Journal for the History of Analytical Philosophy

Volume 10, Number 2

ISSN: 2159-0303

jhaponline.org

Analysis, Decomposition, and Unity in Wittgenstein's Tractatus

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I argue, through appeal to the distinction between analysis and decomposition described by Dummett, that Wittgenstein employs both of those notions in the *Tractatus*. I then bring this interpretation to bear upon the issue of propositional unity, where I formulate an objection to the views of both Leonard Linksy and José Zalabardo. I show that both Linsky and Zalabardo fail to acknowledge the distinction between analysis and decomposition present in the *Tractatus,* and that they consequently mischaracterise Wittgenstein's position with respect to propositional unity.

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1. Introduction

The aim of this article is twofold. I argue for a novel interpretation of Wittgenstein's conception of analysis in the *Tractatus*, and I utilise this interpretation in order to show that a particular view of Wittgenstein's approach to the problem of unity is mistaken. According to several commentators, Wittgenstein's strategy with respect to that problem is to adopt an ontological position on which propositions are fundamental ontological postulates. This interpretation has been endorsed in the form of at least two more specific varieties. Leonard Linsky (1992) has argued that Tractarian propositions are ontologically prior to their constituents, and that Wittgenstein therefore rejects the requirement that the unity of propositions be explained. José Zalabardo (2015, 2018) argues for the more radical view that Tractarian propositions are *without* constituents, are not unities, and that the request for an explanation of the unity of a proposition is therefore inappropriate if levelled at Wittgenstein.

My reading relies upon the appreciation of a conceptual distinction between analytical procedures owed to Michael Dummett (1981a; 1981b). Dummett has argued that Frege employed two different conceptions of analysis, namely 'analysis' and 'decomposition', to separate effect. Here I shall not argue that Dummett is correct with respect to the historical claim he makes concerning Frege.[1] Rather, I argue that Dummett does indeed identify a legitimate conceptual distinction, regardless of whether Frege availed himself of that distinction. Moreover, Wittgenstein *did* employ that distinction, and that he did so is a fact which may be brought to bear on the issue of whether Linsky and Zalabardo are correct in their assessments. I shall claim that both commentators draw conclusions concerning Wittgenstein's ontological views without appreciating the distinction Wittgenstein observes between analysis and decomposition. Linsky and Zalabardo implicitly portray Wittgenstein's ontological views as expressed in passages of the *Tractatus* in which decomposition, rather than analysis, is the operative notion. By contrast, I argue that Wittgenstein's conception of an object is more accurately characterised through the examination of remarks concerned with analysis, rather than decomposition.

[1] See Bronzo (2017, 14–16), Levine (2002), Sullivan (2010), and Sluga (1975; 1977; 1980, 90–95; 1987).

2. Dummett on Analysis and Decomposition

Frege emphasises the priority of judgements over concepts more than once. Frege, for instance,[2] writes 'I start out from judgements and their contents, and not from concepts. . . I only allow the formation of concepts to proceed from judgements' (1979, 16); and

> Now I do not believe that concept-formation can precede judgement because this would presuppose the independent existence of concepts, but I think of a concept as having arisen by decomposition from a judgeable content (Frege 1980, 101).

Dummett offers an interpretation of these remarks in which the epistemological priority of judgements over concepts is consistent with the epistemological priority of words over sentences. Dummett has argued that Frege employs two different conceptions of analysis:

> We recall the distinction we have drawn between two kinds of analysis of a sentence into constituents. A sentence is constructed out of

[2] See also Frege (1979, 253).

> component words. . . This kind of analysis relates to the sense of the sentence, and the constituents of the sentence, with respect to an analysis of this kind, are just the primitive component words. . . The other kind of analysis is needed in order to determine the validity of inferences in which the sentence may be involved, and it is unnecessary, for someone to understand the sentence, that he be aware of the possibility of an analysis of this kind: in this sort of analysis, the 'constituents' into which the sentence may be analysed may be complex incomplete expressions which we form from the sentence itself by omitting some other expression or expressions from it. . . (Dummett 1981a, 65).

Elsewhere Dummett terms the former notion 'analysis' and the latter 'decomposition' (1981b, 271). Analysis consists in identifying those constituents from which a sentence has been *constructed*, and which must be grasped by a speaker if they are to understand the relevant statement. That sentences are complexes composed of constituents which are epistemologically prior to the entities they combine to form is a presupposition of the possibility of analysis. Decomposition, by contrast, involves the formation of expressions through the replacement of others by free variables. The products of decomposition are not, according to Dummett, epistemologically prior to the items from which they have been decomposed; they need not be grasped by a competent speaker in order that sentences involving them be understood. Decomposition furnishes us with *features* capable of being shared by sentences, rather than constituents of those sentences:

> The complex predicate 'ξ killed ξ' cannot be regarded as literally a *part* of the sentences in which it occurs: it is not a word or a string of words, not even a discontinuous string. There is no part in common to the sentences 'Brutus killed Brutus' and 'Cassius killed Cassius' which is not also part of the sentence 'Brutus killed Caesar': yet the predicate 'ξ killed ξ' is said to occur in the first two and not in the third. Such a complex predicate is, rather, to be regarded as a *feature* in common to the two sentences. . . (Dummett 1981a, 31, emphasis in original).

Features, Dummett holds, are not literal components, or 'parts' of the items of which they are features. Zalabardo's description of features is concurrent with that of Dummett in this respect: 'people share heights, incomes, hobbies, and character traits without being compounded from these items' (2015, 112). The features arrived at through the decomposition of sentences include, on Dummett's view, functional expressions. The sentence 'Brutus killed Caesar' may, for example, be conceived of as the value of 'Brutus killed ξ' for argument 'Caesar', the value of 'ξ killed Caesar' for argument 'Brutus', or the value of 'ξ killed ζ' for arguments 'Brutus' and 'Caesar'. We shall see, below, that these alternatives are not exhaustive. Importantly, examples abound of functions which do not figure as components of their values.[3] The number 6, for instance, is the value of the function $\zeta + \zeta$ for arguments 3 and 3, though the function $\zeta + \zeta$ does not figure as a component of the number 6.

Decomposition is a procedure necessary for the success of at least two objectives, in Dummett's view. Firstly, what Sullivan calls the 'extraction of concepts' (2004, 694) from sentences may be explained through appeal to the notion of decomposition. Frege, we have seen, says that he thinks of 'a concept as having arisen by decomposition from a judgeable content', and that 'concept-formation' cannot precede judgement. By replacing two constants of 'Brutus killed Brutus' with free variables we may arrive at the functional expression 'ξ killed ξ'. This functional expression is, we have said, a feature of every sentence which is its value; the sentence 'Brutus killed Brutus' then has at least this much in common with all other sentences which are values of the relevant function. The sentence 'Brutus killed Brutus', therefore, exhibits a feature common also to 'Cassius killed Cassius'. One need not, according to Dummett, grasp what it

[3] The canonical example is due to Frege (1979, 255) who points out that while Stockholm is the value of the function 'the capital of ζ' for the argument 'Sweden', neither the function nor argument feature as constituents of the value in question.

is that these sentences have in common in order to understand either of them. It is not necessary, in order to understand 'Brutus killed Brutus', that one be cognisant of its being the value of 'ξ killed ξ' for arguments 'Brutus' and 'Brutus'. Rather, for the understanding of a sentence it is sufficient to understand the sentence's component words and the significance of their mode of combination. If one does grasp the feature shared by these sentences, though, one has thereby identified the concept of *suicide*:

> The proposition that Cato killed Cato shows the same thing. Here, if we think of 'Cato' as replaceable at its first occurrence, then 'killing Cato' is the function; if we think of 'Cato' as replaceable at its second occurrence, then 'being killed by Cato' is the function; finally, if we think of 'Cato' as replaceable at both occurrences, then 'killing oneself' is the function (Frege 1879, 66).

Importantly, our grasp of this concept depends, in Dummett's view, upon our being able to detect commonalities shared by sentences. The extraction of concepts is posterior to the understanding of sentences. To appreciate the fact that 'Brutus killed Brutus', 'Cassius killed Cassius', and 'Caesar killed Caesar' are members of a class, namely that class of sentences which are values of the function 'ξ killed ξ', is to appreciate the fact that they express the same concept. As Sullivan says

> Each of the members of the now distinguished class of sentences (*iv*) says of some individual that he killed himself; otherwise put, in each of these sentences the concept of suicide is expressed. It is the fact that this concept is expressed in each of them that distinguishes the class. Thus anyone who comes to be able to distinguish this class for the first time can be thought of as acquiring a grasp of the distinguishing mark of the class, in this case, a grasp of the concept of suicide. . . [I]t will be a conceptual achievement to recognise that 'Cato killed Cato' belongs to a class of sentences. . . in each of which this concept is expressed (Sullivan 2004, 695).

