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Reviewed by Henri Wagner

Review: *Quine, New Foundations, and the Philosophy of Set Theory*, by Sean Morris

Henri Wagner

Is the iterative conception of set theory as standardly represented by Zermelo set theory and its variants—Zermelo-Fraenkel (*ZF*) and Zermelo-Fraenkel with the axiom of choice (*ZFC*)—the uniquely correct conception of set? From the viewpoint of the great majority of contemporary philosophers and logicians, the question, as it stands, receives an affirmative obvious answer. Indeed, on the standard view, the equivalence of the iterative conception of set theory[1] as standardly represented by *ZF* set theory and set theory as such has become wholly unproblematic.

It is not the least merit of Sean Morris's book to challenge this equivalence by providing a wide range of philosophical, mathematical, and historical arguments against what can be considered a well-entrenched prejudice. Critical as they are, Morris's investigations are ultimately aimed at vindicating and exploring an alternative way of conceiving and practicing set theory. The originality of his argumentation against the standard view lies in the fact that he relies on Quine's philosophical, logical, and mathematical works on set theory and takes Quine's contributions as exemplary of an alternative and non-orthodox way of conceiving and practicing set theory. Morris's main claim is that:

> work in set theory generally, at least in its current state, should be conducted in the more pluralistic and pragmatic way that I take to be characteristic of what I will identify as the approach to set theory as explication. (1)

The central notion is that of explication which Morris borrows from Quine. Providing an explication of a notion—for example, the notion of set—is a twofold operation by which

> [w]e fix on the particular functions of the unclear expression that make it worth troubling about, and then devise a substitute, clear and couched in terms to our liking, that fills those functions.
> (Quine 1960, 258–59)

As in Quine's *Word and Object*, the notion of explication is used by Morris as an alternative to the traditional picture of analysis, that is, as a way to frame an alternative to "the contrasting approach, which aims to discover a single correct notion of set, as set theory as conceptual analysis" (1). Set theory as conceptual analysis is thought of as the standard philosophy of set theory while the iterative conception of set theory as the paradigmatic set theory, embodying the "essence" of set. To grant this status to the iterative conception of set theory is to be committed to the idea of set theory as conceptual analysis. Morris wishes to promote a whole new philosophy of set theory, set theory as explication:

> the idea that there is no uniquely correct notion of a set beyond its being the sort of object that fulfills the [following] minimal criteria: it is an extensional entity, and it must somehow be restricted enough to avoid the paradoxes but not so much so that it ceases to be capable of fulfilling its intended role in mathematics. (5)

So Morris distinguishes three reliability requirements for a set theory to be an explication, i.e., three conditions bearing on what a set theory should be able to do given what it is designed for: first, a reliable set theory should construe the notion of set as an extensional notion and so should incorporate an axiom of extensionality; second, a reliable set theory should revise the unrestricted comprehension principle—"$(\exists x)(y)\,(x \in y \equiv \varphi)$"

[1] According to the "iterative conception of set", sets are collections of elements built out of their members by means of an iterative process such that the universe of sets is a cumulative hierarchy stratified into a series of well-ordered "stages". Starting with the empty set as V_0 (in Zermelo set theory and its variants), one gets V_1 by applying the power set operation on V_1 so that $V_1 = \wp V_0$. Then $V_2 = \wp V_1$ and so until the first limit ordinal ω. The union of all the sets V_n yields V_ω. Then we proceed to $V_\omega + 1 = \wp V_\omega$, $V_\omega + 2 = \wp V_\omega + 1$, and so on. We can construct still more sets in the following way: $V_\omega + \omega = V_0 \cup V_1 \cup V_2 \cup \ldots \cup V_\omega \cup V_\omega + 1 \cup V_\omega + 2 \cup \ldots$, $V_\omega + \omega + 1 = V_\omega + \omega \cup \wp(V_\omega + \omega)$, $V_\omega + \omega + 2 = V_\omega + \omega + 1 \cup \wp(V_\omega + \omega + 1)$, and so on.

where x is not free in "φ"—so as to prevent the possibility of logically deriving set-theoretical paradoxes; third, a reliable set theory should prevent paradoxes but "not so much that it would lack the power it needed to capture ordinary mathematics as well as the mathematics of the infinite" (2–3; see also 172 where the third criteria is subdivided into two). As in the paradigmatic case of ordered pairs for which many incompatible but reliable explications are available, there are many different ways to meet these three reliability requirements (chapter 5, section 2). A successful set theory is not answerable to something like the "essence" or the "intrinsic nature" of sets, rather, Morris argues for a pluralistic philosophy of set theory that is a pluralism "under rigorous restraints".

Morris's pluralism is not adopted from a perspective external to set theory and its history. His first argument in favor of his alternative view is an historical one (Part I). Indeed, on Morris's narrative, the two main approaches elaborated in response to the set-theoretic paradoxes, Zermelo set theory (and its variants) and Russell's theory of types, were both considered to be legitimate approaches to set theories primarily in the light of the three requisites of reliability. The pluralistic orientation and open-mindedness of set theory as explication are exposed by Morris as characteristic of the period extending from the first set-theoretic paradoxes to the 1950s. "It was in this context of pluralism and tolerance that Quine introduced *NF*" (5): by this historical contextualization Morris suggests that Quine's *New Foundations* set theory (*NF*) has been considered to be an anomaly, not even deserving the name of set theory, only insofar as it was approached from the perspective of the contemporary practice in set theory and evaluated in light of an established standard set theory.

For all that, the book's purpose is not to argue in favor of *NF* over other set theories. With regard to Quine, the book's two main purposes are: first, to argue that *NF* is better motivated than often assumed and offers a philosophically and mathematically coherent alternative to the various set-theoretic systems implementing the iterative conception of set; second, to maintain that Quine's philosophy of set theory is a pluralistic and non-dogmatic philosophy of set theory alternative to the standard view. The first purpose leads Morris to examine, in particular, whether the third criteria of reliability is fulfilled by *NF* (chap. 7), that is whether *NF* can serve "as a plausible framework for mathematics" (174, see also 172), whereas the second purpose leads him to discuss Boolos's plea for the iterative conception (chap. 6). The conjunction of Morris's claims about *NF* and Quine's philosophy of set theory implies that *NF* itself should not be conceived in a dogmatic way. Morris's defense of an approach to set theory as explication is thus anchored in an interpretation of *NF* and Quine's philosophy of set theory: not only does he take *NF* to be an achievement exemplifying this "more pluralistic and pragmatic way" (1), but he also thinks of Quine's philosophy of set theory as the best example of the approach to set theory as explication.

