‘The relation of the meaning to the denotation involves certain rather curious difficulties, which seem in themselves sufficient to prove that *the theory* which leads to such difficulties must be wrong’ (Russell 1994, 421, emphasis added). The target of the GEA is thus what he calls ‘the theory’ here. But it has been a matter of controversy what the phrase ‘the theory’ refers to.⁶ The controversy is certainly due to the fact that Russell initially attributes the distinction between *meaning* and *denotation* to Frege, while Russell does not seem to discuss Frege’s view in the GEA.

My account of ‘the theory’ is twofold. First, I think the phrase refers to the notion of denoting—the idea that each definite description is correlated with two semantic items, *meaning* and *denotation*, in such a way that when a definite description is simply used in a sentence, the sentence is *about its denotation* rather than its *meaning*. Second, as I will argue below, Russell assumes that if one is to endorse the notion without making the relation of denoting mysterious, one needs to suppose that the relation holds not between two separate entities but *within* a multifaceted object—a single object that has sides. He seems to assume that a definite description corresponds to a multifaceted object that has as a side the denotation of the description. As for the meaning, as we will see shortly, he does not sharply distinguish between two distinct views—he first considers the meaning as a side of the multifaceted object but then abandons this view in favour of the idea that the meaning is the multifaceted object itself. The relation of denoting is thus supposed to hold *either* between two sides of a multifaceted object *or* between a side and a multifaceted object itself. Either way, it holds *within* a multifaceted

⁶Some authors think that the GEA is targeted at Frege’s theory of *Sinn* and *Bedeutung* (Butler 1954; Searle 1958; Hochberg 1976; Blackburn and Code 1978; Rosenkrantz 2017). Others interpret ‘the theory’ as Russell’s own theory of *denoting concepts*, developed in *The Principles of Mathematics*, or a more elaborate theory based on it (Geach 1959; Cassin 1971; Hylton 1990; Turnau 1991; Wahl 1993; Pakaluk 1993; Kremer 1994; Noonan 1996; Landini 1998; Demopolous 1999). Some others hold that Russell intends to repudiate a broader notion embracing both theories (Salmon 2005; Simons 2005).

object. Thus, in my view, though the GEA is an objection to the notion of denoting, it is assumed that if a definite description corresponds to an object or what he calls a 'denoting complex', the object must be a multifaceted object.

This twofold account fits the way Russell introduces Frege's distinction between *Sinn* and *Bedeutung*. According to Russell, Frege 'distinguishes, in a denoting phrase, two elements, which we may call the *meaning* and the *denotation*' (Russell 1994, 418). But, strictly speaking, we cannot find the two elements *in* a definite description; we may do so in an object having meaning and denotation as its sides. Russell also ascribes to Frege the idea that 'denoting phrases, in general, have the two *sides* of meaning and denotation' (Russell 1994, 420, emphasis added). But, again, such expressions do not have any *sides*, while denoting complexes do if they are multifaceted objects. Russell throughout the GEA seems to use the expression 'denoting phrase' to talk about denoting complexes, to which such phrases correspond.⁷ Russell thus attributes to Frege the idea that the relation of denoting holds within a multifaceted object, though Frege arguably did not even envisage such an idea. But this should come as no surprise if Russell assumes that one must accept the idea to endorse the notion of denoting, under which he thinks the Fregean distinction between *Sinn* and *Bedeutung* falls.

The twofold account implies that the target of the GEA is not the theory of *denoting concepts* which Russell presents in *The Principles of Mathematics* (Russell 1903) (henceforth, *PoM*). It is in this book that he introduces the idea of denoting: 'A concept *denotes* when, if it occurs in a proposition, the proposition is not *about* the concept, but about a term connected in a certain peculiar way with the concept' (Russell 1903, 53). But he does not regard a denoting concept as a multifaceted object there. He says denoting concepts denote 'something quite different' from

⁷Levine argues, convincingly in my view, that the idea that a denoting complex is merely an expression is what Russell advocates *after* he concludes the GEA (Levine 2004, 269–70). Levine also points out that 'Russell often uses words for linguistic items...when what he intends to discuss is the non-linguistic correlates of those linguistic items' (Levine 2004, 269; see also Levine 2005, 73). Russell's indifference to the distinction between symbols and their non-linguistic correlates arguably stemmed from the fact that he was concerned exclusively with the latter (Hochberg 1976, 23–24; Hylton 1990, 171; Turnau 1991, 52–54; Makin 2000, 11–12). Hereafter, I replace the term 'denoting phrase' with 'denoting complex' in quotations from the GEA to indicate how naturally it can be interpreted as an argument against the view that the relation of denoting holds within a complex object.

themselves (Russell 1903, 47).⁸ It follows that the theory of denoting concepts as it appears in *PoM* is not the target of the GEA, precisely as Frege's theory of *Sinn* and *Bedeutung* itself is not so.

(B) In this paragraph, Russell introduces an account of denoting complexes. In this account, a denoting complex is a multifaceted object, contributing different sides to a larger complex depending on how it occurs in the complex. He remarks:

When we wish to speak about the *meaning* of a denoting [complex], as opposed to its *denotation*, the natural mode of doing so is by inverted commas. Thus we say:

The centre of mass of the Solar System is a point, not a denoting complex;
 "The centre of mass of the Solar System" is a denoting complex, not a point. (Russell 1994, 421)

I interpret this passage as follows: When we simply use a definite description in a sentence, the sentence expresses a proposition whose constituents include the denotation of the corresponding denoting complex. The plain use of a definite description thus signifies what may be called a *denotative* occurrence of a denoting complex—the mode of occurrence in which a denoting complex makes its denotation a constituent of a larger complex. On the other hand, when we place inverted commas before and after a definite description, they signify what may be called a *connotative* occurrence of the corresponding denoting complex.⁹ When occurring in this manner, the denoting complex contributes, as a constituent, the meaning of the complex to a larger complex. The

⁸Cohen attributes to Russell in *PoM* the idea that when a concept denotes an object, not only the concept but the denoted object (e.g., an 'assemblage' of infinitely many entities) are among the constituents of a proposition (Cohen 2022). It is beyond the scope of this essay to discuss whether this idea is equivalent to the notion of a multifaceted object. But it may be well to note that when Russell employs the notion of a multifaceted object in OF and in OD, he does not seem to be committed to the rather contentious claim that a proposition may contain infinitely many constituents (in the sense I have been using) (see Cohen 2022, fn.19). At least, as we will see in Section 3, the way Russell invokes the notion of a multifaceted object in OF does not commit him to the contentious claim as he can maintain that the denotation is always a single entity. See also footnote 34.

⁹Levine introduces the terms 'denotatively' and 'non-denotatively' for these modes of occurrences. I replace the latter with 'connotatively' because Russell considers other non-denotative modes of occurrences in OF. The idea of non-denoting occurrence can be traced back to *PoM* (see Landini 1998, 59; Makin 2000, 26; Levine 2004, 63).

connotative occurrence of a denoting complex thus enables us to speak about the meaning. I shall use the term 'double-faceted object' to speak of a multifaceted object having meaning and denotation as its two sides and contributing either of these sides to other complexes, depending on whether it occurs *denotatively* or *connotatively*.

As we will see below, Russell sets forth his argument by taking on this view—the notion of a double-faceted object. But, before we proceed, it may be useful to note three things here.

First, in explaining how inverted commas are meant to work, Russell does not sharply distinguish the notion of a double-faceted object from an alternative one. As we have seen, he introduces inverted commas 'to speak about the *meaning* of a denoting [complex]'. But when he illustrates how they are meant to work, he says '“The centre of mass of the Solar System” is a *denoting complex*' (emphasis added). In other words, though he proposes to use inverted commas to speak about a side of a multifaceted object, when actually using them, he speaks about the multifaceted object itself. It seems as though he identifies a denoting complex with its meaning. Such a complex may be called a 'single-faceted object' because it is considered to have only one side—its denotation. He does not sharply distinguish this notion from the notion of a double-faceted object. As we will see below, he introduces the former as 'a right phrase', which suggests he thinks of the difference between these two notions as a matter of phraseology.¹⁰

Second, Russell uses the capital letter 'C' in two different ways. I owe Levine the idea that Russell employs the letter to speak about an arbitrary denoting complex, not about the denotation of an arbitrary complex (Levine 2004, 268–71; 2005, 73). When used thus, the letter 'C' is not interchangeable with any particular definite description. Russell does not apply the notion of denoting to 'C' but only to actual definite descriptions. Levine maintains that Russell uses the letter consistently to speak about complexes, but I think there are exceptions.¹¹ One is found

¹⁰Another possible reason for the ambiguity is that in OF, the passage quoted above comes after his introduction of the 'right phrase' (see footnote 47).