The priority of judgements over concepts consists in the fact that one must *already* understand a sentence in order for it to be decomposed into a functional expression such that the features it possesses common to other sentences are brought to light. Although one need *not* acknowledge a sentence's belonging to a particular class which includes other sentences in order to grasp its meaning. That whole sentences are prior to concepts does not, on Dummett's view, imply that sentences are epistemologically prior to the words contained in them, for the concept expressed by a sentence is not, in the present context, to be identified with the meaning of any word(s) which combine to form that sentence.

Second, decomposing sentences into functional expressions containing free variables is necessary if we are to capture the validity of inferences involving those sentences.[4] The validity of the inference from '$a > b$' and '$b > c$' to '$a > c$', for instance, is capable of being rendered perspicuous only by construing each sentence as decomposing into a two-place, rather than one-place, functional expression, along with its arguments. Observation of this point figured prominently in Russell's (1903, 13–15; 1900, §§214–16) criticisms of subject-predicate logic, and his concern to establish a logic of relations. That one and the same sentence admits of more than one decomposition into a functional expression and argument(s) is a detail of crucial importance, to which I shall shortly return.

3. Decomposition

In the quotations given at the beginning of the preceding section, Frege describes a relation of priority in which judgeable contents stand to concepts. There, according to Dummett, he is expressly employing the notion of decomposition, rather than that of analysis. Whether or not Dummett is correct in attributing to Frege the employment, on distinct occasions, of analysis and decom-

[4] Linsky elaborates: 'The predicate 'ξ killed ξ' must be distinguished from the predicate 'ξ killed η' for the latter is the predicate which by double quantification yields 'Everyone kills someone.' This double quantification cannot arise from the former predicate' (1992, 269).

position is not my present concern. Rather, my contention is that Dummett has identified a legitimate conceptual distinction, and that this distinction may be brought to bear upon issues of *Tractatus* exegesis. I will, however, employ passages of Frege's in order to precisify the conceptual distinction I have in mind; instances in which I do so ought to be read as contributing to this effort of precisification, rather than as staking out historical claims regarding Frege. Below, I shall claim that Linsky and Zalabardo both subscribe to an implicit understanding of a Tractarian object which centrally involves the notion of decomposition, rather than that of analysis. In order to establish this claim, however, it is necessary to conceive of the conceptual distinction Dummett makes as somewhat broader in application than I have so far described it as being. In order to show that the notion of decomposition may be legitimately extended beyond application to those items Dummett is concerned with, I shall here discuss the precedent set by Frege. Frege, in at least one instance, evidently conceives of decomposition in such a way that it applies to items distinct from those Dummett discusses in the passages I have quoted. To clarify, my aim here is to derive a conceptual distinction from Frege's work which is broader in application than that discussed by Dummett. I am not concerned to defend a historical claim with respect to the issue of Frege's endorsing that distinction. My interpretation of Frege here serves a philosophical, rather than historical, purpose.

Dummett clearly takes the distinction between analysis and decomposition to concern linguistic items. It is *sentences* to which the procedures of analysis and decomposition apply, as Dummett presents the matter in the quoted passages. Frege, meanwhile, emphasises the possibility of decomposing *thoughts* in 'On Concept and Object':[5]

> . . . [A] thought can be split up in many ways, so that now one thing, now another, appears as subject or predicate. The thought itself does not yet determine what is to be regarded as the subject. If we say 'the subject of this judgement', we do not designate anything definite unless at the same time we indicate a definite kind of analysis; as a rule, we do this in connection with a definite wording. *But we must never forget that different sentences may express the same thought*. . . Language has means of presenting now one, now another, part of the thought as the subject. . . (Frege 1892, 188, emphasis added).

The presentation of a thought takes place, 'as a rule', through the employment of language, according to Frege. One and the same thought may be decomposed in numerous ways, where each decomposition of the thought in question finds its expression in a *whole* sentence. Where Dummett emphasises the possibility of decomposing sentences into further expressions containing free variables, Frege, at least here, appears more clearly to construe multiple decomposition as consisting in the presentation of a single thought by different sentences. What is common to both the decomposition of a single sentence into functional expression and argument(s), and of a Fregean thought into multiple sentences employing, for instance, either passive or active voice, is that in both cases the products of decomposition are not conceived of as representing a wholly faithful route to the inner metaphysical structure of the items from which they have been decomposed.[6] The aim of decomposition is not, in either case, that of revealing the metaphysical composition of a thing. As we have seen, the value of decomposition lies in its facilitating the extraction of concepts from sentences, and the systematisation of logical inferences. The work of unveiling metaphysical structure is, on the present view, carried out by analysis.

Both instances of decomposition, namely that of a single sentence into different expressions containing free variables, and

[5] I am not here claiming that Frege does not conceive of decomposition as applicable to sentences at all, but only that his concerns are clearly otherwise in the passage quoted. Indeed, we will see below that Frege does appear to conceive of sentences as admitting of multiple decomposition.

[6] Frege writes, 'We should mention that, strictly speaking, it is not in itself that a thought is singular, but only with respect to a possible way of analysing it' (1979, 187).

that of a Fregean thought into multiple complete sentences, contribute to our achieving the same ambition. I have already alluded to the necessity of decomposing relational statements into functional expressions of two arguments for the illumination of valid inferences involving those sentences. The decomposition of a thought into more than one complete sentence likewise aids us in the acknowledgment of inferential patterns; for, depending on whether a sentence is voiced passively or actively, we shall be encouraged to decompose it into one functional expression and argument(s) or another. Assume, for example, that both 'Socrates is mortal' and 'Mortality is instantiated by Socrates' express the same Fregean thought. The thought in question possesses both the feature of *being capable of being expressed by the sentence 'Socrates is mortal'*, and that of *being capable of being expressed by the sentence 'Mortality is instantiated by Socrates'*. Vitally, each sentence into which the relevant thought might be decomposed encourages us to view it as the value of a different function for an argument. The sentence 'Socrates is mortal' is naturally conceived of as the value of the first-level function 'ξ is mortal' for the argument 'Socrates'. The sentence 'Mortality is instantiated by Socrates', by comparison, suggests its being conceived of as a value of the second-level function 'ζ is instantiated by Socrates' for the (first-level functional) argument 'ξ is mortal'. Conceiving of these two different sentences as values of different functions for different arguments enables us to extract different concepts from them through recognition of their each belonging to classes of sentences of which being the value of one or other of the relevant functions is the defining feature, as well as to acknowledge different inferential relations in which each sentence may stand. Frege describes the change of active to passive voice as relevant to our choice of functional analysis in his *Begriffsschrift*:

> The subject [of a proposition] is usually intended by the speaker to be the principal argument; the next most important often appears as the object. Through the choice of [grammatical] forms such as active and passive. . . ordinary language has the freedom of allowing whatever part of the proposition it wishes to appear as the principal argument, a freedom, however, that is limited by the paucity of words (Frege 1879, 68).

Although Frege does not, in the *Begriffsschrift*, draw a distinction between the sense and reference of an expression, he does conceive of the shift from active to passive voice as influencing our choice of decomposition. Consequently, where Frege later draws the distinction between sense and reference, and where he conceives of thoughts as expressible by distinct sentences employing either active or passive voice, we may conclude that the difference in voice between two sentences expressing the same thought contributes to our choice of decomposition with respect to either sentence. The example of decomposition Frege describes, namely that of a single thought into distinct complete sentences, may therefore be viewed as preparatory for, and contributory to, that described by Dummett, namely that of a single sentence into expressions containing free variables. We may therefore conjoin both examples into a process in which decomposition is performed *twice*: first on a thought in order to produce a sentence expressing it, and second on the sentence in question in order to produce functional expressions containing free variables. Dummett's discussion of the decomposition of a sentence takes place downstream, as it were, of Frege's own example in 'On Concept and Object'.

What the foregoing considerations suggest is that distinguishing between analysis and decomposition does not, in and of itself, determine the items to which those instruments apply. In other words, we need not, merely in virtue of adopting the relevant distinction, proceed under the impression that we cannot analyse or decompose items other than linguistic ones.[7] In my view we may intelligibly extend application of both analysis

[7] Levine argues that Russell's conception of function-argument analysis, discussed below, constitutes a commitment to decomposition. If Levine is correct, the issue of whether Russellian decomposition operates with linguistic entities or not is thorny indeed. The status of propositional functions in Russell

and decomposition beyond Dummett's and Frege's examples to items which are neither linguistic entities nor Fregean senses. Analysis, as we have seen, constitutes an investigation into the components from which an item is compounded; anything with components, therefore, admits of analysis. Similarly, anything which possesses features is thereby capable of being decomposed. Insofar as being the value of $\Phi(x, \ldots\ x_n)$ for some argument(s) is a feature of any item at all, any item whatsoever admits of decomposition. I will, in later sections, argue that Wittgenstein conceives of facts, including those facts which count as propositions, as capable of being decomposed. Furthermore, I shall claim that the commentators mentioned at the outset of this article train their attention on those passages of the *Tractatus* in which Wittgenstein discusses decomposition, to the exclusion of those in which he is more clearly concerned with analysis. Moreover, this partial treatment of the text results in a mistaken characterisation of Tractarian objects.