Before going into more critical comments, I would like to elaborate the relevance of Morris's work from another perspective. In "The Ways of Paradox" (Quine 1961), Quine interprets Russell's paradox as an antinomy. Antinomies differ from veridical paradoxes—paradoxes whose conclusion is true despite their air of absurdity—and falsidical ones—paradoxes whose conclusion "not only seems at first absurd but also is false, there being a fallacy in the purported proof" (5). In the case of antinomies, there is no present way to dissipate it either by convincing us that its conclusion is true or by finding the fallacious step in the argument. Hence, antinomies are paradoxes

> that bring on the crises in thought. An antinomy produces a self-contradiction by accepted way of reasoning. It establishes that some tacit and trusted pattern of reasoning must be made explicit and henceforward be avoided or revised . . . A veridical paradox packs a surprise, but the surprise quickly dissipates itself as we ponder the proof. A falsidical paradox packs a surprise, but it is seen as

> a false alarm when we solve the underlying fallacy. An antinomy packs a surprise that can be accommodated by nothing less than a repudiation of part of our conceptual heritage.
>
> (Quine 1961, 7–11; see also Quine 1987a, 146)

In the case of Russell's paradox, this part of our conceptual scheme to be revised is what Quine variously called "principle of class existence", "principle of comprehension", or "law of abstraction": "for any condition you can formulate, there is a class whose members are the things meeting the condition" (Quine 1961, 13). What Russell's paradox has shown, according to Quine, is that "common sense is bankrupt, for it wound up in contradiction" (Quine 1941, 27). As Morris notes, in *The Principles of Mathematics* Russell affirms "that no peculiar philosophy is involved in the above contradiction [Russell's paradox], which springs directly from common sense, and can only be solved by abandoning some common-sense assumption" (Russell 1937, 105; see also Morris 2015, 137, and, in his book, 51). Yet what is logically involved in Russell's paradox and should be revised is nothing less than a philosophical heritage as old as Port Royal's *Logique ou l'art de penser*: the idea, lying at the core of the traditional view of concepts, that each concept or predicate has an extension associated with it. The unrestricted comprehension principle is a reframing of this traditional idea. Inherited from Port-Royal through Leibniz, Kant, and Mill, this idea plays a central role in both Frege's definition of number and the collapse of his logicist project. In the *Foundations of Arithmetic* (Frege 1884/1953), introducing his definition of "the number which belong to the concept *F*" according to which "the number which belongs to the concept *F* is the extension of the concept 'equinumerical to the concept *F*'", Frege adds in a footnote that he "assume[s] that it is known what the extension of a concept is" (Frege 1884/1953, 80). After summarizing the achievement of *Foundations* and stating the definition of "the number which belongs to the concept *F*" in §107, he simply notes that "in this definition the sense of the expression 'extension of a concept'" is assumed to be known (Frege 1884/1953, 117). Frege was all the more justified to assume that his readers would know what he means by "extension of a concept":

> This was a traditional logical term, first used, not in *Foundations*, but in an influential seventeenth-century work, *La logique ou l'art de penser*. The term was widely used by logicians in Frege's time (and continues to be used today). Frege means to rely on his readers' familiarity with the traditional assumption that there must be some object (an extension) associated with all concept that hold of exactly the same things. Moreover, he believes that this assumption is fundamental to logic; the notion of extension, he claims must simply be taken as a primitive logical notion . . . The introduction of the notion of extension of a concept leads to a disaster—an inconsistency in his logic. Although Frege goes on to say, in *Foundations*, that he attaches no decisive importance to bringing in the extensions of concepts, he later came to see that the introduction of extensions of concepts is of crucial importance for his ability to offer the appropriate sorts of definitions. (Weiner 1999, 62–63)[2]

Since Frege takes concepts to be functions, "the traditional logical view that there is an object, an extension, associated with

[2]Actually the term "extension" and the related notion of an extension of an idea do not appear as such in *La logique ou l'art de penser*. Arnaud and Nicole distinguish between the "compréhension" of an idea and its "étendue" (the term "extension" was then understood, in conformity with the Cartesian doctrine, as the main attribute of matter), and define the "étendue" of an idea *A* as the collection of ideas which occur as subjects in all the true (universal or particular) affirmations whose *A* is the predicate. Three features of Port-Royal's concept of "étendue" are absent from the later concept of "extension" and its variants. First, the extension of a concept does not consist in ideas. Second, the extension of a concept consists in entities of the same logical level whereas the "étendue" of an idea includes singular ideas ("Socrate is a man"), that is ideas of individuals, as well as general ideas ("Every philosopher is a man"). In other words, the logical relations of subsomption and subordination are not clearly distinguished. Third, an idea may be vacuous or empty without this idea being without "étendue": "an idea is any object that can be contemplated by a thinking being without existential commitment to anything except that being" (Hacking 1975, 29). Of course, nothing of what precedes should lead us to underestimate the revolutionary anti-psychologistic and anti-Aristotelian tone of Frege's concept of concept.

each concept is now a special case of the view that there is an object associated with each function" (Weiner 1999, 122).[3] Frege calls these new objects associated with each function "value-ranges". Extension of concepts are value-ranges of concepts. In "Function and Concept" (Frege 1891), he introduces a new Begriffsschrift symbol to designate a second-level function which takes, as arguments, first-level functions and gives, as values, their value-ranges. This symbol gives us the means to name the value-range of any function: if f is a function, then $\dot{\varepsilon} f(\varepsilon)$ is the value-range of f. The problem is that the introduction of this means for forming a name designating the value-range of a function presupposes that any function that can be named has a value-range. It is this presupposition and, derivatively, the basic law V[4] that were shown to be unwarranted prejudices by Russell's paradox.

A whole logical tradition is thus condensed into the admission of the notion of an extension of a concept. This notion was taken as "commonsensical" by Frege and others in the sense that it is an inherent characteristic of common-sense thought to deny its cultural and historical anchoring and to affirm that its tenets are among what is immediately known and beyond any doubt. Russell's antinomy shows not only that the admission of this notion results in an inconsistency but also that what goes without saying, what is the more obvious is still historically and culturally shaped even if it appears as such only in light of the emergence of a crisis or the emergence of an alternative framework.[5]

It is often emphasized that Quine conceives the alternative axiomatizations of set theory elaborated in response to Russell's antinomy (and other set-theoretic paradoxes) as artificial, *ad hoc* and unnatural relatively to the naturalness and the commonsensicality of the unrestricted principle of comprehension. It is seldom noticed and commented that Quine somewhat anticipates what will be the "new commonsense" in set theory:

> Russell's paradox is a genuine antinomy because the principle of class existence that it compels us to give up is so fundamental. When in a future century the absurdity of that principle has become a commonplace, and some substitute principle has enjoyed long enough tenure to take on somewhat the air of commonsense, perhaps we can begin to see Russell's paradox as no more than a veridical paradox, showing that there is no such class as that of the non-self-members. One man's antinomy can be another man's veridical paradox, and one man's veridical paradox can be another man's platitude. (Quine 1961, 14, see also 6 and 11)

Being an antinomy is relative to a conceptual scheme and its presuppositions. Finding a way to resolve an antinomy consists in revising the presupposition that is thought to lead to inconsistency. Russell's antinomy and the subsequent revision of the conceptual scheme implied by its resolution constitute a transition internal to our conceptual scheme which is important enough to modify the status of Russell's paradox from an antinomy to a veridical paradox, and then to a "platitude". On the standard view, Russell's paradox has indeed become the theorem that there is no set of all sets, that is, a *reductio ad absurdum* in the same sense that the Barber paradox is a *reductio* proof showing that there is no barber who shaves all and only those men in his village who do not shave themselves. Whereas the unrestricted axiom of comprehension allows Russell's set $R = \{x : x \notin x\}$, the separation axiom "$(\exists x)(y)(y \in x \equiv (x \in z \,.\, Fy))$" was designed to prevent the derivation of R. However, it does allow, for any set A, the formation of the corresponding set $R_A = \{x \in A : x \notin x\}$. So it can be asked whether or not $R_A \in R_A$. If $R_A \in A$, then we can conclude that $R_A \in R_A \equiv R_A \notin R_A$. So, by *reductio*, $R_A \notin A$. Since this pattern of proof could be used for any set A, it is shown that

[3] Here I follow in part Weiner's remarkably perspicuous exposition of how Russell's paradox arises in Frege's logical systems (see Weiner 1999, 122–28).