¹¹Turnau and Simons among others point to those two ways of using the letter (Turnau 1991, 63; Simons 2005, 126–27). Salmon also indicates them though he thinks that Russell also employs the letter to speak about an arbitrary definite description (Salmon 2005, 1084). This idea is common among other authors, but we can make sense of the GEA without attributing to Russell the use of 'C' as a metalinguistic variable (see footnote 7).

in this paragraph, where Russell says ‘taking any denoting [complex], say, C , we wish to consider the relation between C and “ C ”’ (Russell 1994, 421). It seems to me that he uses the letter first to speak of a denoting complex but then of its denotation, for what he is concerned with is the relation between the denotation and the meaning (Russell 1994, 421). The other exceptions appear in paragraphs (D) and (E), where he employs ‘ C ’ as a scheme for definite descriptions. As we will see below, when he uses the expressions ‘the meaning of C ’ and ‘the denotation of C ’, he takes ‘ C ’ to be interchangeable with actual definite descriptions. In what follows I shall use the letter ‘ C ’ as an expression for an arbitrary denoting complex unless otherwise specified.

Third, Russell’s use of the symbol “ C ” is even more complicated because its role changes as his argument unfolds. For one thing, as we have seen, he uses inverted commas either to speak of a denoting complex or to speak of a meaning. This ambiguous use of the symbol can also be found in paragraphs (C) and (D). Moreover, in paragraphs (F) and (G), he uses the symbol to speak of a separate object that denotes the given complex. I discuss these different uses of “ C ” in greater detail below.

(C) In this paragraph, Russell undertakes three tasks. First, he restates the view of a denoting complex as a multifaceted object: ‘when C occurs it is the *denotation* that we are speaking about; but when “ C ” occurs, it is the *meaning*’, leaving it unclear whether the meaning is identified with the complex itself (Russell 1994, 421).

Second, Russell introduces the relation of denoting explicitly. He remarks, ‘the relation of meaning and denotation is not merely linguistic through the phrase: there must be a logical relation involved, which we express by saying that the meaning denotes the denotation’ (Russell 1994, 421). As some authors have pointed out, he is rejecting the merely linguistic account of the relation of denoting as the one holding between meaning and denotation of a given *expression*.¹²

The third task Russell undertakes in this paragraph is to outline two difficulties with the notion of denoting:

¹²See Russell (1903, 47). For some existing accounts of Russell’s rejection of such a linguistic bypass, see Blackburn and Code (1978, 70–72), Turnau (1991, 61–62), Pakaluk (1993, 44), Makin (2000, 18–19), Levine (2005, 78), and Simons (2005, 129).

...the difficulty which confronts us is that we cannot succeed in *both* preserving the connection of meaning and denotation *and* preventing them from being one and the same; also that the meaning cannot be got at except by means of denoting phrases. (Russell 1994, 421)

The first difficulty can be paraphrased as follows: *if* 'the connection of meaning and denotation' is preserved, *then* they become 'one and the same'. What does he think it takes to *preserve the connection* between meaning and denotation? The idea that the relation of denoting holds within a multifaceted object. Indeed, as we will see below, if we thus attribute to him the notion of multifaceted objects, we can make sense of the GEA as a whole. The second difficulty is: 'the meaning cannot be got at except by means of denoting [complexes].' This is itself a problem, but it also constitutes a step in his argument for the first difficulty, or so I will argue in the following section.

2.2. The 'difficulty in speaking of meanings'

Russell explains what he calls the 'difficulty in speaking of meanings' in paragraphs (D), (E) and (F). In this section, I will explain how we can make sense of the difficulty by attributing to him the idea that the relation of denoting must hold within a multifaceted object.

(D) In this paragraph, Russell begins his argument by assuming that the connection between meaning and denotation is preserved. He then illustrates the difficulty using some examples.

As we have seen, Russell intends to show that if the connection between meaning and denotation is preserved, then they end up being 'one and the same'. It is then natural for him to begin his argument by assuming that the connection between meaning and denotation is preserved (with a view to drawing the conclusion that they are 'one and the same'). I think this is precisely what he does when he remarks, 'The denoting [complex] C was to have both meaning and denotation' (Russell 1994, 421). He says 'was' because he already introduced the view in paragraph (B). He thus assumes that the connection between meaning and denotation is preserved, by taking denoting complexes to be multifaceted objects.

Russell proceeds to discuss the ‘difficulty in speaking of meanings’, which can be stated as follows.¹³ Suppose the description ‘the first line of Gray’s *Elegy*’ corresponds to a certain denoting complex *C*. We cannot speak about the meaning of *C* by combining the expression ‘the meaning of ...’ with the description, which corresponds to *C*. If we use the expression ‘the meaning of the first line of Gray’s *Elegy*’, we speak about the meaning of the denotation of *C*, not about the meaning of *C*. Whenever we use the description ‘the first line of Gray’s *Elegy*’ without inverted commas, the corresponding complex *C* occurs *denotatively* and makes its denotation a constituent of a larger complex in which the complex occurs. Similarly, consider the description ‘the denoting complex which we are dealing with’.¹⁴ As we are dealing with the complex *C*, this description corresponds to a denoting complex *D* which has *C* as its denotation. In other words, *C* is the denotation of *D*. But we cannot speak of *C* by combining the expression ‘the denotation of ...’ with the description ‘the denoting complex which we are dealing with’, which corresponds to *D*. Using the expression ‘the denotation of the denoting complex we are dealing with’, we cannot speak about *C* but about the denotation of *C*, namely, ‘what is denoted by the denotation we want’ (Russell 1994, 422).¹⁵

Russell’s own exposition of the difficulty is rendered unclear partly by his twofold use of the single letter ‘*C*’ in paragraph (D). We noted earlier that he primarily uses, as we have been doing, ‘*C*’ to speak about a denoting complex itself, not about the denotation of it. But in this paragraph he also employs it schematically so it can be replaced by particular descriptions such as ‘the first line of Gray’s *Elegy*’ and ‘the denoting complex we are dealing with’. This may be somewhat

¹³To make sense of the following examples in terms of the notion of a multifaceted object, we only need to assume that if a denoting complex occurs in a larger denoting complex denotatively (or connotatively), the denotation (or meaning) of the former is a constituent of the meaning of the latter. In OF, however, Russell is confronted with a wider variety of cases (see Section 3.2).

¹⁴I am using this description instead of the original expression ‘the denoting complex occurring in the second of the above instances’.

¹⁵Noonan, Makin, Levine, Salmon and Simons also understand the ‘difficulty in speaking of meanings’ in this way. But those authors go further to urge that the difficulty leads to an explanatory circle: we use the expression ‘the meaning of “*C*”’ to explain what “*C*” is (Noonan 1996, 93–95; Makin 2000, 33–37; Levine 2004, 275; Salmon 2005, 1101, 1104; Simons 2005, 128). Yet, I do not think there really is a circularity here (see footnote 19).

excusable as he can thereby state the difficulty in a general form: we cannot speak about the meaning and denotation of a given complex using such expressions as 'the meaning of *C*' and 'the denotation of *C*'.

Another reason for Russell's unclear exposition lies in his ambiguous use of inverted commas. He is considering the view that we can speak of the meaning of a denoting complex by using inverted commas. But he also employs them when he wants to speak about a denoting complex itself. For instance, he remarks:

"The meaning of the first line of Gray's Elegy" is the same as "The meaning of "The curfew tolls the knell of parting day""... (Russell 1994, 421)¹⁶

Here he employs inverted commas to speak about denoting complexes so he can assert the identity between them.¹⁷ The two different uses of inverted commas can also be found in the following sentence:

Thus in order to get the meaning we want, we must not [use] "the meaning of *C*", but... "the meaning of "*C*"" , which is the same as "*C*" by itself. (Russell 1994, 421)¹⁸

He uses the expression "the meaning of *C*" to speak about a denoting complex that has as its denotation the meaning of the denotation of a target complex. He uses the expression "the meaning of "*C*"" to speak about another denoting complex that has as its denotation the meaning of the target complex. Thus, in those expressions, inverted commas are used to speak about denoting complexes. On the other hand, when he says "the meaning of "*C*"" ... is the same as "*C*", he seems to claim that what the denoting complex "the meaning of "*C*"" has as its denotation (rather than the complex itself) is identical to the meaning of the target

¹⁶I have replaced single quotation marks with double quotation marks (inverted commas). Russell uses the former to avoid nesting the latter in OD.