4. Multiple Decomposition

We have just seen that in Frege's view the decomposition of one and the same thought into distinct sentences encourages our decomposing those sentences in ways different from one another. Employing a certain grammatical form may, according to Frege, aid one in the cerebral feat of construing a sentence as the value of a particular function for an argument. Frege describes an application of decomposition which is of psychological assistance to the effort of grasping a thought's inferential relations. Through application of greater mental effort, though, we are free to decompose any sentence expressing a thought into functional expression and argument(s) more obviously suggested by an alternative expression of the very same thought. In other words, we may legitimately proceed to ignore the recommendation of surface grammar, and decompose 'Socrates is mortal' into the second-level functional expression 'ζ is instantiated by Socrates' and (first-level functional) argument 'ξ is mortal'. In the *Begriffsschrift* Frege offers the following example:

> Indeterminate functions of several arguments are expressed in a corresponding way.
>
> $$\vdash \Phi(A)$$
>
> can be read: 'A has property Φ'
>
> $$\vdash \Psi(A, B)$$
>
> may be translated as 'B stands in the Ψ-relation to A' or 'B is the result of an application of the procedure Ψ to the object A'.
>
> Since the symbol Φ occurs in the expression $\Phi(A)$ and can be thought of as replaced by other symbols Ψ, X, by means of which other functions of the argument A are then expressed, $\Phi(A)$ *can be regarded as a function of the argument* Φ (Frege 1879, 69, emphasis in original).

Frege here insists that one and the same sentence, in this case namely '$\Phi(A)$', may be decomposed in different ways, depending on which function the sentence is construed of as a value of. Frege describes an additional example:

> Consider now the example: 'the circumstance that the centre of mass of the solar system has no acceleration, if only internal forces act on the solar system'. Here 'solar system' occurs in two places. We can therefore take this as a function of the argument 'solar system' in different ways, depending on whether we think of 'solar system' as replaceable at its first occurrence or at its second or at both (but in the last case by the same argument both times). These three functions are all different (Frege 1879, 66).

We have, in these two separate examples, quite distinct instances of a sentence being multiply decomposed. In the first example,

is a question which has engendered great controversy, and I remain neutral on it for present purposes. See Quine (1967, 151–52), in contrast with Stevens (2005, 81–89).

one and the same sentence is shown to be capable of being decomposed into either: *i*) the functional expression '$\Phi(x)$' and argument 'A', *or*: *ii*) the functional expression '$\Phi(A)$' for argument 'Φ'. In the case of *i*) the functional expression '$\Phi(x)$' may be conceived of as first-level, while in the case of *ii*) the functional expression '$\Phi(A)$' may be conceived of as second-level. The different decompositions of '$\Phi(A)$' therefore involve the construal of that sentence as the value of functions differing in level.

In the second case, namely that of the sentence 'the circumstance that the centre of mass of the solar system has no acceleration, if only internal forces act on the solar system', Frege describes the available options for decomposition differently. Here Frege describes the relevant sentence as the value of three different functions, where each of these functions is of the *same* level, though two are functions of one argument and the other is a function of two arguments. In other words, the options Frege describes for decomposing the relevant sentence do not include functional expressions of a level higher than one. Where, in Section 2, I mentioned the necessity of decomposing relational statements into functions of two arguments rather than one if the validity of certain inferences involving them is to be captured, I was describing options for decomposing relational sentences in a way comparable to that Frege describes in the present case, rather than that discussed in relation to the sentence '$\Phi(A)$'.

Crucially, sentences *always* admit of multiple possibilities for decomposition in which they are conceived of as the value of functional expressions of *differing* level. Whether or not sentences always admit of multiple possibilities for decomposition in which they are conceived of as the value of functional expressions differing in *adicity* is a separate question. I shall not answer that further question here.[8]

[8]An answer to this question conceivably involves deciding whether the copula in simple subject-predicate sentences may function as indicating a relation. This question was of continual interest to Russell, though a thorough examination of his views will take us too far from the present issue.

Dummett focuses his attention on the case of decomposition in which a sentence is conceived of as the value of distinct functions of different level:

> Now, with respect to an analysis of the second type, it is indeed true that, on Frege's own principles, we must admit not only of the analysis of 'Socrates is wise' as resulting from putting the proper name 'Socrates' in the argument-place of the first-level predicate 'ξ is wise', but also the analysis of it as resulting from putting the first-level predicate 'ξ is wise' in the argument place of the second-level predicate 'Φ(Socrates)' (Dummett 1981a, 65).

Sentences may, as we have seen, be decomposed in multiple ways. We may decompose 'Socrates is wise' into either the proper name 'Socrates' and the first-level predicate 'ξ is wise', or the first-level predicate 'ξ is wise' and the second-level predicate 'ζ is instantiated by Socrates'.[9] Decomposing the sentence in the first way, but not the second, facilitates our grasping the validity of the inference from 'Socrates is wise' to 'something is wise'. Decomposing the sentence in the second way, but not the first, facilitates our grasping the validity of the inference from 'Socrates is wise' to 'There is something Socrates is'. We may extend this view to the case of facts. Given the fact *that Socrates is mortal*, we might decompose it into the feature *Socrates is* φ, and the first-level feature ξ *is mortal*. We might also decompose the relevant fact into the first-level feature ξ *is mortal*, and the second-level feature ζ *is instantiated by Socrates*. The decomposition of an item may proceed in various ways. Moreover, no one decomposition is privileged over any other. Rather, our choice of decomposition with respect to a given sentence is determined by which inference involving that sentence we are concerned to

[9]See Bronzo (2017, 4). Hodes (1982, 167–68) has argued that infinitely many decompositions are possible for any given sentence. MacBride (2005a, 15–16) points out that, owing to Montague's (1965) demonstration of the reduction of third and higher-order logic to second-order logic, the postulation of an infinite hierarchy need not follow from the mere possibility of more than one decomposition for a given item.

shed light on. Our choice of decomposition with respect to any item at all is determined by the features possessed by that item in which we are interested. There are not, by contrast, multiple *analyses* of an item which are all equally accurate. An analysis revealing more of the constituents from which an item is compounded than another is *thereby* more accurate than that which reveals fewer of those constituents; and an analysis revealing all of the constituents from which an item is compounded is necessarily identical to any analysis of equal accuracy.

5. Tractarian Analysis

In this section I show that Wittgenstein was committed to a conception of analysis according to which the possibility of analysis presupposes the possession of constituents by the item analysed. I draw the conclusion, therefore, that in Wittgenstein's view propositions do possess constituents and are composite. This conclusion will be important for my discussion of Zalabardo's opposing view, namely that Tractarian propositions are *simple*.

Wittgenstein writes

> A proposition has one and only one complete analysis (TLP 3.25).

Given what has so far been said, what remark 3.25 demonstrates is that Wittgenstein's conception of analysis in remark 3.25 of the *Tractatus* is *not* that of decomposition. This is a crucial detail to notice, for it serves to dispel the impression that Wittgenstein conceived of the analysis of propositions as consisting in the discovery of features, where features are understood as the products of decomposition. Application of decomposition to a proposition cannot secure for it a unique analysis, for any item may, as we have seen, be multiply decomposed into functional expressions of different levels. We must therefore look elsewhere to identify the conception of analysis Wittgenstein adopts in 3.25. We saw, above, that Dummett describes two different analytical procedures: analysis and decomposition. We have just eliminated decomposition from counting as the analytical procedure employed in 3.25. On the assumption that Dummett's bipartite classification exhausts the plausible available options, the remaining candidate notion with which 3.25 might be claimed to operate is that of analysis. Analysis, recall, consists in an investigation into the constituent items out of which a complex entity is composed. In other words, that the conception of analysis with which 3.25 operates is not that of decomposition suggests that Wittgenstein held that propositions possess constituents. Wittgenstein held that propositions can be analysed, in a sense of the word 'analyse' relevantly similar to that intended by Dummett,[10] and *that* Wittgenstein held to this position suffices also to show that he conceived of propositions as composite.