[4] It can be stated as follows: $\dot{\varepsilon} f(\varepsilon) = \dot{\alpha} g(\alpha) \equiv (x)(f(x) = g(x))$. The special case of Frege's Basic Law V that applies to concepts asserts: the extension of the concept F is identical to the extension of the concept G if and only if all and only the objects that fall under F fall under G.

[5] In this paragraph I adapt an anthropological reflection on commonsense from C. Geertz, see Geertz (1975, 75).

for every set A there is a set that is not an element of A. In brief, in *ZF*, Russell's paradox is turned into a proof of the following theorem: there is no universal set, i.e., not set that contains all sets as elements. To classify Russell's paradox as telling the truth about the universal set, as Zermelo does, amounts to favor one way of resolving the paradox over other ones (50–51, for a presentation of Zermelo's proof). What was a theorem specific to Zermelo's axiomatization of set theory has later become a truth about the universe of sets according to the standard view of set theory. But this theorem is relative to specific axiomatic theories of set theory, Zermelo set theory and its variants. The only thing that is shown is that one cannot have a universal set while assuming Zermelo's axiom of separation.

In Quine's eyes, the point of Russell's antinomy produced by the "class of of all classes that are not members of themselves" is not that there is no universal set V. It is neither that there is no sense to talk of classes that are not members of themselves or properties that are not instantiated by themselves. Quine's interpretation of Russell's antinomy departs also from Russell's own. Russell's diagnosis is that the antinomy (and other kindred paradoxes) proceeds from a vicious circle: it violates the principle according to which "[w]hatever involves *all* of a collection must not be one of the collection" (Russell 1908, 30). In other words, the definition of the paradoxical class R is "impredicative". However, "[i]mpredicative specification is not visibly more vicious than singling out an individual as the most typical Yale man on the basis of averages of Yale scores including his own" (Quine 1963, 243, 1987a, 93–94, and Morris's book, 57–58, 101–102, 129). Accordingly, the only logical consequence of Russell's antinomy is that "there is no class, empty or otherwise, that has as members precisely the classes that are not members of themselves" (Quine 1961, 13; compare with Russell 1937, §101), thus that "not all open sentences determine classes" (Quine 1963, 3). Then, "a major concern in set theory is to decide what open sentences to view as determining classes" (Quine 1961, 13; compare with Russell 1937, §102). As it is masterly explained by Morris, with *NF*, Quine aims to maintain as much as possible of the naïve unrestricted axiom of comprehension while preventing the emergence of paradoxes by balancing it out with a stratification procedure. This procedure consists in imposing syntactical conditions on a formula so that it appears as an instance of the stratified comprehension axiom "$(\exists x)(y)(y \in x \equiv \varphi)$" (where "$\varphi$" is stratified and x is not free "φ") and as a *NF* theorem. A formula A is said to be stratified if and only if there is an assignment of numerals to their variables such that each occurrence of "$\in$" is to be found in contexts of the form $n \in n^{+1}$. This procedure blocks Russell's paradox: if A is an unstratified formula as "$x \notin x$", then the formula "$(\exists x)(y)(y \in x \equiv x \notin x)$" is not a *NF* theorem. Contrary to an axiomatic set theory with a separation scheme but not unlike set theory with unrestricted comprehension, *NF* does not prevent the formation of the universal set: the stratified formula "$x = x$" may occur in an instance of the stratified comprehension axiom in place of "φ" so that the existence of V is an *NF* theorem. From a Quinean viewpoint, to consider that there is no universal set as the main truth disclosed by Russell's paradox turns out to be a fallacious retrospective interpretation of the paradox rooted in the acceptance of Zermelo's axiom of separation which is one among many ways out of Russell's paradox. One of the most important contributions of Morris's book is to show why what has become natural, what has become the putative new commonsense in set theory is no more conceptually and mathematically justified than what has since been recurrently regarded as an anomaly, that is, *NF*. Through historical contextualization and comparative exercises, Morris highlights how the standard view of set theory is, in some sense, ideological, meaning that it attempts to erase its parochial dimension.

The renewal of Quine's scholarship during the last twenties years has been primarily focused on the metaphysical and epistemological aspects of his naturalism. So Morris's contentions that "[Quine's] early work in logic and the foundations of mathematics shaped his general approach to philosophy" (ix) and that the "core concern's of Quine's philosophy emerg[es] from his logical work" (61) are all the more important in that they bring to the foreground the basic idea that Quine's philosophy can not be properly understood without giving to his views on logic and mathematics their proper and rightful place. In that respect, *Quine, New Foundations, and the Philosophy of Set Theory* is a major contribution to our understanding of Quine's philosophy. Among the most significant and insightful analyses to be found in Morris's book are the following: a detailed examination of how *NF* is able to prevent the emergence of set-theoretical paradoxes (section 3.2); an invaluable discussion of Quine's way out of the problem of the unity of proposition in his earliest logical works (105; see also Morris 2015); a fruitful approach of Quine's masterpiece *Set Theory and its Logic* interpreted as a paradigmatic instance of Quine's philosophical program as exposed in *Word and Object* (1960, 125–34); a detailed commentary of Quine's pluralism in *Set Theory and its Logic* (1963), both in its philosophical and logical aspects (130–34, 185–94).

In what follows, I will focus my critical remarks on three issues central to Morris's argumentation: (1) the logic of set theory; (2) Quine's departure from Russell's theory of types; (3) the nature of naturalism.

1. Set Theory and Its Logic

Echoing some of Dreben's seminal remarks (Dreben 1990), Morris argues that the core problems of Quine's philosophy emerge from his early logical works and that the naturalist mode of philosophical activity favored by Quine is first illustrated in his early logical and set-theoretical contributions. In comparison to the detailed and perceptive analyses devoted to Cantor (chaps. 1 and 2), Russell (chaps. 2 and 4), or Boolos (chap. 6) and given that Quine's philosophy of set theory is internally connected to his philosophy of logic and mathematics, one may nevertheless be surprised how little is said about many of Quine's central theses in philosophy of logic and mathematics.