¹⁷He understands the sentence 'The curfew tolls ...' as corresponding to a complex having meaning and denotation. He thus understands complexes in some manuscripts (see Section 3.2).

¹⁸I have made two modifications to clarify how I understand the sentence. First, I have replaced the expression 'speak of' with 'use' to avoid ambiguity. In this paragraph Russell employs the phrase 'speak of' both to indeed speak about something and to use an object in the sense we employ the expression corresponding to the object so that we can assert propositions in which the object occurs. Second, I have replaced single quotation marks with double quotation marks (inverted commas) (see footnote 16).

complex. If so, he uses the symbol “‘C’” to speak about the meaning of the target complex here.¹⁹

(E) In this paragraph, Russell restates the ‘difficulty in speaking of the meaning of a denoting complex’ before he abandons the notion of a double-faceted object in favour of the notion of a single-faceted object (Russell 1994, 421).

Russell does not state why he gives up the view of a denoting complex as a double-faceted object. But the view is indeed undermined by the ‘difficulty in speaking of meanings’. As we have seen, he argues that we cannot speak of the meaning and denotation of a denoting complex by combining the functional expressions ‘the meaning of ...’ and ‘the denotation of ...’ with a given definite description which corresponds to the complex. The plain use of the definite description never allows us to speak about the corresponding denoting complex. Therefore, we need some way of speaking about denoting complexes themselves in order to use those functional expressions and therewith state the relation between meanings and denotations in general terms. But the notion of a double-faceted object gives us no explanation as to how we can speak about denoting complexes.²⁰ It only provides us with the ways in which we can speak about the meaning and denotation of a given denoting complex, but not about the complex itself.

In light of this problem, Russell moves on to consider a natural account of how we can speak about denoting complexes—so natural that he himself has already been adopting occasionally. The idea is that we can use inverted commas to speak about denoting complexes. So far, his use of inverted commas has been ambiguous, as he also employs them as a way to speak about meanings. Yet, if he gives up the notion of a double-faceted object, he is entitled to maintain that denoting complexes are identical to meanings so we can use inverted commas to speak about

¹⁹Hence, as long as a denoting complex is distinguished from its meaning, there is *no* circularity here (see footnote 15).

²⁰Thus, the difficulty Russell points to in paragraph (D) boils down to the difficulty in speaking about denoting complexes or multifaceted objects themselves. In my view, he calls it the ‘difficulty in speaking of meanings’ for two reasons. First, the difficulty appears when we consider how we can generally speak of the meaning of a denoting complex. (Note that the notion of denoting provides us with a general way of speaking of the denotation of a denoting complex.) Second, he maintains, as we will see below, that if we adopt the ‘right phrase’, denoting complexes are identical with what he calls meanings.

denoting complexes/meanings unambiguously. This is precisely what he does in adopting the 'right phrase':

... when we distinguish meaning and denotation, we must be dealing with the meaning: the meaning has denotation and is a complex, and there is not something other than the meaning, which can be called the complex, and be said to *have* both meaning and denotation. The right phrase, on the view in question, is that some meanings have denotations. (Russell 1994, 421)

The view he rejects here is the view of a denoting complex as a double-faceted object because on this view a denoting complex is precisely 'something other than the meaning, which can be called the complex, and is said to *have* both meaning and denotation'. He then adopts the 'right phrase': 'some meanings have denotations'. If he uses the word 'have' here in the sense that a double-faceted complex has meaning and denotation, then the 'right phrase' refers to the view that a denoting complex is identical to a meaning and has denotation as its sole side. In this way Russell is led to examine the notion of a single-faceted object.

Thus, even when he adopts the 'right phrase', Russell does not abandon the view of a denoting complex as a multifaceted object. This is a point where my interpretation diverges from those of Noonan, Makin, Levine, Salmon and Simons, and indeed from any other existing ones. Turnau maintains that the 'right phrase' refers to the theory of denoting concepts (Turnau 1991). But in my view the theory as it appears in *PoM* is precluded from the discussion. Some others point out that the phrase refers to the view that a denoting complex is identical to its meaning (Pakaluk 1993; Kremer 1994; Levine 2004, 2005). But they do not understand a denoting complex as a single-faceted object. They assume a denoting complex and its denotation are separate entities. I contend that by adopting the 'right phrase', Russell does not abandon the idea that denoting complexes are multifaceted objects but only the idea that they have more than one side. In fact, he has begun the GEA by assuming that the connection between meaning and denotation is preserved or that the relation between meaning and denotation holds within a multifaceted object. But he is yet to conclude that meaning and denotation are one and the same. He is still working on the assumption.

(F) Russell moves on to argue that the adoption of 'the right phrase' 'makes our difficulty in speaking of meanings more evident' (Russell 1994, 422). Indeed, the view of a denoting complex as a single-faceted

object gives us no clue as to how a given denoting complex/meaning is related to ‘what we use to speak of the meaning’.

Suppose that a definite description corresponds to a denoting complex C , identified with the meaning. This denoting complex is considered a single-faceted object, having the denotation as its sole side. Then, to speak about the denoting complex/meaning, we need a proposition in which not the complex itself but ‘something which denotes C ’ occurs (Russell 1994, 422). This is because the complex C is supposed to have only one side, and so it *always* occurs denotatively in larger complexes. The symbol ‘“ C ”’ no longer signifies the connotative occurrence of the complex/meaning C . Hence, if we still want to use the symbol to speak of the complex/meaning, the symbol must now correspond to some separate object that denotes C , not to C itself. Consequently, if “ C ” is to be ‘what we use when we want to speak of the meaning’, we can only say it is ‘something which denotes C ’ (Russell 1994, 422).

Russell adds that a denoting complex C cannot occur in “ C ”—the corresponding object that somehow denotes C .²¹ As we have just seen, if C is a single-faceted object, the symbol ‘“ C ”’ does not signify the connotative occurrence of C but corresponds to a separate object that somehow denotes C . Suppose C occurs in this object “ C ”. Since C occurs denotatively there, “ C ” must be something that contains the denotation of C as a constituent and thereby singles out the complex/meaning C as its denotation. But we have no idea how it can thus single out C , because the denotation of C can be denoted by many other meanings than C . That is, we know of ‘no backward road from denotations to meanings, because every object can be denoted by an infinite number of different denoting [complexes]’ (Russell 1994, 422). It follows that the complex/meaning C cannot occur in “ C ”. This in turn means that we have no general way of obtaining an object with which we can speak of a given complex/meaning.²²

²¹He also says the former ‘must not be a constituent of’ the latter. He thus does not distinguish sharply between the two relations, *being a constituent of* and *occurring in*, even though the notion of a multifaceted object involves the distinction or an analogue of it. He does not do so probably because, as I argue in the following section, the distinction breaks down once the notion of a single-faceted object is adopted.

²²See Noonan (1996) Makin (2000); Levine (2004); Levine (2005); Salmon (2005); Simons (2005).

2.3. The 'inextricable tangle'

Russell indicates an 'inextricable tangle' in paragraphs (G) and (H). The notion that the relation of denoting obtains within a multifaceted object helps us grasp these otherwise enigmatic parts of the GEA.

(G) In this paragraph, Russell draws two conclusions from the preceding discussion. 'Thus it would seem', he first remarks, 'that "C" and C are different entities, such that "C" denotes C' (Russell 1994, 422). He thus *concludes* that "C" (what we use to speak about a denoting complex C) and C itself are different entities. This has puzzled many commentators because they have taken for granted that a denoting complex and what we use to speak of it are separate entities. But Russell does not think so. On the contrary, he assumes that the relation of denoting would be left mysterious unless it is supposed to hold within a multifaceted object. In his view, the mere claim that "C" denotes C 'cannot be an explanation' and 'the relation of "C" to C remains wholly mysterious' (Russell 1994, 422).²³ It is thus one of his conclusions that a denoting complex "C" and C must be *separate* entities in the sense that the former is not a side of the latter or *vice versa*.