It might at this stage be objected that I have concluded more from 3.25 than the remark in question can support.[11] It has been an implicit assumption of my interpretation that 3.25 concerns elementary, rather than non-elementary, propositions. I have therefore held that what 3.25 asserts is the possibility of analysing elementary propositions into their constituents. Remark 3.25 has, however, been the subject of an alternative interpretation:

> The core tenets of Wittgenstein's logical atomism may be stated as follows: (i) Every proposition has a unique final analysis which reveals it to be a truth-function of elementary propositions (TLP 3.25...)... (Proops 2017a).

Proops here claims that 3.25 concerns the analysis of a molecular, or non-elementary,[12] proposition into a form which reveals its

[10] Where the relevant issue is the *composite* nature of propositions. In other words, I do not maintain that in each and every possible respect Wittgenstein's conception of analysis is identical to that which Dummett ascribes to Frege. I do maintain, though, that Wittgenstein's conception of analysis, like the Dummettian Frege's, presupposes the complexity of what is analysed.

[11] Thanks to Tom Smith for bringing this to my attention.

[12] Here and elsewhere I use the terms 'molecular' and 'non-elementary' interchangeably.

truth-functional structure. If Proops' reading is correct, we need not ascribe to Wittgenstein the view that *elementary* propositions admit of unique analysis, and we need not therefore draw the conclusion that elementary propositions are, in Wittgenstein's view, composite.

In defence of my contention that 3.25 concerns elementary propositions rather than molecular ones is the following consideration: immediately following 3.25, in a remark which, according to the numbering system of the *Tractatus*, is a direct comment on 3.25, Wittgenstein writes:

> What a proposition expresses it expresses in a determinate manner, which can be set out clearly... (TLP 3.251).

We must assume, on the basis that 3.251 comments on 3.25, that those propositions which admit of 'one and only one' complete analysis are the same propositions which express what they express in a 'determinate manner'. If it can be established that propositions which express what they express in a determinate manner must be elementary, it will follow that those propositions which admit of a unique analysis are likewise elementary, for 3.25 and 3.251 must discuss the same items.

Wittgenstein makes the connection between 'determinacy' and elementary propositions in 3.23:

> The requirement that simple signs be possible is the requirement that sense be determinate (TLP 3.23).

The requirement that simple signs be possible is simultaneously the requirement that there be elementary propositions, for elementary propositions are 'concatenations' of simple signs (TLP 4.22). Anscombe, in an influential discussion, gives the following description of what it is for something to be 'determinate' in this context: '*Elementary propositions are such that for them there are no two ways of being true or false but only one*' (1959, 34, emphasis in original). Anscombe contrasts elementary propositions with claims involving definite descriptions:

> One kind of indefiniteness in a proposition might be that there was more than one way of its being false: the complex might exist, but what was said of it might not hold; or the complex might not exist (Anscombe 1959, 34).

A proposition involving a definite description does not 'settle' everything (1959, 34), for knowledge that the proposition in question is false leaves it open just *how* it is false. Elementary propositions contrast with those involving definite descriptions insofar as the former but not the latter may not be true or false in more than one way. An elementary proposition's determinacy[13] consists in its not admitting of multiple ways of being true or false. Wittgenstein, immediately prior to 3.25, describes propositions whose elements signify complex items as indeterminate: 'In such cases we *know* that the proposition leaves something undetermined' (TLP 3.24, emphasis in original). Propositions whose elements signify complex items are not elementary, in Wittgenstein's view, but molecular.[14] Molecular propositions in general exhibit indeterminacy of the relevant kind, for to know that '$p \vee q$' is true is not in and of itself to know *what is the case*.

A natural question here is that of why, in Wittgenstein's view, there must be determinate propositions. In other words, why must there be propositions of which a grasp of their truth is a *direct* route to a grasp of what is the case? Propositions, Wittgenstein says, *show* what is the case if they are true (TLP 4.022). There is nothing more to understanding a proposition than knowing what is the case if it is true (TLP 4.024).[15] Crucially, there must be determinate propositions, for if there were not, we should have to go and *find out* what is the case when a given proposition is true through some means which do not appeal to the proposi-

[13] Anscombe, following Ogden, translates '*Unbestimmtheit*' as 'indefiniteness'. I have followed Pears and McGuinness in speaking of 'determinacy' rather than 'definiteness'.

[14] Recalling here that descriptive, quantified propositions reduce to molecular truth-functional combinations, on Wittgenstein's view; see TLP 5.52.

[15] See Wittgenstein (1961b, 93–94).

tion itself. What could 'finding out' here involve, if appeal to the proposition itself is ruled out on grounds of indeterminacy? On the assumption that 'p' and 'q' are determinate, finding out what is the case if '$p \vee q$' is true involves adverting to what is determinately shown by 'p', and determinately shown by 'q'. It is unclear how it could be established what is the case if a proposition is true, where the proposition itself does not settle the matter, and where no further proposition figuring in an analysis of the original contributes to our settling it. If the sense of a proposition is no guide to what is the case if it is true, it is hard to imagine what else could serve as such a guide. Crucially, in the case of ineliminable indeterminacy the connection between a proposition's sense and what is the case if it is true is severed, for in that case the sense of the proposition doesn't determine what is the case if it is true. Wittgenstein's conception of sense, though, makes essential appeal to the notion of understanding what is the case if a proposition is true. For any proposition of which neither it nor any of the propositions figuring in its analysis is determinate, the sense of that proposition cannot be a route to what is the case if it is true. Sense, though, just is the route to what is the case if a proposition is true, on Wittgenstein's view. Accordingly, we must conclude that no proposition of the kind just described possesses a sense at all. In other words, ineliminable indeterminacy collapses into meaninglessness, on the conception of sense Wittgenstein subscribes to. Indeed, indeterminacy at the molecular level is only possible because there is determinacy at the elementary one. On the assumption that Anscombe is correct in her assessment of what Wittgenstein means by '*Unbestimmtheit*', we must conclude that those propositions described as determinate by 3.251 are elementary, and that therefore 3.25 likewise concerns elementary propositions exclusively.

A further objection which may be raised against the reading of 3.25 I propose is as follows.[16] Elementary propositions are the *end* result of the analysis of non-elementary ones. Consequently, there is nothing left to analyse once we have arrived at elementary propositions. This objection is supported by Wittgenstein's saying that an elementary proposition is 'completely analysed' (TLP 3.201). That no further analysis of elementary propositions may take place is, it is argued, further supported by the consideration that elementary propositions are 'concatenations of names' (TLP 4.22), and that 'names cannot be dissected any further' (TLP 3.26). Winch, for instance, expresses this view where he says, on the basis of an appreciation of Ishiguro's discussion of the context principle, that 'one cannot further analyse an elementary proposition by splitting it up into its names' (1969b, 8). One cannot, according to Winch, analyse a proposition by splitting it up into its names for, as Ishiguro (1969, 20–50) argues, there are no such things as names conceived of as capable of treatment independent of the propositions in which they occur. To respond here requires that we identify a kind of analysis which does not entail analysing the names of an elementary proposition, nor 'splitting' a proposition into its names, such that those names appear, as it were, free-floating. Wittgenstein hints at such a conception of analysis in TLP 5.55 and 5.557:

> We now have to answer a priori the question about all the possible forms of elementary propositions.
> Elementary propositions consist of names. Since, however, we are unable to give the number of names with different meanings, we are also unable to give the composition of elementary propositions (TLP 5.55).

> The application of logic decides what elementary propositions there are... (TLP 5.557).

Here Wittgenstein talks of the difficulty involved in giving the *form* of elementary propositions, as well as that of giving the *composition* of elementary propositions *a priori*. Wittgenstein says, though, that the *application of logic* decides what elementary propositions there are. Johnston writes:

[16] My thanks to an anonymous referee for raising this point.

> Wittgenstein asserts that we can know nothing a priori about the forms of atomic propositions, that the unbiased logician will not be concerned to make any a priori distinction between different types of things. This leaves open the possibility of a posteriori knowledge of atomic forms. Indeed, Wittgenstein's Tractarian claim that what elementary propositions there are is decided by the application of logic (. . . TLP 5.557), and his 1929 claim that '[o]nly when we analyse phenomena logically shall we know what form elementary propositions have' ([Wittgenstein 1979, 42]), would appear to endorse this possibility as genuine (Johnston 2009, 158–59).

Johnston interprets 5.557 as suggesting that in Wittgenstein's view the form of an elementary proposition is something discoverable only *a posteriori*. MacBride directs us to the kind of inquiry Wittgenstein felt must be carried out in order that the forms of propositions be revealed:

> What Wittgenstein meant was that only analysis of what we say about the world as we find it will lead us to the logical forms of the elementary propositions, what Wittgenstein later described as 'the logical investigation of the phenomena themselves, i.e., in a certain sense *a posteriori*, and not by conjecturing about *a priori* possibilities'. . . The only epistemological access to the logical forms of elementary propositions available to us is via the analysis of what we say about the world (truly or falsely) (MacBride 2018, 197).