To give an example in support of this assessment, I will focus on Morris's analyses on the relationships between *NF*, Russell's theory of types and Zermelo set theory. As Morris shows, Quine explicitly devised *NF* by combining insights of both Russell's theory of types and Zermelo set theory (section 3.1). From Russell's theory of types, he gains the insight of typical ambiguity—the variables of a formula are typed but it is not yet specified which types they are indexed to—and reinterprets it so that "what the typing restriction does is to provide a syntactic test for which formulas actually determine classes" (62), that is, for which formulas are stratified. Thus, Quine resolves Russell's antinomy and devises a new and original set theory while avoiding the undesirable consequences of the non-cumulative hierarchy of the universe into levels in Russell's simple theory of types—reduplication of arithmetic at each level of the hierarchy of types and lack of "big classes", notably the universal class *V* and absolute complement classes—and rejecting the axiom of reducibility. From Zermelo, Quine gains the insight "that a meaningful open sentence may or may not determine a class, and that it can be left to the axioms to settle which ones do" (Quine 1987b, 288; see Morris's book, 64–65). Instead of excluding all the unstratified formulas from the language on the grounds of their alleged typical or categorial meaninglessness, Quine leaves it open that they may not determine a class according to which classes the axiom of stratified comprehension yields. Since Quine rejects Russell's hierarchy of types, variables are not typed and restricted in range to a domain of significance but are general in Zermelo's sense. Yet the axiom of stratified comprehension does not rule out, as Zermelo's axiom of separation does, the existence of a universal class and absolute complements.

Morris points out that, in addition to this genesis which traces *NF* back to Russell's simple theory of types and Zermelo set theory, Quine examines the structural relations between theory of types and Zermelo set theory to show that the "hierarchical structure [*of the theory of types*] leads to Zermelo theory and then finally to his own *NF*" (189). The "easy slip into Zermelo's system by successive deliberate refinements of the Theory of types, as seen in *Set Theory and Its Logic*" (Quine 1986b, 590) consists in translating a many-sorted logic with typed variables—"theory of types with special variables" (Quine 1963, 270)—i.e., indexed variables ranging only over entities of a determinate type, into a first-order set theory with general variables restricted to the appropriate typed predicates—what Quine calls a "standardized theory of types" or a "theory of types with general variables". Instead of providing each variable with a specific index of type, it is possible to resort only to one style of variable and to introduce a typing predicate T_n. Logical schemata as "$(x_n)Fx_n$" and "$(\exists x)Fx_n$" can be rewritten, respectively, as "$(x)(T_n x \supset Fx)$" and "$(\exists x)(T_n x \, . \, Fx)$". This procedure of translation consists in eliminating typed variables in favor of typing predicates, showing then how to derive Zermelo set theory by converting typed variables into a cumulative type hierarchy with general variables (130–34, 189–94; Quine 1956, 1963, chap. XII, 1987b, 287). Morris comments on this procedure as follows:

> Quine moves to Zermelo set theory with its unrestricted (with regard to type) variables not merely as a reaction to the undesirable features of types but rather as a natural generalization of types' hierarchical structure. Whereas other philosophers of set theory have ruled out types as set theory, often by fiat, Quine engages with both theories to demonstrate how we might see the interconnections between them. . . .
>
> Here, we see just the sort of philosophy of set theory in which Quine engages. Rather than looking at differences and attempting to privilege one set theory over another, he tries to see their similarities. For Quine, coming to understand the realm of sets means investigating sets from the various perspectives that different set theories allow for. His endeavor in set theory, as in much of science, is cooperative rather than exclusionary and aims to broaden our knowledge through pluralism about set theory. This moving from noncumulative to cumulative types perhaps allows us to better understand the apparent hierarchical structure of sets. It was always present in both types and Zermelo set theory, and Quine has now shown us explicitly how the idea connects the two approaches to set theory. (190–92)

What is underemphasized in Morris's account is the interplay between Quine's philosophy of logic and his philosophy of set theory. Indeed, two central features of Quine's conception of logic are involved both in the procedure for translating Russell's theory of types into Zermelo set theory described above and in his philosophy of set theory: the first one relates Quine's philosophy of set theory to the important turn in the history of 20th-century logic "which led to the demise of type-theory as the fundamental background logic in favor of first-order theories" (Mancosu 2010, 370); the second feature is Quine's conception of the variable.

In *Set Theory and Its Logic*, while commenting on Russell's theory of types, Quine asserts:

> In general it is convenient in presenting formal systems of set theory to be able to assume the standard logic of truth functions and quantifiers as a fixed substructure requiring only the addition of axioms appropriate to the special set theory in question. (Quine 1963, 248)

At the moment of the publication of *STL*, the idea that mathematical and logical work in set theory should be pursued in a first-order framework or, at least, that the axiomatization of set theory requires only a first-order framework was not new to someone involved in the field of logic (even though the claim was contested). However, the evolution was slow until the so-called "triumph of first-order languages". Without going into the details of the history of the emergence of first-order logic as such or as an underlying logic[6], a few remarks will help to

[6]By "underlying logic" I mean "a logic applicable to any particular mathematical area in the following manner: one specifies a vocabulary and particular axioms in this vocabulary, and uses instances of quantificational axioms and inferences rules to obtain the results peculiar to the particular area" (Goldfarb 1979, 352).

convey a sense of how important is this historical turning point to understand Quine's philosophy of set theory.

The 1930s was the scene of a progressive and important shift from theories of types as the most widespread underlying logic to first-order logics as an underlying logic for axiomatizing set theory.[7] Quine's 1936 article "Set-Theoretic Foundations for Logic" was a contribution to this shift. It might be thought that Gödel's metalogical theorems (most notably Gödel's completeness theorem and first incompleteness theorem) were taken as decisive arguments. Still, these metalogical results did not settle the issue of which underlying logic to adopt: even after Gödel's main metalogical theorems, logicians such as Gödel, Tarski, Hilbert, and Carnap continued to use higher-order variants of the simple theory of types. More than that, the Upward Löwenheim-Skolem Theorem—according to which any first-order theory with an infinite model has models of arbitrarily large cardinalities—implies that, contrary to higher-order theories (under conditions), no first-order theory with infinite models can be categorical, that is to say, no first-order theory can specify a unique model up to isomorphism. Accordingly, no first-order axiomatized set theory is able to rule out unintended or "nonstandard" models and thus to specify a unique model even for the natural numbers. It is a noticeable fact, though, that during the 1930s, logicians and philosophers such as Tarski, Quine, Bernays, and Gödel emphasized the fact that with suitable axioms for set theory, first-order logic is sufficient for regimenting mathematical proofs. As a first-order theory, *NF* exemplifies this actual possibility: Quine uses only a single style of variable and membership is his only primitive predicate—identity is eliminable, being defined in terms of membership and first-order logic. In his early logical works, one of his main justifications for adopting first-order logic as an underlying logic is that "with the help of the special relation '∈' we can translate set theory and mathematics into the lower functional calculus [i.e., first-order logic]" (Quine 1940,147). This justification is of a "pragmatic" sort, close to Morris's notion of set theory as explication, insofar as it has to do with the application of logic. The first-order notation so elaborated can then be expanded by adding to it extralogical predicates specific to distinct scientific areas (Quine 1960, 160, and 1986a, 97–98). Logic in this sense becomes the "common denominator of all not wholly trivial theories" (Quine 1942/2018, 13, see also 1951a, 2). Quine's promotion of first-order logic leads to a demarcation between logic and set theory and, derivatively, to a demarcation between logic and mathematics. This twofold demarcation underlies the distinction, in his early works, between logic in a strict sense—i.e., including only truth-functional and quantificational logic—and logic in a wide sense—i.e., including also set theory. However hesitant Quine seems to be about the domain of logic in his early works, it is all the more important to keep in mind that what he calls "general theory of quantification" is not a subsystem of a higher-order logic system—contrary, for instance, to Hilbert and Ackerman's "restricted functional calculus" which was a subsystem of an "extended functional calculus". Quine rejects even the idea that ultimately a notion of order can be coherently applied to the notions of variable and logic. All the notations of logics whose order is higher than 1 are therefore reinterpreted as languages of a first-order theory of sets. To the letter, it should then be said that there is nothing like a "first-order" logic. The label "first-order logic" is misleading since higher-order logic proves to be, in Quine's phrase, "set theory in sheep's clothing" (Quine 1986a, 66; see also 1963, 257–58). More generally, Quine does not agree with the interpretation of the general logical principles that progressively imposed itself, and which was closely related to the emergence of the model-theoretic paradigm.