Russell does not, however, state why "C" and C must now be separate entities. One might think this is because, as Russell has shown in paragraph (F), C cannot occur in "C". Yet, this claim is compatible with the idea that C is a side of "C". As is the case with Sir Walter Scott and the denoting complex corresponding to 'the author of *Waverley*', a denotation can be a side of a complex without occurring in it. To see why "C" and C must be separate entities, we need to understand the other conclusion he draws from the preceding discussion.

The conclusion is that if a denoting complex C is a single-faceted object having a denotation as its sole side, 'C is only the denotation', that

²³Levine argues that Russell is here raising an epistemological issue concerning the notion of acquaintance (Levine 2004, 278–80; 2005, 66–67). Some authors have gone even further to interpret the whole of the GEA in terms of the notion (Kremer 1994; Demopolous 1999; Noonan 1996, 79–82). But I think the notion plays no role in the GEA. Bostock remarked that Kremer's account, albeit 'the most plausible one' in the literature, 'does not appear to improve the very disputed argument that we actually find in his text, for that argument never mentions the notion of acquaintance' (Bostock 2012, 39–40). Neither does the notion appear in the corresponding part of OF (see Noonan 1996, 92). Russell certainly mentions it in other parts of OD and OF, but this only makes it even more mysterious why he does not do so in the GEA or in the corresponding part in OF. See also footnote 38.

is, *C* is identical to its denotation (Russell 1994, 422). He does not state why this is the case. But how can an object having only one side differ from its sole side? If a denoting complex is a single-faceted object, then, *whenever* it occurs in a larger complex, the denotation is a constituent of the complex. There is no way left to differentiate a denoting complex from its denotation. In other words, given the notion of a single-faceted object, the distinction between *occurring in* and *being a constituent of* becomes virtually non-existent because whenever a denoting complex occurs in a larger complex, it makes its denotation a constituent of the larger complex. Thus, a single-faceted object collapses into its sole side.

If each denoting complex is identical to its denotation, then it follows that “*C*”—what we use to speak about a denoting complex *C*—cannot be a multifaceted object having *C* as a side. For if “*C*” is identical to *C*, the notion of denoting becomes trivial—we use “*C*” to speak about *C*, namely, about “*C*” itself. The view of a denoting complex as a single-faceted object thus implies that “*C*” and *C* must be separate entities.

This explains why Russell says that the meaning is now ‘wholly relegated to “*C*”’ (Russell 1994, 422). Since the denoting complex/meaning *C* is identical to its denotation, “*C*” (a separate object we use to speak of *C*) is, if there is such a thing, the only object that denotes the denotation in a non-trivial sense. In this sense, “*C*” may be called a *meaning* though it is a separate entity from the original meaning of *C*, which is now identified with *C* itself.

It is also clear why meaning and denotation are said to be ‘one and the same’ (Russell 1994, 421). Given the view of a denoting complex as a single-faceted object, a denoting complex *C* is identical to its meaning, while the complex is now shown to be identical to its denotation. We have thus finally reached the conclusion Russell has been trying to draw from the assumption that the connection between meaning and denotation is preserved. This conclusion indeed leads immediately to the ‘inextricable tangle’.

(H) In this paragraph, Russell explains the ‘inextricable tangle’. On the one hand, if the connection between meaning and denotation is to be preserved, meaning and denotation must be ‘one and the same’. This implies that the propositions “Scott was the author of *Waverley*” and “Scott was Scott” are ‘identical propositions’. They have exactly the

same constituents and we can no longer differentiate them by claiming that the identity of a proposition depends on those objects which occur in it and that a denoting complex occurs in the former proposition but not in the latter. On the other hand, unless meaning and denotation are distinguished, the notion of denoting cannot be used to explain why George IV wanted to know whether or not Scott was the author of *Waverley* but not whether Scott was Scott. Assuming that the relation of denoting must hold within a multifaceted object, Russell introduced the notion of denoting to the effect that '[if] we say "Scott is the author of *Waverley*", we assert an identity of denotation with a difference of meaning' (Russell 1994, 419). The idea was that 'the meaning is *relevant* when a denoting [complex] occurs in a proposition' in the sense that the identity of the proposition is not determined by its constituents including the denotation of the denoting complex but by those objects that occur in the proposition including the denoting complex that has or is the meaning (Russell 1994, 422, emphasis added). This idea presupposes that meaning and denotation must not be 'one and the same'. This dilemma is what I take to be the 'inextricable tangle', which shows that 'the point of view in question must be abandoned' (Russell 1994, 423).

We can understand the 'inextricable tangle' as a conclusion of the discussion which Russell began by assuming, in paragraph (D), that the relation between meaning and denotation is preserved. He maintains that if the relation of denoting is not to be 'wholly mysterious', it must hold within a multifaceted object. But if a definite description 'C' corresponds to a denoting complex conceived as a double-faceted object, then we cannot speak about the two sides—meaning and denotation—of the complex using such expressions as 'the meaning of ...' and 'the denotation of ...'. We can still use, as Russell sometimes does, the symbol "'C'" to speak of the denoting complex itself, while we are supposed to be able to speak about the meaning of the complex using the same symbol. We are thus led to identify the denoting complex with its meaning, viewing the complex as a single-faceted object. But this view not only implies that there is no general way of speaking about a denoting complex/meaning but also that each denoting complex/meaning is identical to its sole side—the denotation. Thus, the assumption that the relation of denoting holds within a multifaceted object implies the identity between meaning and denotation. This conclusion leads

us to the ‘inextricable tangle’: we distinguish between meaning and denotation so as to resolve the George IV puzzle, but if we attempt to do so in a non-mysterious manner, we conclude that meaning and denotation are ‘one and the same’.

Levine views the ‘inextricable tangle’ as ‘the culmination of the GEA’ (Levine 2004, 282). According to him, the George IV puzzle requires that a denoting complex should occur otherwise than denotatively in some propositions, while Russell rejects such occurrences through his discussion on the difficulty of speaking of meanings (Levine 2005, 73–76). But this interpretation alone does not explain why Russell concludes that the two sentences ‘Scott was Scott’ and ‘Scott was the author of *Waverley*’ express the same proposition—it seems as though he simply confuses Scott with the denoting complex that denotes him. In fairness, Levine argues that Russell did not rely on this alleged confusion for his conclusion because he had an alternative argument to the same effect (Levine 2005, 75–76). But, in my view, Russell does not confuse *C* and its denotation but rather *argues* that they are identical once denoting complexes are understood as single-faceted objects.

Wahl offers an account of the ‘inextricable tangle’ similar to mine, holding that ‘[t]he whole argument ...is designed to show that any view that makes the distinction between the meaning and denotation of a complex will be forced to this two entity view, and this view is untenable’ (Wahl 1993, 91). What he calls ‘the two entity view’ is in effect the view that the sides of a denoting complex are indistinguishable and hence a denoting complex *C* and “*C*” (what we use to speak of *C*) must be two separate entities. He has thus observed that the ‘inextricable tangle’ involves the identity between meaning and denotation. But he does not derive the identity from the view of a denoting complex as a single-faceted object. He instead appeals to the idea that the relation of denoting holds between a denoting complex and an object if and only if the denoting complex can be replaced *salva veritate* by any denoting complex that denotes the object or by the object itself (Wahl 1993, 89–90; Hylton 1990, 251–52). Given this idea, two sentences ‘Plato is human’ and ‘the teacher of Aristotle is human’ have the same truth value, and similarly, ‘the teacher of Aristotle is human’ and ‘the denoting complex “the teacher of Plato” is human’ also have the same truth value, implying

that the denoting complex “the teacher of Plato” is human.²⁴ In OF, however, Russell repudiates this idea even before he discovers the difficulty in speaking of meanings or an analogue of it, as we will see in Section 3.2.