Discovering an elementary proposition's form constitutes an analysis of that proposition. The form of a proposition is the possibility of its structure (TLP 2.15); and the structure of a proposition is, in turn, the 'connexion of its elements' (TLP 2.15). Discovering the form of an elementary proposition, therefore, involves grasping the possibility of that proposition's structure through appeal to the combinatorial capabilities possessed by its elements.[17] Such an inquiry neither attempts to 'dissect' constituent names into further components, nor does it attempt to treat those names as capable of appearing independently of propositions. Grasping the combinatorial capabilities of some propositional elements involves unearthing their forms, and doing *this* involves attending to the ways in which those elements may or may not significantly combine with others *in propositions*.[18] Elementary propositions will have a unique analysis, on this reading, for they do not possess more than one form. The form of a proposition depends upon the forms of its constituents; what a proposition is capable of representing depends upon the combinatorial potential of its elements. The possibility of an analysis revealing the form of a proposition, therefore, presupposes the possession of constituents capable of combining with others. What analysis consists in, on this view, is an inquiry into the combinatorial potential possessed by some constituent(s), such that the character of that potential be more explicitly brought into view, and the form of the proposition whose constituents they are be determined. The analysis of a proposition involves, on this conception, commitment to propositional constituents conceived of as prior to the form of the proposition itself, for the latter depends upon the former. Accordingly, we may not construe propositional elements as mere features of propositions, where such a construal serves to withhold from a propositional element the status of a genuine ontological commitment. If propositional elements were nothing more than posterior extractions from propositions, the attempt to grasp a proposition's form *through appeal to* the forms of those items extracted from it would clearly be circular, for those extractions in turn depend entirely upon the character of the whole from which they have been extracted.

In summary, then, we may not construe propositional elements as features for two reasons: (i) decomposition of a proposition into features cannot secure for propositions the unique analysis Wittgenstein asserts is possible, and (ii) conceiving of

[17] Campbell writes: 'Now, the structure of an elementary proposition is the way in which its elements, names of simple objects, are combined, and so the possibility of its structure is inseparable from the forms of those names, that is, from their respective ranges of possible combination with other names' (2014, 143).

[18] See Wittgenstein (1961a, 70) for an example of just such an exercise.

propositional elements as features prohibits us from appreciating the way in which the form of a proposition depends upon the forms of its component parts. Note that none of this is to deny that there is a kind of analysis Wittgenstein describes according to which analysis reveals the truth-functional structure of a molecular proposition. Rather, my aim is to draw our attention to a separate activity, the possibility of which presupposes that propositions are composite items.

6. Tractarian Decomposition

I have, throughout the preceding discussion, repeatedly claimed that remark 3.25 does not concern the notion of decomposition, and that it must therefore constitute commitment to analysis, where the possibility of analysis presupposes the possession of constituents by propositions. I have concluded from these observations that elementary propositions are composite, rather than indivisible. I have not, however, claimed that *nowhere* in the *Tractatus* does Wittgenstein employ decomposition. Here I shall argue that Wittgenstein did, indeed, operate with that notion, and that Wittgenstein distinguishes between items discovered by decomposition and those revealed through analysis. In the sections which follow I draw the conclusion that those who have inferred from Wittgenstein's remarks concerning items discovered by decomposition claims about the constituent names of a proposition have failed to observe the distinction between analysis and decomposition here described.

Wittgenstein expresses a commitment to the possibility of decomposition in the following remarks:

> An expression presupposes the forms of all the propositions in which it can occur. It is the common characteristic mark of a class of propositions (TLP 3.311).
>
> Thus an expression is presented by means of a variable whose values are the propositions that contain the expression.
>
> (In the limiting case the variable becomes a constant, the expression becomes a proposition.)
> I call such a variable a 'propositional variable' (TLP 3.313).
>
> If we turn a constituent of a proposition into a variable, there is a class of propositions all of which are values of the resulting variable proposition. In general, this class too will be dependent on the meaning that our arbitrary conventions have given to parts of the original proposition. But if all the signs in it that have arbitrarily determined meanings are turned into variables, we shall still get a class of this kind. This one, however, is not dependent on any convention, but solely on the nature of the proposition. It corresponds to a logical form—a logical prototype (TLP 3.315).
>
> To stipulate values for a propositional variable is *to give the propositions* whose common characteristic the variable is... (TLP 3.317, emphasis in original).

Propositional variables are arrived at through decomposition. Wittgenstein identifies propositional variables as 'the common characteristic mark of a class of propositions', as well as those things which propositions may share with one another. Wittgenstein here describes propositional variables as 'expressions'. Crucially, 'expression' is a broad term in the *Tractatus* referring to both propositional variables and names (TLP 3.31).[19] Propositional variables are the products of decomposition, while names, on this reading, are those items investigated by analysis.

Given the sentence 'aRb' we may replace the name 'a' with a variable in order to form the propositional variable 'ξRb'. That the propositional variable 'ξRb' is conceived of as functional is shown by Wittgenstein's claiming that it has values, and that those values for appropriate arguments are propositions. We may replace the remaining constants of 'ξRb' through successively exchanging them for variables, arriving at '$\xi R \zeta$', and finally '$\xi \Psi \zeta$', where this last expression corresponds to a logi-

[19] My thanks to an anonymous referee for clarifying this point.

cal form. Each variable expression is a 'common characteristic mark of a class of propositions' (TLP 3.311). By this Wittgenstein cannot mean that each expression containing a free variable is literally a *constituent*, or component, of each proposition which is its value, for what distinguishes these functional expressions from their values is precisely that the former include Greek letters while the latter do not. Wittgenstein cannot therefore hold that the values of propositional variables *contain* those variable expressions, in anything other than a figurative sense of 'contain'. Rather, propositional variables may be more accurately described as features which their values have in common, inasmuch as it is a feature common to both '30+2 = 32' and '4+2 = 6' that they are both possible values for the function expressed by '$\xi + 2 = \zeta$'. Wittgenstein says, 'The propositional variable signifies the formal concept' (TLP 4.127), and:

> . . . The expression for a formal property is a feature of certain symbols.
> So the sign for the characteristic of a formal concept is a distinctive feature of all symbols whose meanings fall under the concept. . . (TLP 4.126).

Wittgenstein here clearly describes functional expressions as *features* of their values. Of features Wittgenstein says, 'An internal property of a fact can also be called a feature of that fact (in the sense in which we speak of facial features, for example)' (TLP 4.1221). Facial features, such as the weight of one's brow, or the luminescence of one's eyes, are not naturally construed as literal *constituents* from which a face is compounded. Rather, facial features are characteristics which may be shared or inherited, in contrast to one's literal components, which are neither shared nor inherited. Functional expressions therefore are features of their values, according to Wittgenstein, without thereby counting as constituents of them. It should be clear that Wittgenstein, throughout these remarks, describes a process satisfying the description of decomposition we saw given by Dummett, above.

That propositional variables and names are not identical can be gathered from Wittgenstein's insistence that names are *simple signs*: 'The simple signs employed in propositions are called names' (TLP 3.202). Wittgenstein, moreover, says 'Names are the simple symbols: I indicate them by single letters ("x", "y", "z")' (TLP 4.24). Propositional variables produced by replacing elements of propositions with free variables, however, are plainly not simple. The expression 'ξRξ', for instance, contains *two* Greek letters. That 'ξRξ' contains two Greek letters indicates a functional expression which outputs a value for two arguments.[20] Above, I alluded to the importance of this feature of functional expressions for Russell. Recall, it was Russell's view that relational statements must be conceived of as values of functions for two arguments if the validity of inferences involving those statements is to be captured. That the functional expressions into which sentences decompose are not simple is essential if the effort of systematising inferential patterns is to succeed. Furthermore, Wittgenstein says that the limiting case of decomposition is an expression 'corresponding' to a logical form. Expressions composed entirely of free variables do not, however, *name* logical forms, for logical forms cannot, in Wittgenstein's view, be named:[21] 'There is no *thing* which is the *form* of a proposition, and no name which is the name of a form' (1961b, 99, emphasis in original). From what I have said so far, it should be clear that expressions containing free variables are not indicated by *single* letters. Rather, functional expressions are indicated by complex symbols, where the complexity of these expressions serves to facilitate recognition of the inferential relations holding between their values.

Wittgenstein stakes out a vital role for propositional variables in the following passage:

[20] Hence Dummett's description of these expressions as *complex* predicates (1981a, 33).

[21] See also TLP 4.12–4.121.