It is often argued that not only was the very idea of an underlying logic foreign to the logical universalism of Frege and Russell, but also the distinction between first-order logic and

[7] For references to classical works by Tarski, Hilbert, Gödel, and Bernays, see Ferreirós (2001).

higher-order logics. Even if Quine's conception of logic is, in this respect, at odds with basic tenets of logical universalism, it involves a conception of variable close to that usually ascribed to partisans of logical universalism according to which variables are intrinsically and absolutely unrestricted in range. Quine's conception of the variable is at the center of his conception of logical notation. That "the quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality" (Quine 1960, 160) does not mean that the canonical notation should be built according to predetermined categories, be they be categories of variables or categories of predicates. The only categorial distinctions allowed by Quine are between variables and predicates and between names and sentences. In particular, canonical notation does not include distinctive styles of variable but only "general variables", variables that are "regarded as taking as values any objects whatever; and among these objects we are to reckon classes of any objects, hence, also classes of any classes" (Quine 1937, 81). Variables are intrinsically unrestricted in range and each of them confers the same unrestricted generality.

This conception of the variable is, to a certain extent, partially justified by Quine's putative observation that "notations with one style of variables and notations with many are intertranslatable" (Quine 1969a, 92) so that the style of variable is an arbitrary matter. Indeed, combined with the stratification device, this mutual translatability is regarded by Quine as showing that "even under the theory of types the use of distinctive styles of variables, explicitly or even implicitly, is the most casual editorial detail [since] it is a distinction which is not invariant under logically irrelevant changes of typography" (Quine 1951b, 132–33). One remarkable consequence, notably put forward against Carnap, is that the subclass/category distinction is not "invariant under logically irrelevant changes of typography". Category questions are "questions of the form 'Are there so-and-so's?' where the so-and-so's purport to exhaust the range of a particular style of bound variables" while subclass questions are "questions of the form 'Are there so-and-so's?' where the so-and-so's do not purport to exhaust the range of a particular style of bound variables" (Quine 1951b, 130). When a many-sorted notation is changed into a notation with a general variable, categorial questions turn into subclass questions; when a notation with a general variable is changed into a many-sorted notation, subclass questions turn into categorial questions.

Hintikka complains that Quine's conception of the variable prevents the very formulation of the "problem of categories in the original Aristotelian sense of the word, for those categories were primarily the irreducibly largest classes of entities that can be considered together" (Hintikka 1997, 220). Quine's method of "relativization" (Quine 1963, 235) at work in the procedure of translation described above does not undermine Hintikka's judgment since "the very point of categorial distinctions is that several ranges of quantifiers can not be obtained by relativization from an absolute all-comprehensive class of entities" (Hintikka 1997, 220). The method of "relativization" consists in restricting the range of values of the variable of a quantificational schema by introducing the appropriate predicate in the restrictive clause so that the limitation is extrinsic as it does not bear on the fact that the variables are taking any objects as values whatsoever. Russell already knew that any variable restricted by relativization would simply occur in the antecedent clause of a conditional whose variables are unrestricted (or if not, they would in turn require a new antecedent clause involving unrestricted variables). He came to conclude that the restriction of the range of a universally quantified variable by a relativization procedure cannot account for distinctions of types (Gandon 2013). Hintikka's concurs with Russell in holding that a method of relativization such as Quine's can only yield subclasses of a domain of quantification.

So Morris's emphasis on the conception of set theory as explication leads him to underemphasize two crucial aspects of

Quine's philosophy of set theory: first, the conception of logic as "first-order logic" involved in *NF*; second, the conception of the variable that relates Quine's conception of logic to logical universalism. Furthermore, Morris's exegetical perspective leads him to underestimate one of the main reasons why Quine rejects Russell's theory of types.

According to Russell, even though a typed variable is a restricted variable, a type is not to be thought of as the extension of a predicate—i.e., the predicate in the restrictive clause. So the limitation by typification is not to be understood as an extrinsic limitation by hypothesis on the range of the variable. The whole point of Russell's reasoning is to show that a type cannot be assimilated to an extension and then that type limitation cannot be assimilated to hypothetical restriction. The distinction between unrestricted variable and restricted variable and the distinction between absolute variable and relative variable should not be conflated:

> From Russell's perspective, a relative restriction would correspond to a restriction set via a hypothetical clause, while an absolute restriction would correspond to a type restriction. In the latter case, any entity which does not belong to the range of the variable cannot replace *salva significatione* any entity of the domain. This is the reason why type restriction can be called absolute.
>
> (Gandon 2013, 216)

Quine's departure from Russell's theory of types is premised, at least in part, on his rejection of this absolute/relative distinction as being based on "categorial fences" (Quine 1960, 229), that is, boundaries of sense. Hintikka's criticisms addressed to Quine's conception of the variable make it explicit that among his reasons for rejecting Russell's theory of types is his wholesale dismissal of the traditional project of a theory of categories. This brings me to my second critical comment.

2. Types, Categories, and Nonsense

Morris often points out that in *NF*, stratification is not a condition of meaningfulness (63–64, 130–32) and that this feature of *NF* constitutes a decisive departure from Russell's theory of types. In Russell's theory of types, for a set-theoretic formula or statement to be meaningful, variables on the left- and right-hand sides of the membership relation had to be of consecutive ascending types. Thus $(\alpha \in \beta)$ is a formula only if the values of β are of type $n + 1$ and the values of α are of type n. Otherwise, $(\alpha \in \beta)$ is neither true nor false but meaningless (Russell 1937, Appendix B). Theory of types relates types, propositional functions, and nonsense through the notion of "range of significance". A type is characterized as a range of significance of a propositional function, i.e., as the collection of the arguments for which the function in question is significant, that is, has a value (Russell 1908, 236). Every propositional function has a range of significance and that ranges of significance form types. The vicious circle principle implies that the different ranges of significance corresponding to different logical types are hierarchized, mutually exclusive and collectively exhaustive.