3. The notion of multifaceted objects

The reader may still wonder whether Russell ever envisaged the notion of what I call a multifaceted object—an object that contributes different sides to a larger complex depending on how it occurs there. To show he did, I will make three points by looking into some of the manuscripts he wrote in 1904 and in 1905. First, his attempt to philosophically motivate the zigzag theory naturally led him to envisage the notion. Second, he invoked the notion to account for the nature of complex objects. Third, he was examining the view of a denoting complex as a multifaceted object when he discovered some of the constitutive arguments of the GEA.

3.1. The zigzag theory

In this section, I argue that Russell's attempt to develop the zigzag theory with some ‘intrinsic plausibility’ naturally resulted in his understanding of a second-order variable as a multifaceted object (Russell 2014, 77).²⁵ A second-order variable can occur either as a function applied to a first-order object or as an argument of a higher-order function. He once advanced, or so I argue below, the account of such a variable as an object that can occur in complexes either *as entity* or *as meaning*.²⁶

²⁴The idea in question makes the relation of denoting transitive. For critical discussions of this line of interpretation, see Kremer (1994, 260–61) and Noonan (1996, 82–87). Searle (1958) and Cassin (1971) do not appeal to the idea but simply assume that the relation between meaning and denotation would be transitive.

²⁵Russell found no such plausibility when he drafted ‘On Some Difficulties in the Theory of Transfinite Numbers and Order Types’ in November 1905. As we will see below, the attempt had indeed failed because of various difficulties with the notion of a multifaceted object, including the ones that constitute the GEA.

²⁶It is contentious to claim that Russell uses the expression ‘variable’ to speak about some sort of object. But I will argue below that if we thus understand the expression, we can make sense of his remarks about variables in OF (see footnotes 33 and 40).

The aim of the zigzag theory is simple. It aims to resolve the set-theoretic paradox by preventing such propositional functions as “ $x \notin x$ ” from determining a corresponding class.²⁷ Russell called those propositional functions that determine a class *legitimate* and the others *illegitimate*.²⁸ If a propositional function ϕx is illegitimate, it exhibits ‘a certain characteristic which we may call zigzaginess’: for each class u , there must be either some $a \in u$ such that ‘ ϕa ’ is false or some $b \notin u$ such that ‘ ϕb ’ is true (Russell 2014, 74; compare Russell 1994, 120–21).

Russell might have attained a tenable formal system based on this idea if he had been content to specify a sufficiently narrow range of legitimate propositional functions.²⁹ But, as Klement puts it, Russell’s ‘standards were high; he did not want a formal dodge, he wanted a philosophically, even metaphysically, motivated explanation for the avoidance of the contradictions’ (Klement 2003, 15). Indeed, Russell sought a demarcation between the two kinds of propositional functions with ‘intrinsic plausibility’. He thought he would be able to distinguish between legitimate functions and illegitimate ones if he could find a non-trivial reason why illegitimate functions should fail to determine a class. He was thus led to examine what is wrong with the function “ $x \notin x$ ”.

In so doing Russell focused on the form of illegitimate functions. Suppose we define, as Frege did, the class-membership relation \in in terms of second-order quantification and class-abstraction $\{x : \phi x\}$:

$$x \in u =_{\text{Df}} (\exists \phi)(u = \{z : \phi z\} \ \& \ \phi x).$$

Then, the self-membership function “ $x \in x$ ” amounts to

$$(\exists \phi)(x = \{z : \phi z\} \ \& \ \phi x).$$

The problematic function “ $x \notin x$ ” may then be defined as

²⁷Crudely put, propositional functions are functions whose values are propositions, which are in turn conceived as mind-independent complexes. In what follows I use double quotation marks to talk about propositions and propositional functions thus understood.

²⁸In ‘On Some Difficulties in the Theory of Transfinite Numbers and Order Types’, Russell uses the terms ‘predicative’ and ‘non-predicative’ to talk about these two kinds of propositional functions.

²⁹This is in effect what Quine did by developing his well-known formal system NF. Its (slight) modification NFU is proven to be consistent relative to ZF (Jensen 1968).

$$(\forall\phi)(x = \{z : \phi z\} \rightarrow \sim\phi x).$$

Replacing the class abstraction $\{z : \phi z\}$ with an arbitrary second-order function $f(\phi)$, we can generalise this into

$$(\forall\phi)(x = f(\phi) \rightarrow \sim\phi x).$$

Using a notation Russell adopts in 1904 manuscripts, we can rewrite this formula thus: ' $x = f\prime(\phi) . \supset\phi . \sim\phi\prime x$.'³⁰ I shall call the functions of this form *diagonal*.

Now, each diagonal function involves two kinds of occurrences of a second-order variable ϕ —occurrence *as an argument* of f and occurrence *as a function* applied to x . Russell sought a reason why any function should be illegitimate that involves a second-order variable occurring in these two distinct ways.

In some 1904 manuscripts, Russell tries to answer this question by developing the view of a (propositional) function as a *mode of combination*.³¹ The idea is that a mode of combination is not a constituent of a complex in which it connects other entities, while it can be a constituent of other complexes.³² It should be noted that this idea already presupposes that an object may be involved in a complex—in the sense it glues its constituents—without being a constituent of it. I take this to be a precursor of the distinction between *occurring in* a complex and *being a constituent of* it.

Russell appeals to the view of a function as a mode of combination to obtain two possible accounts as to why the two kinds of occurrences of ϕ in a function make it illegitimate. One is simply to think we

³⁰In 1904 manuscripts, Russell places an inverted comma to express functional application.

³¹The view is found in some later parts of 'On Functions' (Russell 1994, 6), 'Fundamental Notions' (Russell 1994, 7) and 'On the Nature of Functions' (Russell 1994, 9). Prior to this view, he tried to explain the illegitimacy of diagonal functions by endorsing a view of functions analogous to the one presented in *Principia Mathematica*. We can find this attempt in 'On Functions, Classes, and Relations' (Russell 1994, 5) and in some earlier parts of 'Fundamental Notions'. It is worth noting that, in 'Fundamental Notions', he employs inverted commas to speak of a function (that is, a meaning) itself (Russell 1994, 114, 124, 128–29).

³²In endorsing this idea, Russell counts a mode of combination as an entity (Russell 1994, 268–69). This contrasts with his own notion of *logical form* in later writings (e.g., Russell 1984, 98).

cannot quantify over functions occurring as function: ‘if ϕx occurs in a complex, the ϕ must not be varied, because it is not a constituent of ϕx ’ (Russell 1994, 100). According to the other account, we cannot use a single variable ϕ to quantify both over functions occurring as function (e.g., “ ϕ/x ”) and over functions occurring as argument (e.g., “ $\hat{z}(\phi/z)$ ”) at the same time, because ‘ ϕ/x designates the compound of x with other entities according to a certain mode of composition’, while ‘ $\hat{z}(\phi/z)$ designates that mode of composition itself’ (Russell 1994, 265). However, he eventually abandoned the view of a function as a mode of combination mainly because what we normally call functions have some constant parts, whilst a mode of combination is ‘got by making every constituent variable’ (Russell 1994, 255).

Russell makes another attempt to explain the illegitimacy of diagonal functions in OF. In this manuscript, he draws a distinction between *occurrence as meaning* and *occurrence as entity*, which arguably corresponds to the one between *occurrence as function* and *occurrence as argument*. He uses the new distinction to propose two similar accounts on the illegitimacy of diagonals. One is that ‘what occurs as meaning can’t be varied’ because ‘we must be able to specify what varies, and this can only be done if what varies occurs as entity, not as meaning’ (Russell 1994, 362). On this account, we cannot quantify over functions occurring as meaning or as function. He introduces the other account by claiming that ‘meaning-variation must be distinguished from entity-variation, and that two variables of which one means and the other is can only be equal by accident, and can’t be kept equal throughout variation’ (Russell 1994, 360). In OF he calls what the variable *is* the *being* of the variable. He thus uses the distinction between *occurrence as meaning* and *occurrence as entity*, or the distinction between *meaning* and *being*, to explain why diagonal functions are illegitimate.