> When a bracketed expression has propositions as its terms—and the order of the terms inside the brackets is indifferent—then I indicate it by a sign of the form '$(\overline{\xi})$'. 'ξ' is a variable whose values are terms of the bracketed expression and the bar over the variable indicates that it is the representative of all its values in the brackets. (E.g. if ξ has the three values P, Q, R, then $(\overline{\xi}) = (P, Q, R)$.)
> What the values of the variable are is something that is stipulated. The stipulation is a description of the propositions that have the variable as their representative.
> How the description of the terms of the bracketed expression is produced is not essential.
> We *can* distinguish three kinds of description: 1. direct enumeration, in which case we can simply substitute for the variable the constants that are its values; 2. giving a function fx whose values for all values of x are the propositions to be described; 3. giving a formal law that governs the construction of the propositions, in which case the bracketed expression has as its members all the terms of a series of forms (TLP 5.501).

Infinitely many operands may potentially be inputted to Wittgenstein's 'operator N' simultaneously; and for instances in which the number of operands exceeds our capacity to enumerate them, they may be described, according to Wittgenstein, through the presentation of a functional expression of which those operands are values.[22] In other words, the presentation of a defining feature of the relevant class of operands may determine the items upon which we should like to perform the N operation. Such functional expressions therefore play an important role in the reduction of propositions involving quantifiers to those whose only 'logical constant' is the N operator.

[22]Connelly clarifies Wittgenstein's position: 'The inputs, or arguments, to the operation N would thus be the various distinct outputs, or values, which result when each of these individual constants is independently substituted in for "x" in "fx". Importantly, here already it should be clear that what ultimately ends up as an argument to the N operator is not an open sentence which contains a [free] variable, but rather a proposition which results from *replacing* a variable with an individual constant' (2017, 3, emphasis in original). Connelly's description of Wittgenstein's view concords with my claim, below, that no Tractarian proposition contains free variables.

The decomposition of one and the same proposition into different propositional variables may be carried out in order to make perspicuous certain inferences. Symbolising 'Socrates is mortal' as 'Fa', we may perform decomposition in order to arrive at the propositional variable 'Fx'. The variable 'Fx' may be employed in order to determine a collection of sentences in which mortality is ascribed to an item, such that the N operator may be applied to them. The sentence '$N(Fx)$' therefore will, on Wittgenstein's view, be equivalent to '$(\forall x) \sim Fx$', and '$N(N(Fx)$' will be equivalent to '$(\exists x)Fx$'. The inference from 'Socrates is mortal' to 'something is mortal' is therefore capable of expression through use of the propositional variable 'Fx' to determine those operands to which the N operator may be applied. The sentence 'Fa' may also be decomposed into the functional expression 'ξa', which may be translated as 'ξ is true of Socrates'. The expression 'ξa' is a second-level functional expression which determines the collection of sentences in which first-order properties are ascribed to Socrates. The sentence '$N(N(\xi a)$' is equivalent to asserting Socrates' possession of every first-order property. From '$N(N(\xi a)$', 'Fa' follows. It is a *desideratum* on Wittgenstein's formal language that it be at least capable of being employed in such a way as to make the validity of the relevant inference plain. The decomposition of one and the same sentence into different propositional variables is here, as it was in the case of Dummett's Frege, vital to the exercise of illuminating inferences.

That propositional variables are not identical to names may be concluded from the following consideration. Wittgenstein famously remarks that 'only in the context of a proposition does a name have meaning' (TLP 3.3),[23] as well as claiming, 'The name occurs in the proposition only in the context of the elementary proposition' (TLP 4.23). The expression '$\xi \mathrm{R} \zeta$', though, does *not* appear in any proposition. The expression '$\xi \mathrm{R} \zeta$' does not appear in any proposition because it contains free variables, and no

[23]Here I quote from the C. K. Ogden translation of the *Tractatus*.

proposition, in Wittgenstein's view, contains free variables. That no proposition contains free variables was a point insisted upon by Wittgenstein in his criticism of Russell's chosen formulation of the axiom of reducibility:

> Your axiom of reducibility is
> $\vdash : (\exists f) : \varphi x \equiv_x f!x$;
> now is this not all nonsense as this proposition has only then a meaning if we can turn the φ into an apparent variable. For if we cannot do so no general laws can ever follow from your axiom. The whole axiom seems to me at present a mere juggling trick. Do let me know if there is more in it. The axiom as you have put it is only a schema and the real Pp ought to be
> $\vdash : .(\varphi) : (\exists f) : \varphi(x) \equiv_x f!x$,
> and where would be the use of that?— (Wittgenstein 1961c, 122).

Russell's formulation of the axiom of reducibility is, Wittgenstein suggests, merely *schematic*; it does not *say* anything at all because it is an open sentence awaiting either the replacement of a free variable by a constant, or the binding of that variable to a quantifier. The relevant formulation is, consequently, neither true nor false. Indeed, Wittgenstein asks if it is not all nonsense. The expression '$\xi R\zeta$' is not a name because names have meaning only in the context of a proposition, and '$\xi R\zeta$', owing to its containing free variables, is incapable of appearing in a proposition.

We have seen that remark 3.25 expresses a commitment to analysis, rather than decomposition, but that Wittgenstein clearly does employ the latter notion elsewhere in the *Tractatus*. Analysis of a proposition consists in discovering the forms of its constituent names, while decomposition delivers functional expressions more accurately characterised as features of their values.[24]

[24]McGinn fails to appreciate this distinction where she writes, 'I suggested earlier that we should understand Wittgenstein as holding that propositions contain two kinds of names: names of the form "*a*", "*b*", "*c*", and so on and functions of these ("Fx", "xRy", *etc.*)' (2006, 121); see also McGinn (2006, 115). McGuinness, by contrast, denies that functional expressions are names; see (1956, 72–73).

Remarks of Wittgenstein's to the effect that we may derive functional expressions from propositions through the replacement of constants with free variables, and that such expressions indicate shared features rather than constituents proper, must not therefore be taken as evidence for the conclusion that the *names* of the *Tractatus* likewise indicate the presence of features. It is a mistake to argue, on the basis of Wittgenstein's remarks concerning propositional variables, that the objects to which names refer are not constituents of facts, and that facts therefore entirely lack constituents. This mistake consists in both conflating names with propositional variables, and, concomitantly, failing to acknowledge the distinction between analysis and decomposition. A potential challenge to this view, on which propositional variables do not include names, is raised by the following remark:

> An expression has meaning only in a proposition. All variables can be construed as propositional variables. (Even variable names) (TLP 3.314).

The objection to my view consists in the suggestion that names, according to 3.314, *are* propositional variables.[25] In response I would like to point out that Wittgenstein's concern in 3.314 is to insist that every variable may be construed of as propositional, even variable names. What it means for a variable to be propositional in this context is for it to determine a range of propositions which it commonly figures in. Propositional functions, the result of replacing a name by a Greek letter, clearly determine a range of propositions which are its values for different arguments. Names may also be viewed as common elements to a number of different propositions. Insofar as a name is a common component of several items, we can view that name as a propositional variable determining a range of propositions. This does not entail, however, that names are to be identified with compound signs which include Greek letters unbound by quantifiers. There is, in other

[25]My thanks to an anonymous referee for raising this point.

words, a sense in which names may be construed of as propositional variables, but the sense in which they may be so construed does not simultaneously license the identification of functional expressions containing free variables with names. Functional expressions including free variables, unlike names, do not literally occur in propositions. Here I draw from MacBride who expresses the point as follows:

> The expression 'xRb' enables us to collect all the propositions 'aRb', 'bRb', 'cRb', etc. because 'xRb' is the form common to them all. In this way, expressions like 'xRb' enable us to grasp the distinctive range of propositions about which we wish to make an assertion. Of course a name can perform this role too. A name can be used to collect together the range of propositions that result from combining it with other names. But some expressions won't be names because their identification isn't a feature of our picturing practice. By contrast to names, we don't rely upon expressions to picture or model the logical multiplicity of facts; expressions are merely the rest of a proposition in which a name occurs. The expression 'xRb' is the result of a semantic subtraction, (e.g.) the propositional sign 'aRb' minus the name 'a'. We don't identify 'xRb' as a constituent of 'aRb' when we make use of this propositional sign to model the fact that aRb—rather we identify '*a*', '*b*' and the relation we make between '*a*' and '*b*' by writing 'aRb' (MacBride 2018, 214).

There is, MacBride points out, a role which names can perform which is relevantly similar to that capable of being performed by expressions including free variables; both types of expression can be used to determine ranges of propositions. There is, though, a role capable of being performed by names which is not capable of being performed by expressions involving free variables. Names can occur in pictures, but expressions involving free variables cannot, for, as we have seen, no item which includes a free variable possesses a truth-value. Moreover, names remain, once 3.314 is accounted for, genuine constituents of propositions, while expressions containing free variables are better conceived of as *features* of their values.