On the contrary, "[i]n *NF* there are no types. Nor is it required that formulas be stratified to be meaningful. Stratification is simply an ultimate, irreducible stipulation to which a formula is to conform *if* it is to qualify as a case of 'Fx' in the particular axiom schema [of comprehension]" (Quine 1963, 289). Formulas that do not pass the syntactic test of stratification are simply false since, differently from the vicious circle principle implemented in Russell's theory of types, stratification does not consist in a condition of meaningfulness.

At the core of Russell's theory of types lies a view of nonsense that is rejected by Quine. In other words, not only, as Morris emphasizes, does Quine disagree with Russell in not conceiving stratification as a condition of meaningfulness but he also dismisses the view of nonsense presupposed by Russell's verdicts of nonsensicality applied to formulas or statements that infringe the vicious circle principle. Although central to Quine's philosophy of set theory, this last point is, on the whole, neglected by Morris. The violation of the restrictions resulting from the non-cumulative hierarchy of types produces a "substantial" or "positive" nonsense: a "nonsense got by combining terms whose meaning is such that nonsense results from putting them together" (Diamond 1991, 112). This variety of "positive" nonsense may be called "categorial nonsense" and characterized as follows: the categorial nonsense of a grammatically well-formed statement or formula is due to an *a priori* incompatibility between the categories to which some of its meaningful constituents belong (Diamond 1991, 95–96; see also Ryle 1938, 188). In such a view, "Caesar is a prime number" is neither true nor false but meaningless inasmuch as its subject falls outside the range of significance of the predicate in that it does not belong to the right type or category in order for the predicate to be ascribed to the subject. Quine sees this view of nonsense at work in Ryle's doctrine of "category-mistakes" ("type-trespasses" or "type-errors") according to which an assertion including at least one expression that is not "of the right type" to be combined with the other expressions in the required and legitimate manner—for example, "Saturday is in bed"—is neither true nor false but "absurd" (Ryle 1938, 188).[8] Ryle's doctrine finds support in Russell's theory of types:

> There is a recurrent notion among philosophers that a predicate can be significantly denied only of things that are somehow homogeneous in point of category with the things to which the predicate applies; or that the complement of a class comprises just those things, others than members of the class, which are somehow of the same category as members of the class. [This point of view] is part and parcel of the doctrine that "This stone is thinking about Vienna" (Carnap's example) is meaningless rather than false. This attitude is no doubt encouraged by Russell's theory of types, to which, by the way, Mr. Strawson seems to think modern logic is firmly committed.
>
> (Quine 1953, 153; see also 1960, 220, 1969a, 96, and 1987a, 191)

Against this view, Quine argues that, from a logical point of view, such statements should be treated as false statements in exactly the same manner as the rest of the false statements. Yet his repudiation of any view of nonsense admitting a variety of categorial nonsense does not lead him to elaborate an alternative *theory* of nonsense according to which *all* cases of alleged categorial nonsense would be reducible to false statements. There is no "falsidical" theory of nonsense in Quine.[9] As it is now well established, Quine's reflections on meaning, synonymy and nonsense do not amount to a theory of meaning, that is, to the formulation of necessary and sufficient conditions for meaningfulness, but to a denial of the necessity and possibility of such a project.

The alleged categorial nonsense of statements like "Virtue is not square" or "Caesar is a prime number" follows directly from the application of Russell's theory of types. For Ryle, Carnap, or Strawson, one of the most compelling reasons for holding the first statement to be nonsense rather than a truism or the second to be nonsense rather than an obvious falsehood or a "silly sentence" (Quine 1960, 229) is the need to dissolve the antinomy to

[8] The "absurdities" to which type-trespasses give rise are conceived by Ryle to be symptoms of differences in logical or conceptual categories. Ryle's claims that they are such things as category-mistakes and that category-mistakes are positive absurdities became a widespread view in the 1940s and 50s so much so that Arthur Prior came to argue that anyone who thinks of category-mistakes as false rather than meaningless "must nowadays count themselves among the heretics" (Prior 1954, 26).

[9] On the relation between Quine's position and the "no-nonsense" or "falsidical" view, see Horn (2001, 110–21), Prior (1954, 31, 1962a, 118), and Diamond (1991, 96). It is worth mentioning that Diamond defends Quine's reading of the Fregean principle of context against Dummett's criticisms, showing that his reading implies the rejection of the substantial conception of nonsense and amounts then to a radical reorientation of semantics (Diamond 1991, 108–10). On the different topics surveyed in this section, see Narboux (forthcoming, chap. 5).

which "the class of all classes that do not belong to themselves" gives rise (Prior 1954, 26–27). Four elements of response to this line of thought can be offered. First, as Morris lucidly spells it out, *NF* is the demonstration that neither Zermelo set theory nor Russell's theory of types (either in its simple or in its ramified version) are logically required to settle Russell's antinomy. Second, "the trouble is that the restriction [related to the hierarchy of types] is far more severe than needed in barring the paradoxes" (Quine 1987a, 95). Russell's antinomy does not show that it is illegitimate and paradoxical to admit unrestricted variables and that only typed variables should be admitted but only that it is paradoxical to consider that the values of an unrestricted variable form a class in the set-theoretic sense. Third, if "even under the theory of types the use of distinctive styles of variables, explicitly or even implicitly, is the most casual editorial detail [since] it is a distinction which is not invariant under logically irrelevant changes of typography" (Quine 1951b, 132–33), then Russell's theory of types is superfluous both as a solution to the antinomy and as a logical and notational framework. Fourth, "there is no evident standard to what to count as a category, or category word" (Quine 1969a, 91): either the notion of category is explained by the notion of the range of a distinctive style of variable, however, "the style of variable is an arbitrary matter" (Quine 1969a, 92); or the notion of category is explained by the notion of a substitution class—"expressions belong to the same substitution class if, whenever you put one for the other in a meaningful sentence, you get a meaningful sentence" (Quine 1969a, 92)—but there is no available criterion of meaningfulness on which to ground this notion of substitution class.

The *a priori* delineation of categories forming as many subuniverses, subdomains, or "ranges of significance", whether they be hierarchized in non-cumulative levels as in Russell's theory of types or not, is extrinsic to what is required from a logical point of view:

> All in all, I find an overwhelming case for a single unpartitioned universe of values of bound variables, and a simple grammar of predication which admits general terms all on an equal footing. Subsidiary distinctions can still be drawn as one pleases, both on methodological considerations and on considerations of natural kind; but we may think of them as distinctions special to the sciences and unreflected in the structure of our notation. (Quine 1960, 230)

Quine's departure from Russell on this point is reflected in two other elements of his philosophy relevant to his views on set theory.