It is through developing these accounts that Russell comes to endorse the notion of multifaceted objects. Considering the second account, he remarks:

...if we assert a connection between a variable in a meaning-position and a variable in an entity-position, we must avoid denoting complexes, since these will stand for their meaning in the one position and for their denotation in the other. (Russell 1994, 361)

Diagonal functions assert 'a connection between a variable in a meaning-position and a variable in an entity-position'.³³ He thinks those functions may be dismissed as illegitimate on the grounds that they attach two different domains of quantification to a single second-order variable. It is crucial here to understand a second-order variable as a multifaceted object. If the two occurrences of a variable letter ' ϕ ' are directly correlated with two different domains of quantification, then we should simply use two different variable letters. In order to preserve the 'connection' between the two occurrences of the letter ' ϕ ' and thereby explain why diagonal functions are illegitimate, we need to interpret the letter as corresponding to a single object somehow associated with two different ranges.³⁴ He obtains a natural account of such an object by means of the notion of a multifaceted object. He understands second-order variables and first-order propositional functions as multifaceted objects having two sides, meaning and being. When a second-order variable occurs as meaning in a complex, it contributes to the complex its *meaning*, which ranges over the *meanings* of first-order functions; and when one and the same variable occurs as entity, it contributes its *being*, which ranges over the *beings* of first-order functions.

Russell's attempt to explain the illegitimacy of diagonal functions in terms of the distinction between *meaning* and *being* thus involves the application of the distinction to propositional functions in general. In OF, he goes further to understand propositions—the values of propositional functions—as complexes having these two sides.³⁵ As we will see in the following section, he indeed applies the distinction to complexes in

³³The phrase 'stand for' in the passage above suggests that what Russell calls 'denoting complexes' are expressions. But we need not think thus, because the notions of meaning-position and entity-position are ontological notions, defined in terms of the distinction between *meaning* and *entity* (see footnote 40). In OF, Russell does not understand denoting complexes as expressions but as what they correspond to (see Wahl 1993, 85).

³⁴It is unclear how Russell in OF understands the relation between a variable and its values. But he need not commit himself to the claim that a proposition may contain infinitely many constituents, if he understands the relation in the way he does in *PoM*, namely, if he understands a variable as '*any term* as entering into a propositional function' (italics mine) and thinks that '*any term* does not denote, properly speaking, an assemblage of terms, but denotes one term, only not one particular definite term' (Russell 1903, 94).

³⁵In OF, Russell indeed includes propositions among complexes. He remarks that some complexes are 'propositional' in the sense that they can be asserted and may possess the property of being true (Russell 1994, 366, 369, 384).

general in OF. The account of complexes he offers will give us another reason to attribute the notion of a multifaceted object to him.

3.2. Russell's account of complexes in 'On Fundamentals'

In OF, Russell develops the distinction between *meaning* and *being* into an account of denoting complexes and other complexes in general.³⁶ He holds, or so I will claim, that those two sides of complex objects account for the dualistic nature of them—a complex is essentially *one* entity composed of *many* entities. To explain the nature of complexes thus, it is crucial to understand complexes as multifaceted objects.

In OF, Russell introduces the account of complexes in general by identifying two things with one another: (i) a complex occurring as entity (the *being* of the complex) and (ii) the *denotation* of the complex.³⁷ He claims that every complex has two sides: a *meaning*, which is complex, and a *denotation* or what he calls *being*, which is simple:

I think the line to take is this: Every complex has meaning and being. *Quâ* meaning, it is not one entity, but a compound of several. A complex may occur in two ways, as meaning or as entity. Complexes may differ as meaning without differing as entity. What the complex *is* is what we have called the denotation. There is no entity which *is* the complex as meaning, because the complex as meaning is not one entity. (Russell 1994, 366)

The idea is that the *meaning* of the complex is not a single entity but a plurality of its constituents, while the *being* of a complex is what the complex is when taken as a single entity. It is considered characteristic of a complex to have those two sides: 'An entity which is not a complex does not have the two sides, but only has being' (Russell 1994, 366).

In order to explain the dualistic nature of complexes in this way, it is crucial to understand complexes as multifaceted objects. One must not think of those two aspects of a complex—meaning and being—as two separate entities, for the two aspects are meant to account for the

³⁶Russell understands complexes as having both *meaning* and *denotation* in a 1903 manuscript 'Points about Denoting' (Russell 1994, 13).

³⁷By thus merging the distinction between *meaning* and *denotation* and the distinction between *meaning* and *being* into one, Russell attaches to the former distinction a type-theoretic implication of the latter. His association of the type-theoretic distinction among objects with the semantic distinction between *meaning* and *denotation* can also be found in a 1903 manuscript 'Dependent Variable and Denotation' (Russell 1994, 12).

complexity and unity of one and the same complex. One needs to view those aspects as sides of multifaceted objects. In the passage quoted above, Russell seems to put forward the idea that whether a complex contributes its meaning or being to a larger complex depends on how the complex occurs there. When it occurs as meaning, it contributes its meaning—the constituents of the complex—and when as entity, its being—the complex itself taken as a single object. He thus views a complex in general as a multifaceted object having the two sides of *meaning* and *being*.

It is, however, questionable whether we can consistently put together the two notions with each other: the distinction between *meaning* and *denotation* and that between *meaning* and *being*. Russell himself is quick to point to a problem. There are cases where a complex occurring as entity—thereby being simple—still needs to be seen as being composed of its constituents. In the proposition “People were surprised that Scott was the author of *Waverley*”, we cannot substitute *salvo veritate* Scott for the denoting complex to which ‘the author of *Waverley*’ corresponds even though the complex appears to occur as entity there. In such cases, ‘a proposition as entity must depend upon its constituents, and be changed by the substitution of other constituents with the same denotation’ (Russell 1994, 368).³⁸ He then finds that he cannot simply identify the being of a denoting complex with its denotation and that there are cases where the meaning of a complex should be treated as a single object, not as a plurality of entities (Russell 1994, 369).

Russell does not, however, abandon the account of the nature of complexes in terms of the distinction between *meaning* and *being*. He retains it as a ‘broad rule’:

The broad rule is that when complexes occur as *meaning*, their complexity is essential, and their constituents are constituents of any complex containing the said complexes; but when complexes occur as *entities*, their unity is essential, and they are not to be split into constituents. (Russell 1994, 373)

³⁸Russell distinguishes this problem from an epistemological one that arises from the same observation. The former is an ontological one concerning the identity of a proposition, while the latter is related to the fact that in some propositions in which a denoting concept is used, ‘we know what we are saying, but we don’t know what this concept denotes’ (Russell 1994, 368; compare Cohen 2022, 13). In OF, he calls the discussion of the latter a ‘digression’ (Russell 1994, 369). This fits the fact that he formulates the ‘inextricable tangle’ in terms of the identity of two propositions in OD.

Of course he does not rest content to state the ‘broad rule’. He first attempts to clarify the distinction between *meaning* and *being*. He now maintains that an entity *A* occurs *as entity* in a complex *B* if and only if ‘any entity, simple or complex, may be substituted for *A* in *B* without loss of significance’ (Russell 1994, 374).³⁹ On the other hand, *A* occurs *as meaning* in *B* if and only if ‘it can only be significantly replaced by an entity of a certain sort, e.g. a proposition, or a type, or a relation’ (Russell 1994, 374). He makes the same point using the notions of ‘entity-position’ and ‘meaning-position’: ‘If something which is not a complex is put in a meaning-position, the result is nonsense’ (Russell 1994, 370).⁴⁰ There is of course no question of whether or not a complex object is *nonsense*, or of whether it has or lacks *significance*. I think what he has in mind is the question of whether the result of such a substitution has the *unity* required to count as a complex (compare Russell 1992, 56). He then goes on to introduce further kinds of occurrences including the one between *primary occurrence* and *secondary occurrence* (Russell 1994, 374–75). A complex *A* occurs in another complex *B* *primarily* if any complex with the same denotation, or the denotation itself, may be substituted for *A* in *B* *salva veritate*.⁴¹ With all those distinctions, however, he does not succeed in defending the account of complexes in general. He discovers, as we will see in Section 3.3, that even with those distinctions he is unable to explain how we can speak of meanings.

3.3. The original arguments in ‘On Fundamentals’

In this section, I introduce various arguments Russell presents in the part of OF that contains some passages re-used almost verbatim in OD (Russell 1994 from 381, l40 to 383, l23). As his remarks in OF may themselves be subject to interpretation, I will not claim that Russell in OF presents exactly the same argument as the GEA. But it is safe to

³⁹In OF, Russell initially thinks that when a complex occurs as entity in a larger complex, it may also be substituted for its being *salva veritate*. But he rejects this idea due to the George IV puzzle and analogous ones.