We will see below that several commentators, where they infer from certain remarks in the 3's which concern propositional variables, conclusions about the ontological status of those items to which names refer, are in error. We must not expect the behaviour of propositional variables[26] to reveal the character of Tractarian objects, for, as Wittgenstein says, 'Objects can only be *named*' (TLP 3.221, emphasis in original).

7. Unity

We are now at last in a position to assess the claims made by Linsky and Zalabardo with respect to Wittgenstein's treatment of the problem of the unity of the proposition. It will be worth our while to briefly describe the relevant problem, and to establish the fact that Wittgenstein was aware of it.

The so-called problem of unity is that of explaining how the constituents of a complex are capable of combining with one another such that the product of their combination counts as one rather than many. This difficulty was famously articulated by F. H. Bradley (1893, chapters 2-3), and discussed on several occasions by Russell.[27] It has been widely argued that Wittgenstein, like Russell, was concerned with the problem of unity, and that the *Tractatus* contains resources designed either to solve or avoid it.[28] In what follows I am chiefly interested in the unity of *propositions*. Portions of the ensuing discussion, however, involve remarks of Wittgenstein's more expressly aimed at facts. I take it that my drawing conclusions about Tractarian propositions on the basis of remarks of Wittgenstein's directed at facts is

[26] In what follows I use 'propositional variable' to mean an expression including a free variable, with my explanation of remark 3.314 borne in mind.

[27] See for instance Russell (1899, 146; 1903, §53; 1910, 137–45; 1924, 263–64; 1927, 263–64).

[28] See Candlish and Damnjanovic (2012), Gaskin (2008, 318, 327–28), Gibson (2004), Johnston (2007), Linsky (1992, 264–67), MacBride (2018, 195), Morris (2008, 118), Potter (2008, 109), Spinney (2018), and Zalabardo (2015, 2018).

justified on the grounds that Wittgenstein conceived of propositions as facts, and that it is partially due to this characteristic of propositions that they are capable of representing. Zalabardo sets a precedent here:[29] '[W]hat goes for facts in general goes, in particular, for the facts that play the role of propositions' (2015, 108).

Evidence that Wittgenstein was aware of the problem of unity can be seen in the following passages:

> That is why the point in the above cases is to say how propositions hang together internally. How the *propositional bond* comes into existence (Wittgenstein 1961a, 5, emphasis in original).

And

> It is obvious that the analysis of propositions must bring us to elementary propositions which consist of names in immediate combination.
> This raises the question how such combination into propositions comes about (TLP 4.221).

According to Linsky, Wittgenstein discharges himself of the obligation to offer an explanation of what the unity of a proposition consists in by conceiving of propositions as prior to their constituents:[30]

> . . . Wittgenstein, in the *Tractatus*, following Frege, reverses the order of explanation. The constituents of the proposition, names, are only arrived at by extraction from the unified proposition, just as the various organs of an animal body can be extracted from it. These organs only perform their function in the whole healthy animal body, and not in separation from it. Just so, a name has a meaning only in the context of a proposition. A bit of mechanism is a break only provided the rest of the mechanism is in place. It is only in the 'unity' of the whole mechanism that a part functions as a break. The unity of the mechanism—the animal body, the proposition—is not derivative. Rather the parts are the parts they are only in the functioning whole containing them (Linsky 1992, 269).

According to this view, unities are prior to their constituents insofar as constituents are essentially *extractions* from the unities in which they figure. Wittgenstein, Linsky claims, need not provide an explanation of how independently available items possess the capacity to combine with one another, for he is not committed to any such items, and does not conceive of propositions as having resulted from a process of combination. Rather, we arrive at constituents only through an understanding of the role they play in unities. Unities are prior on Wittgenstein's position, according to Linsky, because they are fundamental postulates whose existence is not explained in terms of the combination of constituents. Instead, the existence of constituents is explained through reference to their occurrence in unities. On this position, the difficulty of explaining how propositional constituents combine with one another is exchanged for that of explaining criteria of identity for constituents, where those constituents are conceived of as essentially dependent upon the items from which they have been extracted. Linsky claims that Wittgenstein's holding to this position is supported by the fact that Frege adopted a similar strategy, and that Wittgenstein was influenced by Frege in this respect.

The vital section of the just-quoted passage for our purposes is: 'The constituents of the proposition, names, are only arrived at by extraction from the unified proposition'. In order to appraise Linsky's proposal we must understand what is meant here by the word 'extraction'. A clue may be found earlier in the same paragraph as that including the quoted passage. Linksy, immediately prior to his description of the constituents of propositions as arrived at *via* extraction, approvingly discusses Dummett's conception of Fregean decomposition:

> Dummett explains in this way why isolated function symbols are not to be employed. A function symbol 'cannot literally be removed

[29] See also Linsky (1992, 266) for the view that Wittgenstein's remarks pitched at an ontological level may be legitimately employed in order to interpret those more clearly aimed at describing linguistic items.

[30] See also Spinney (2018) for an extended defence of Linsky's proposal.

> from a sentence and. . . displayed on its own: we can only indicate the common feature of various sentences which we have in mind by the use, together with words or symbols belonging to the language, of the Greek letters which represent argument-places. And it is, in turn, just because the complex predicate is thus not really an expression—a bit of language—in its own right, that we are compelled to regard it as formed from a sentence rather than as built up of its components.' Dummett here both gives a quite unproblematic account of Frege's metaphors of completeness, unsaturatedness, *Unselbständigkeit*, as applied to function symbols, and indicates how we can connect this to Frege's account of functions by the context principle. By taking the context principle seriously, Wittgenstein, in the *Tractatus*, following Frege, reverses the order of explanation (Linsky 1992, 269).

Linsky here clearly holds that Wittgenstein's reversing the explanatory order of proposition and constituent(s) such that propositions are explanatorily prior to their constituents is a view inherited from Frege. Moreover, Linsky suggests, through his employment of Dummett's interpretation of Frege, that Wittgenstein's adoption of the relevant view constitutes a position in which Frege's conceiving of complex predicates as formed *from* whole sentences figures as the chief influence on Wittgenstein's endorsing that position. From what we have seen, above, it should be clear that Dummett, in the passages Linsky cites, describes decomposition, rather than analysis. Linsky therefore argues, in effect, that Wittgenstein's conception of propositional constituents as posterior to propositions is a conception modelled on Frege's conception of complex predicates as posterior to judgeable contents. In other words, Wittgenstein, according to Linsky, conceives of *names* as extracted *via* decomposition. I have shown, though, that Wittgenstein did not conceive of names as extracted *via* decomposition. Rather, Wittgenstein conceived of propositional variables as extracted *via* decomposition. Linsky here fails to appreciate the distinct roles played by analysis and decomposition in the *Tractatus*. Insofar as Linksy's description of Wittgenstein's conception of explanatory priority with respect to propositions and constituents depends upon the conclusion that names are the products of decomposition, his interpretation is mistaken. Absent some further argument to the effect that the names which figure in *analyses* are explanatorily posterior to the items in which they figure, Linsky's account of how Wittgenstein treats the problem of unity in the *Tractatus* fails to be persuasive.

Zalabardo, while making a proposal in keeping with the spirit of Linsky's suggestion, argues for a more extreme conclusion:

> [T]he *Tractatus* puts forward an account of facts according to which they are not composite items. They are ultimate indivisible units, not the result of a process of composition. Hence Wittgenstein doesn't face the need to explain their unity. And what goes for facts in general goes, in particular, for the facts that play the role of propositions. I am going to argue that, on this point, Wittgenstein was following Frege's lead. Wittgenstein's account of the relationship between states of affairs and objects, and between propositions and names, is an extension of Frege's account of the relationship between judgments and concepts (Zalabardo 2015, 108).

Zalabardo's suggestion clearly shares characteristics with that of Linsky, insofar as Wittgenstein is interpreted by both as avoiding the requirement that the unity of propositions be explained by his taking propositions as ontologically fundamental. Unlike Linsky, however, who holds that constituents are extractions from unities, Zalabardo argues that Tractarian propositions do *not* possess constituents. Tractarian propositions are not unities. Wittgenstein, according to Zalabardo, therefore faces no explanatory task which might be described as a 'problem' of unity. Wittgenstein need not provide an explanation of how propositional constituents combine, for he conceives of propositions as entirely *without* constituents. Recalcitrant remarks concerning the composition of constituents[31] constitute 'vulgar talk', according to Zalabardo, which we ought to translate into 'learned thought' in order to faithfully represent Wittgenstein's position

[31] E.g., TLP 2–2.01; 2.011.