The first element is Quine's conception of logical notation and the way it differs from Russell's "logically perfect language". Whereas Russell takes for granted that there are and need to be such things as "categorial fences" (Quine 1960, 229) to be built in the syntax of a "logically perfect language", Quine's canonical notation is not constructed according to categories of variables or categories of predicates that would be answerable to categories of entities. No categorial framework precedes a canonical notation. It follows that, in accordance with Quine's naturalism (Quine 1960, 274–76), if there were to be "categorial" distinctions to be drawn between classes of entities likely to count as values of variables, these distinctions would result from the application of logic in regimentation. Indeed, just as it is only through the application of logic in regimentation that one can determine the ontological commitments carried by a given discourse, so it is only through regimentation that one can distinguish between different varieties of objects that are said to exist. Yet varieties of entities (e.g., physical objects or classes) are not categories of entities. One reason is that, in virtue of the unrestricted generality of the variables, varieties of entities represent at best only subclasses of anything that is likely to count as a value of variable. The criterion of ontological commitment is thus dependent on the notion of general variable and the univocal sense of the notion of object it conveys.

The second element is Quine's view that being is univocal. As Prior argues, Russell's theory of types and its philosophical variants are nothing but a modern version of the traditional philosophical view according to which being is not univocal and its various senses have at best a unity of analogy:

> The theory of types, or the theory of categories as it is now often called, is essentially a Thomist theory. When we are told that there is a type-fallacy or category-mistake in saying that virtue and my left eye are both of them not square, we are irresistibly reminded of the way the Thomists tell us that we must not say that God and (for example) Mr. Grave are both intelligent, because nothing that is predicable of God is predicable of Mr. Grave in the same sense. It cannot even be said (according to this story) that God and Mr. Grave both are in the same sense of "are".
>
> (Prior 1954, 29; see also 1962a, 118, 1962b, 137)

Quine would accept this diagnosis as he opposes one of the consequences of Ryle's doctrine of category-mistakes (Ryle 1949, 11–12) according to which the verb "exists" has one sense or another according to the category of the purported object, concrete or abstract, so that "to apply the verb jointly to something abstract and something concrete [is] to use it simultaneously in two senses, and hence [is] meaningless" (Quine 1987a, 191; see also 1960, 130–31). That Ryle's doctrine of category-mistakes has such a consequence is not incidental since Ryle's doctrine is in line with Russell's theory of types. Indeed, as Russell acknowledges it, the theory of types implies the equivocality of the concept of meaning and of the concepts of existence, being, and entity. On the one hand, it follows from the definition of logical types that there are as many senses of "meaning", each of a different logical type, as they are logical types among the objects for which there are words (Russell 1924, 137–38). Given the indefinite extensibility of the non-cumulative hierarchy of types, "there are infinite numbers of different ways of meaning, i.e., different sorts of relation of the symbol to the symbolized, which are absolutely distinct" (Russell 1918, 109). On the other hand, entities of distinct types are not entities in the same univocal sense and can not be said to be in the same univocal sense: since there are as many different senses of being as there are different types of entities, entities should be said to be in as many senses as there are different types of entities. The "systematic ambiguity" of "there is", i.e., his possession of "a strictly infinite number of different meanings which it is important to distinguish" (Russell 1918, 108) implies that if "there is", "entities", or "exists" are applied to certain entities in conformity with their category or type, the application of these expressions, taken in the same sense, to entities belonging to a different category or type does not yield anything true or false but only a meaningless string of words (Russell 1924, 142, 1949, 11–12). Morris repeatedly comments on the reduplication of arithmetic within the types and Quine's objections to this default (63–67, 107–10, 190). The hierarchy of types is such that the number of cardinal virtues is different from the number of musicians in a jazz quartet. The reason is not that there are more (or fewer) cardinal virtues than there are musicians in a jazz quartet but that virtues and musicians are entities of distinct types, so that there are as many numbers four as there are types. In brief, there are as many distinct arithmetics as there are distinct types. As Morris points it out, the reduplication of arithmetic within types is a consequence of the non-cumulative feature of the hierarchy of types into levels. But it also can be thought of as consequence of the equivocality across types. To abandon the non-cumulativity of the hierarchy of types—and not only to amend it by the introduction of an axiom of infinity for each type—is to pave the way for the rejection of the equivocality across types. Morris should have insisted on the connection between Quine's criticisms of this drawback of Russell's simple theory of types and his views on the univocity of being and existence as they do not appear to be merely incidental.

3. The Nature of Naturalism

My third and last critical remark concerns a central assumption of Morris's argumentation. Morris claims that Quine's early concerns in logic and foundations of mathematics shaped his general approach to philosophy (ix, 61, 81, 114). This book as well as some of his essays therefore aim to "bring to light the origins of Quine's philosophy from the perspective of Quine as logician rather than as epistemologist" (Morris 2015, 134). What, according to Morris, inchoately emerges in Quine's early logical works is his naturalism (87, 100, 106, and Morris 2015, 135). Morris thus apparently subscribes to the current standard reading of Quine's philosophy, whose core principle is formulated as follows by Hylton: "at the heart of Quine's system is his naturalism" (Hylton 2007, 2). While the distinctiveness of Morris's approach lies in the fact that he apprehends Quine's naturalism from the perspective of Quine's early logical concerns, all along his book, Morris seems to take for granted what is meant by "naturalism" and never spells out what he means by such a label. However, there are many different, conflicting ways to understand the nature and significance of Quine's naturalism.

Three main reasons support Morris's reading according to which Quine's early logical works foreshadow his "late" naturalism. Each of these reasons relates to an aspect of Quine's "mature" naturalism.

First, as Morris argues, "it is in fully committing himself to the idea that mathematics can solve philosophical questions that . . . we have the origins of Quine's naturalistic philosophy. In short, we have Quine taking the best methods of the science of his day—in this case, those of the new mathematical logic—and using them to address philosophical concerns" (100). A naturalistic philosophy of mathematics is then a philosophy of mathematics developed within mathematics itself. One may legitimately be doubtful about this understanding of the "methodological" aspect of Quine's naturalism. That mathematics can have a philosophical significance or, more precisely, that an epistemology of mathematics can be developed from within mathematics is, in a sense, what mathematicians and philosophers of mathematics learned from Gödel's celebrated incompleteness theorems.[10] But Gödel was no naturalist and the incompleteness theorems do not convey any naturalistic commitment. So much more has to be said. In addition, the expression "the best methods of the science of his day" (see also 106, 114–15, and 143–44) is, as far as I know, absent from Quine's published writings. It would not be problematic to use it if this expression was not equivocal (on this point, see Burgess 2014, 290–91). But even though this expression was not equivocal, it would not be appropriate for describing Quine's naturalism:

> The fact is that though Quine in "Epistemology Naturalized" pointed philosophers in a certain direction, he did not himself follow that direction to the end, or even very far. Quine never became deeply involved in interdisciplinary research with psychologists or linguists; and it does not even appear that he followed very closely the progress of those sciences since they turned away from behaviorism. Quine's approach to philosophy of science in general and philosophy of mathematics in particular remained one that focused on very general features, and did not much concern itself with the details of the current state either of physics or of mathematics . . . As Moses himself never entered the promised land, so Quine's own epistemology was never naturalized in the fullest sense. He nonetheless remains, for many contemporary philosophers who describe themselves as naturalists, an inspiring prophet of naturalism. (Burgess 2014, 293–94)

Second, one of the main task of philosophy is to simplify, refine, and clarify our conceptual scheme from within the conceptual scheme of science (chap. 5 and Morris 2015, 135). Third, it is a fundamental tenet of Quine's naturalism that there is no first

[10]The first incompleteness theorem states that, in any consistent formal system S within which elementary arithmetic is expressible, there are sentences of the language of S which can neither be proved nor refuted in S. In short, no consistent formal system can capture all arithmetical truths. The second incompleteness theorem states that, in such a formal system, it cannot be proved that the system itself is consistent.

philosophy separate from science (79, 104, 188). Fourth, following Russell and Carnap, Quine is committed to the idea that "the fundamental approach to philosophy should be a scientific one" (134; see also 194 and Morris 2020, section 1 for a commentary on the notion of "scientific philosophy").