⁴⁰This is how Russell *defines* the notions of entity-position and meaning-position. Landini argues that Russell confuses the ontological notion of ‘occurrence as entity/meaning’ with the linguistic one of ‘entity/meaning-position’ (Landini 1998, 61). But we need not attribute such confusion to Russell if we understand both distinctions as ontological.

⁴¹As Noonan (1996, 92) points out, Russell in OF is thus aware of, and even envisages a solution to, the problem that Hylton and Wahl attribute to the GEA.

say he somehow turned those arguments in the corresponding part of OF into the GEA. I will argue below that those arguments are directed against the idea that definite descriptions (among other expressions) correspond to multifaceted objects. This will in turn suggest that when he discovered the GEA, he was examining the notion of a multifaceted object and hence that the GEA was itself an objection to the idea.

Russell in OF suggests in various ways that he understands a denoting complex as a multifaceted object having the denotation as a side. As we saw in the previous section, he puts forward the view that '[w]hat the complex *is* is what we have called the denotation' (Russell 1994, 366). If the being of a complex can be a constituent of a larger complex, so can the denotation of a (denoting) complex. The denotation must be a side of the denoting complex so that the denotation can be a constituent of a larger complex in which the denoting complex occurs. He also remarks that 'the difference of a complex from a simple concerns its meaning, not its being' (Russell 1994, 366). He seems to think that two complexes with the same constituents may still differ from each other if a simple entity occurs in one of them and a complex whose denotation is the simple entity occurs in the other. If so, he effectively draws the distinction between *occurring in* and *being a constituent of*. It is true that he gives up identifying the being of a denoting complex with the denotation of it when he considers the *Waverley* case and other similar ones. He comes to think that '[w]henver a denoting concept occurs in a proposition, it is the meaning, not the denotation, that occurs' (Russell 1994, 368). But this does not mean that he no longer understands a denoting complex as a multifaceted object having the denotation as a side. For instance, he remarks that 'to affirm a proposition is not to say that it is true, but to say something about the *constituents* of the proposition' (Russell 1994, 381). If this remark is strictly applicable to those propositions in which a denoting complex occurs, the denotation of a denoting complex must be capable of being a constituent of the propositions in which the complex occurs in a certain manner—otherwise, one cannot say anything about the denotation using the denoting complex. Russell in OF thus seems to understand a denoting complex as a multifaceted object having its denotation as a side. This can also be seen in his discussion of difficulties in speaking about meanings, to which we are turning.

In OF, Russell initially holds two opposing ideas about how we can speak of meanings of complexes.⁴² He assumes that we cannot speak of a meaning of a complex by asserting a proposition in which the complex occurs as meaning. In his view, when a complex occurs as meaning, 'we merely mean it, and do not say anything about it' (Russell 1994, 382). Can we then speak of a meaning by making it occur in an entity-position? On the one hand, he states we cannot: 'if we wish to put a denoting meaning in an entity-position, and say something about the meaning itself, we can only do so by means of a denoting concept...' (Russell 1994, 363). He also remarks that inverted commas 'give a denoting concept which denotes the meaning of what is between the inverted commas' (Russell 1994, 363). On the other hand, he also suggests that using inverted commas, we can make the meaning of a denoting complex occur as entity, and talk about it: 'Verbs not in an infinite mood and prepositions, and conjunctions, when they occur in sentences, normally occur as *meanings*; to make them occur as entities, it is necessary to employ inverted commas or italics' (Russell 1994, 380). Although he mentions such linguistic items as verbs here, I think he has the corresponding objects in mind since the linguistic items do not 'occur as *meaning*'. Further, in considering 'the nature of *undetermined* or *ambiguous* denotation' and thereby 'the essence of a *variable*', he remarks that '*x* means "anything" and denotes anything' (Russell 1994, 381).⁴³ He seems to think that by using inverted commas, one can make the meaning occur as entity in the corresponding proposition, thereby speaking about it.

It is when he notes the conflict between these two ideas that Russell finds a range of problems with the idea that definite descriptions correspond to multifaceted objects. To be precise, he does not find the conflict itself problematic. In OF he simply decides that we speak of the denotation of a complex *whenever* it occurs as entity.⁴⁴ But this decision

⁴²In OF, as in OD, Russell does not make it clear whether complexes are considered identical to meanings or not until he comes to introduce the 'right phrase'.

⁴³He goes on to remark, "'any man" means a certain ambiguously denoting concept and denotes any man' (Russell 1994, 381). It is unclear whether he is using inverted commas to speak of a meaning or a denoting complex. Yet, he does not seem to mean by '*x*' anything linguistic but an object, for he also says, 'a concept has meaning and denotation' (Russell 1994, 381).

⁴⁴He added a marginal note 'Imp[ortant] principle' to a passage where he remarks that 'a denoting meaning can only be spoken of by means of denoting concepts that denote the meaning in question' (Russell 1994, 363, 649, RA230.030710, Fo1, folio 7).

makes him wonder how he can explain the role that inverted commas are supposed to play:

If we say "any man" is a denoting complex, "any man" stands for 'the meaning of the complex "any man"', which is a denoting concept. But this is circular; for we use "any man" in explaining "any man". And the circle is unavoidable. For if we say "the meaning of any man", that will stand for the meaning of the denotation of any man, which is not what we want. (Russell 1994, 382)

The circularity would, if it is genuine, undermine the explanation of the expression "C" as synonymous with 'the meaning of the complex "C"' (see footnote 15). But what he finds problematic is rather the observation underlying it: 'if...we put [the meaning] in an entity-position,...we get the meaning (if any) of what the complex denotes, not of what the complex means' (Russell 1994, 382).⁴⁵ Having thus observed, he admits, 'The phrase "the meaning of a denoting complex" is wrongly formed; for suppose C is a denoting complex; then "the meaning of C" puts C in an entity-position, and therefore means "the meaning of the denotation of C"' (Russell 1994, 382). He then adopts, as in OD, the 'right phrase': 'some meanings have denotations' (Russell 1994, 382). He now seems to view a complex as an object having its denotation as its sole side. He thinks, again as in OD, that this view 'makes our difficulty in speaking of meanings more evident' because it entails whenever a complex occurs as entity, we speak about the denotation. He also observes, as he does in OD, that 'a concept which denotes C must not contain C as entity (as in the case, e.g., with "the meaning of C"), for then we get the denotation of C occurring where we meant to have the meaning' (Russell 1994, 382). Note that if we take this remark literally, he assumes that the denotation of a denoting complex may occupy the place 'where we meant to have the meaning', namely, that the denotation may be a constituent of the proposition with which we meant to talk about the meaning.

So far, Russell's argument in OF is mostly parallel to the corresponding part of the GEA. But, in OF, he moves on to make the following remark, which makes it explicit that he is concerned with the view of a denoting complex as a multifaceted object:

⁴⁵Strictly speaking, what we 'put...in an entity position' must be a denoting complex, rather than a meaning.

- (1) If *C* is a denoting complex, “the meaning of *C*” does not *denote* the meaning of *C*, but the meaning of the denotation of *C*.
- (2) If *C* is a denoting complex, “the denotation of *C*” does not *mean* the denotation of *C*, but “the denotation of *C*”.

These two facts show the indissolubility of meaning and denotation, and the impossibility of inventing a symbolism which will avoid the necessity of distinguishing the two sides of complexes. For “the meaning of *C*” and “the denotation of *C*” both have two sides, and are therefore in no way less two-fold than “*C*” itself. (Russell 1994, 383)

As these remarks are absent in OD, it is arguable whether the two facts he points to above play any roles in the GEA or not.⁴⁶ But what is important for our purposes is the fact that he talks about ‘the two sides of complexes’ and thinks of complexes as being ‘two-fold’.

In OF, Russell concludes the critical discussion on the view of definite descriptions as correlated with meaning and denotation by pointing to yet another problem with it. He argues that if the two sides of corresponding complexes may occur both as entities, they must be different entities:⁴⁷

What is wanted is not a further kind of occurrence. Consider

“The centre of mass of the Solar System” is a denoting complex, not a point.