(Zalabardo 2015, 124). The translation of vulgar talk into learned thought consists, on this view, in our conceiving of the constituents of propositions as *features* which they have in common with other propositions, rather than independent ontological commitments:

> On this view, propositions are not produced by the combination of expressions. Just as people share heights, incomes, hobbies, and character traits without being compounded from these items, propositions share characteristic marks without being compounded from them (Zalabardo 2015, 112).

Support for this view can be found, it is argued, in the following remarks:

> An expression presupposes the forms of all the propositions in which it can occur. It is the common characteristic mark of a class of propositions (TLP 3.311).

> Like Frege and Russell I construe a proposition as a function of the expressions contained in it (TLP 3.318).

It is claimed that the conception of a function Wittgenstein employs in remark 3.318 is essentially that of a propositional function in Russell's *Principia Mathematica* (1910). Russell there conceived of propositional functions as parasitic on their values:

> [T]he values of a function are presupposed by the function, not vice versa. It is sufficiently obvious, in any particular case, that a value of a function does not presuppose the function. Thus for example the proposition "Socrates is human" can be perfectly apprehended without regarding it as a value of the function "x is human" (Russell and Whitehead 1910, 42).

In Russell's view propositions must not, on pain of circularity, contain the propositional functions they are the values of as constituents. Russell's description of propositional functions as epistemologically posterior to propositions clearly echoes the account we saw given by Dummett of Fregean decomposition. On the assumption that Wittgenstein's conception of function was the same as Russell's, the claim that propositions are functions of expressions is, according to Zalabardo's reading, tantamount to the claim that *names* are not constituents of propositions. Just as any two numbers share the feature of *being the value of* $\xi + \xi$ without containing that function as a constituent, names, in Zalabardo's view, are features of propositions which they share with other propositions, without thereby counting as constituents of the propositions they are features of. These remarks concerning the relationship between propositions, names, and function-argument analysis may be generalised, on Zalabardo's view, to facts and objects more broadly. Facts, on the present proposal, are construed of as a function of their features; these features constitute the objects of the *Tractatus*. Features, on this view, are *not* ontological commitments; there are, according to Zalabardo, no objects in the ontology of the *Tractatus* (2015, 116). Wittgenstein's claim, 'The world is the totality of facts, *not of things*' (TLP 1.1, emphasis added), constitutes the primary evidence cited in favour of this view.

Zalabardo, like Linsky, argues that Frege's position with respect to the priority of judgements over concepts constitutes the chief source of influence on Wittgenstein's view. Unlike Linsky, Zalabardo conceives of Tractarian propositions as simple items, entirely lacking in constituents. Zalabardo's view, that names are not constituents of Tractarian propositions, and that Tractarian names are to be conceived of as analogous to Russellian propositional functions, is a view which results from the failure to recognise in Wittgenstein's position a distinction between analysis and decomposition. Wittgenstein does indeed conceive of propositions as the value of a function for some argument(s). Moreover, it is plausible to suppose that Wittgenstein's conception of a propositional function was relevantly similar to Russell's insofar as Tractarian propositions do not contain the functions they are values of as constituents. It does not follow, though, that Tractarian *names* are not constituents of propositions; for, as

we have seen, the functional expressions into which Tractarian propositions decompose are not, on Wittgenstein's view, names. It has been the aim of the preceding discussion to show that we must not infer from Wittgenstein's remarks concerning the products of decomposition conclusions about the character of names. Zalabardo commits just this error where he attributes to Wittgenstein the view that Tractarian propositions are not unities.

Zalabardo is not the first Wittgenstein scholar to claim that Wittgenstein's approach to the problem of the unity of the proposition consists in his denying that propositions are unities. Palmer writes

> The opening sections of the *Tractatus* should have made it clear that in Wittgenstein's view this problem [of unity] is only overcome when we cease to think of propositions as having, in any ordinary sense of the word, constituents at all (Palmer 1988, 49).

We have already seen that remark 3.25 employs a conception of analysis according to which propositions do possess constituents. That 3.25 does not employ the notion of decomposition, we saw, strongly suggests that it concerns a conception of analysis on which the items from which propositions are compounded are investigated such that their combinatorial potential be revealed. Having established that decomposition is inappropriate to serve as the conception of analysis employed in remark 3.25, any motivation for construing constituent talk as 'vulgar' rather than learned, and relatedly for treating Tractarian names as features rather than constituents, dissolves. To put the point slightly differently, since we have established that 3.25 provides us with a strong reason to conceive of propositions as possessing constituents, we are at liberty to treat Wittgenstein's remarks concerning the complexity of propositions at face value. Our examination of remark 3.25 has, as it were, performed the decisive role, and our finding textual evidence to the effect that Wittgenstein conceived of propositions as complex is, while compelling, essentially supplementary. We could not have relied solely on such evidence without begging the question against Zalabardo and Palmer with the respect to the issue of propositional complexity, but having already established my view, we may now avail ourselves of the relevant evidence.

Wittgenstein's commitment to propositions as composed of constituents is clearly expressed in a 1919 letter to Russell:

> . . . "... But a *Gedanke* is a *Tatsache*: what are its constituents and components, and what is their relation to those of the pictured *Tatsache*?" I don't know *what* the constituents of a thought are but I know *that* it must have such constituents which correspond to the words of Language. . . "Does a *Gedanke* consist of words" No! But of psychical constituents that have the same sort of relation to reality as words. What those constituents are I don't know (Wittgenstein 1961c, 130, emphasis in original).

It is implausible to suggest that Wittgenstein here takes himself to be speaking with the vulgar, as Zalabardo suggests, while withholding his real views. On the contrary, Wittgenstein, throughout this letter, is quick to correct what he evidently feels are misapprehensions of his position. Wittgenstein, in his *Notebooks*, describes the analysis of a proposition as complete when that proposition is as complex as the item which exists if it is true: 'When the proposition is just as complex as its reference, then it is *completely* analysed' (1961a, 46, emphasis in original). This remark is echoed in the *Tractatus*: 'In a proposition there are exactly as many distinguishable parts as in the situation that it represents. The two must possess the same logical (mathematical) multiplicity' (TLP 4.04). The same commitment to propositions as possessing constituents is voiced in remarks 2.011, 2.0201, 3.24, 3.315, 3.4, 4.024, 4.025, and 5.5423.[32] It is my contention that these remarks should be interpreted as representing Wittgenstein's considered views, rather than as vulgar expressions which obscure a contrary doctrine.

[32] Proops (2017b) observes the poor textual basis for Zalabardo's reading.

I have criticised several authors for reading into Wittgenstein a particular approach to the problem of unity which I claimed is not supported by the evidence cited in favour of it. A thorough defence of my own, positive proposal with respect to Wittgenstein's treatment of the problem of unity would require more space than is reasonable to use here. I would, though, like to briefly suggest the shape which that proposal may take. In my view Wittgenstein's approach constitutes an attempt to expose expressions of the problem of unity as meaningless. Wittgenstein asks how elements come to *combine* in propositions (TLP 4.221). To ask this question, then, is to inquire after the nature of an item's combinatorial potential. I have identified the combinatorial potential of an item with its form. To ask, then, of some propositional element how it comes to be able to combine with others is to ask a question concerning the form of that element. To ask such a question, then, is clearly to employ concepts which are 'formal'. For reasons which lie far outside of the scope of the current article, Wittgenstein felt that the attempted employment of formal concepts constitutes the expression of nonsense (TLP 4.1272). What we unsuccessfully attempt to say through the illegitimate use of formal concepts is only capable of being expressed by 'features of symbols' (TLP 4.126). The combinatorial capacities of some symbol may be gleaned through our attending to the symbol itself, and what the symbol has in common with other symbols of the same form is shown in the symbol for a variable capable of replacing those symbols. Accordingly, there is no problem of unity which it is sensible to pose, on Wittgenstein's view, for any formulation of the relevant problem must employ concepts of the offending variety. This response is, Wittgenstein admits, unsatisfying to those metaphysically minded. Nonetheless, Wittgenstein insists that such an approach is correct (TLP 6.53).

8. Conclusion

I have applied a conceptual distinction derived from Dummett's reading of Frege to the *Tractatus*. I have argued that Wittgenstein employs both analysis and decomposition in the *Tractatus*, to separate effect, and that he adopts a terminological distinction reflecting those divergent interests. I have further argued that partial readings of the *Tractatus* which do not acknowledge in that work the presence of the relevant distinction, and which subsequently draw the conclusion, from remarks in which decomposition is the operative notion, that Tractarian names are not constituents of propositions, are for that reason mistaken.

Acknowledgements

I would like to thank Fraser MacBride for providing generous comments on several earlier drafts, as well as for the numerous insightful discussions we have had concerning the present subject. I would also like to thank an audience at The University of Manchester for their helpful responses to a presentation of this article. My thanks also to two anonymous referees from this journal for their useful suggestions.

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