Many objections have been addressed to Quine's naturalism and many of them simply are misunderstandings of Quine's philosophy (see Ebbs 2011). One specific difficulty of Quine's naturalism has not really been addressed in the literature. It concerns the relation between two basic claims central to Quine's naturalism: the "abandonment of the goal of a First Philosophy prior to natural science" (Quine 1981, 67) and to science in general; the claim that philosophy is continuous with science. Quine recurrently characterizes his naturalism by means of the first claim. Although the assertion that philosophy is "continuous" with science is a different and stronger assertion, he often seems to identify it with the first claim or, at least, to consider it as implied or presupposed by the first claim. Yet that philosophy should not be conceived as an "*a priori* propaedeutic or groundwork for science" (Quine 1969b, 126) neither means nor implies that philosophy is continuous with science. To reject any view of philosophy as first philosophy is to consider that philosophy is not an inquiry that lays down, on *a priori* grounds—in the traditional sense of the notion of *a priori*—metaphysical and epistemological warrants for knowledge in general. That philosophy should be conceived as continuous with science means positively that philosophical should be done and pursued from within science and that philosophical problems should be reconceived as emerging from within science and addressed from within science. In this sense, philosophy and science are "in the same boat". Even though the negative claim is a necessary condition for establishing the positive claim, it is by no means a sufficient condition. Quine's rejection of philosophy as first philosophy is compatible with a different sort of relation between science and philosophy that preserves the difference in nature between the two. In other words, on the ground of the negative claim, Quine could have reconceived the task and mode of philosophy in many different ways. Quine's claim that science is continuous with commonsense does not fill the gap in the transition from the negative claim to the positive one. Even if philosophy were to lose its "autonomy" and *sui generis* character, why does it follow that it should be continuous with science, even in the broadest sense, and not with art or literature? To put it another way, Quine might be right when he says that science takes care of itself but wrong when he thinks that "philosophy of science is philosophy enough" (Quine 1953, 151).

Morris never confronts this difficulty, even though he assumes that Quine's naturalism involves both the negative and the positive claims. He goes as far as contending that "[i]n adopting an extensional view of logic, Quine pushes aside difficult, if not hopeless, worries that are external to mathematics and thus opens the way to a philosophy of mathematics that takes place *entirely* within mathematics itself" (104; my emphasis). Quine's naturalism is not reducible to naturalized epistemology which is only an aspect of it. But the idea of a reciprocal containment is still valid for characterizing Quine's naturalism. There is a reciprocal containment, though a containment in different senses: philosophy in science and science in philosophy. Whereas Morris elucidates how philosophy is supposed to be "contained" in mathematics and, in particular, in set theory, he does not offer many clues about the other way around, that is the alleged containment of set theory in philosophy. This makes hard to believe that, on Quine's naturalism in philosophy of mathematics, philosophy of mathematics would "take place *entirely* within mathematics itself". This last difficulty relates to what could be legitimately thought of as a retrospective fallacy in many of contemporary interpretations of Quine's philosophy.

Sander Verhaegh has convincingly shown that "Quine does not even label his view 'naturalistic' before the late 1960s" (Verhaegh 2018, 7). He adds that "in the first forty years of his philosophical career, his naturalistic commitments were largely implicit" (Verhaegh 2018, 7) and "although Quine was always a science-minded philosopher, he did not adopt a fully naturalistic perspective until the mid-1950s" (Verhaegh 2018, 11). In different ways, Verhaegh and Morris are both interested in the evolution of Quine's philosophy from his alleged early and non-reflexive naturalism to his late and reflexive naturalism. In both cases, it is argued that, first, some of the main features of this late naturalism would be already implicitly present and at play in Quine's early works; second, Quine's late naturalism would be the key to understanding Quine's philosophical contributions in their diversity.

That Quine came to describe his own philosophical stance by means of the label "naturalism" is one thing. That naturalism, as explicitly endorsed and characterized by Quine, is sufficient to account for Quine's various philosophical contributions is quite another thing. The retrospective fallacy consists in interpreting Quine's many celebrated criticisms, problems, or theses elaborated from the early 1930s to the 1950s in light of his late naturalism. The criterion of ontological commitment, the adherence to a univocal, amodal, and non-relative conception of truth, the substitutional definition of logical truth, the reciprocal containment of logic and ordinary language, the reinterpretation of the traditional *de re*/*de dicto* distinction, all these basic themes integral to Quine's conception of logic would be, in one way or another, exemplifying his naturalism (whether it be by being deducible from it, by instantiating it or foreshadowing it). One may be prone to reply that they first were articulated without being explicitly logically parasitic on self-conscious naturalistic positions. The question is then whether they can be legitimately interpreted through the lens of Quine's late naturalism. This is less than evident, since a naturalistic interpretation of classical pieces like "On What There Is", "Quantifiers and Propositional Attitudes", or "Reference and Modality" seems to distort Quine's arguments and the problems underlying and motivating these arguments (see also Hill 2011, 118). How to account for Quine's criterion of ontological commitment or his conception of truth in a naturalistic manner without distortion is something no one to this day has shown but that is surely one of the most important challenges for whoever claims that the heart of Quine's systematic philosophy is naturalism.

In the end, any "naturalistic" interpretation of Quine's philosophy faces the following dilemma: either Quine's views are interpreted as naturalistic but such an interpretation would lead to serious distortions, omissions, or misconceptions; or, Quine's naturalism is construed in a sufficiently broad sense to be faithful to Quine's philosophy in its diversity but then the description of some of Quine's views as "naturalistic" would prove to be too vague and indeterminate to be philosophically significant. One way out of this dilemma would be to construe Quine's philosophy as naturalistic and systematic but not systematically naturalistic.

None of these preceding remarks are meant to belittle Morris's investigations. Nor it was in my aim to argue for specific interpretations about different aspects of Quine's philosophy relevant to Morris's book. Instead, each of these critical comments is meant to point out that, in a way or in another, some connections between distinct problems, themes, or theses integral to Quine's philosophy of set theory are missing in Morris's argumentation. I have raised these points not to cavil at a work that represent both a significant and timely change in perspective on Quine's work, but in the hope that it will be completed and deepened. Quine's philosophy—even in his naturalistic aspects—can not be properly understood and assessed without taking seriously that it is framed from a logical point of view. Morris's investigations represent one of the most important contributions to the task of bringing to light the distinctiveness and fruitfulness of Quine's logical point of view.

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