The centre of mass of the Solar System is a point, not a denoting complex.

In each of these the subject occurs as entity, not as meaning; in the first, the subject is “*C*”, in the second it is *C*. Thus it would seem that “*C*” and *C* are two different entities. In that case, what is the connection between them? (Russell 1994, 383)

In the examples above, both “*C*” and *C* occur *as entity*, while different properties are predicated of them. In his view this means that “*C*” and

⁴⁶Some authors indeed connect the remarks quoted above with the GEA and with a certain argument in ‘Knowledge by Acquaintance and Knowledge by Description’ (Rodriguez-Consuegra 1992, 202–3; Makin 2000, 36–37).

⁴⁷The passage quoted below appears at the beginning of the GEA. This is I think because Russell in OF finds that we cannot speak about the meaning of a denoting complex unless we give up the idea that we speak of the denotation of a denoting complex whenever it occurs as entity. In OD, Russell uses the same examples to distinguish between two different ways in which a denoting complex occurs as entity, that is, to draw the distinction between the *denotative* occurrence and the *connotative* occurrence. I will expand on this point at the end of this section.

C are two different *entities*, not the two *sides* of one complex. 'We have now', he remarks, 'not one complex with the two aspects of meaning and denotation, but two entities, "*C*", the complex, and *C*, the denotation of "*C*"' (Russell 1994, 383). The passage quoted above also shows that he thinks the 'connection' between meaning and denotation is now lost. Immediately after drawing these conclusions, he turns to a radically different approach to the relation between meaning and denotation, which results in the theory of definite descriptions and an analogous account of class-symbols (see Noonan 1996, 97–101).

Russell thus presents various arguments in the corresponding part of OF. But among those points, he counts the following three as 'fatal' objections: 1) *C* occurring as entity—the denotation of *C*—and "*C*" occurring as entity—the meaning of *C*—are separate entities; 2) we cannot speak of a denoting complex by putting it in an entity-position; 3) we cannot appeal to any complex that contains the complex in an entity-position. He notes that these problems arise not only with definite descriptions but with other sorts of expressions:

We shall have to distinguish between "everything" and everything, *i.e.* we shall have: "everything" is not everything, but only one thing. Also we shall find that if we attempt to say anything about the *meaning* of "everything", we must do so by means of a denoting concept which denotes that meaning, and which must not contain that meaning occurring as entity, since when it occurs as entity it stands for its denotation, which is not what we want. These objections, to all appearance, are as fatal here as they were in regard to *the*. (Russell 1994, 385–86)

Importantly, if the first objection is an objection at all, it must be an objection to the contrary idea, namely, to the idea that the expression 'everything' corresponds to 'one complex with the two aspects of meaning and denotation' (Russell 1994, 383). This in turn suggests that the arguments in the corresponding part of OF are directed against the idea that definite descriptions (among other expressions) correspond to multifaceted objects. Furthermore, the last two objections stated above, together with the observation that there is 'no backward road from denotations to meanings', constitute what Russell calls the 'difficulty in speaking of meanings' in OD. We may, therefore, think that the GEA, which incorporates the discussion of the 'difficulty', is also directed against the view of definite descriptions as corresponding to multifaceted objects.

It is beyond the scope of the present work to determine how exactly Russell turned those three objections into the GEA. But I would like to sketch a possible account before I conclude this paper. In OD, he does not state the first objection; instead, he begins his discussion by assuming that meaning and denotation must not be separate entities. The first objection establishes that if the meaning and being of a complex can both occur as entity and possess different properties, then they must be distinct entities. In OD, he introduces a possible way of avoiding this conclusion. The idea is that there are two ways in which a denoting complex can occur as entity. This is of course the view of a denoting complex as a double-faceted object, according to which a denoting complex can occur either *denotatively* or *connotatively*. This view is different from the view that he abandons at the end of the corresponding part of OF. As we saw above, his discussion of the ‘difficulty in speaking of meanings’ in OF is based on the assumption that we speak about the denotation of a denoting complex *whenever* the complex occurs as entity. Nevertheless, the former view is, just like the latter, susceptible to the ‘difficulty in speaking of meanings’. Those two views are no different in that they both posit ‘something other than the meaning, which can be called the complex’. He in OD is thus led back to the ‘right phrase’ that ‘some meanings have denotations’. In OF, he observes that the ‘right phrase’ implies that ‘in all ordinary propositions in which C occurs, what is said does not hold of C, but of what C denotes’ (Russell 1994, 382). Once he realises that a denoting complex to which a definite description corresponds *always* occurs as entity in other complexes, not just in ‘ordinary’ propositions, he is in a position to draw a more devastating conclusion: once we identify the meaning of a complex with the complex itself, we have no reason to think that the meaning/complex is distinct from what it always contributes to other complexes—the denotation. We must then *identify* meaning with denotation if we are to *distinguish* between them.

4. Concluding remarks

In Section 2, I argued that we can interpret the whole of the GEA as a single, coherent argument against the notion of denoting if we attribute to Russell the idea that the relation of denoting holds within a multifaceted

object. The GEA is an argument that 'we cannot succeed in *both* preserving the connection of meaning and denotation *and* preventing them from being one and the same'. He begins the argument by assuming that we 'preserve the connection', namely, that the relation of denoting holds within a multifaceted object. He first examines the notion of double-faceted objects—the notion that a denoting complex may occur in larger complexes either denotatively or connotatively. When we use a definite description itself, the corresponding complex occurs denotatively, and when we use it together with inverted commas, the complex occurs connotatively. This notion involves, however, the 'difficulty in speaking of meanings'. It leaves us no general way to speak about the meaning and denotation of a denoting complex using such expressions as 'the meaning of ...' and 'the denotation of ...'. These expressions require we should be able to speak about denoting complexes themselves, but the notion of a double-faceted object does not explain how we can do so. If we identify a denoting complex with its meaning, then we can use inverted commas to speak about the denoting complex/meaning itself. This is why he adopts the 'right phrase' or what I call the notion of a single-faceted object. But this view makes the 'difficulty in speaking of meanings' 'more evident' because it also fails to offer a general way to speak about denoting complexes/meanings. Moreover, and more crucially, this view implies that meaning is identical to denotation. We cannot differentiate a single-faceted object from the sole facet. He thus concludes that we fail to 'prevent [meaning and denotation] from being one and the same'.

In Section 3, I made three points for Russell's tentative endorsement of what I call the notion of multifaceted objects. First, in his attempts to establish the 'intrinsic plausibility' of the zigzag theory, he obtained the idea that a second-order variable ϕ can occur either as meaning (as in " ϕx ") or as entity (as in " $f(\phi)$ "). The use of a single letter ' ϕ ' in two different ways in such a sentence as ' $(\forall \phi)(x = f(\phi) \rightarrow \sim \phi x)$ ' led him to think the letter corresponds to a single object that has two different sides. Second, when writing OF, he developed this idea into the general theory of complexes according to which each complex occurs in other complexes either as meaning or as being. This theory presupposes that the meaning and being of a complex are not two separate entities but two sides of a multifaceted object. Third, in the part of OF that contains some

passages of the GEA verbatim, he points to various objections to the view of denoting complexes as having ‘the two aspects of meaning and denotation’. He turned those objections into the GEA, which suggests that the GEA was intended to be an objection to the idea that definite descriptions correspond to multifaceted objects. His remarks in the corresponding part also make it clear that he assumes that the relation between meaning and denotation will be left mysterious unless they are ‘the two aspects’ of one and the same object, not ‘two different entities’.

In my view, the force of the GEA hinges on the plausibility of this assumption. If I am not mistaken, the GEA is *valid* as an argument that *if* we preserve the connection between meaning and denotation via the notion of a multifaceted object, *then* we end up identifying them. The GEA indeed shows what Russell says it does: ‘we cannot succeed in *both* preserving the connection of meaning and denotation *and* preventing them from being one and the same’. But one may wonder whether it is necessary to ‘preserve the connection’ in order to endorse the distinction between meaning and denotation. Whether the GEA is *sound* or not depends on whether it is necessary to account for the connection by means of the notion of a multifaceted object. I cannot claim it is necessary. But I do think it was natural for Russell to think so, given his attempts to find the ‘intrinsic plausibility’ of the zigzag theory and to explain the dualistic nature of complexes.

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