Journal for the History of Analytical Philosophy

Volume 9, Number 7

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In his 1918 logical atomism lectures, Russell argued that there are no molecular facts. But he posed a problem for anyone wanting to avoid molecular facts: we need truth-makers for generalizations of molecular formulas, but such truth-makers seem to be both unavoidable and to have an abominably molecular character. Call this the *problem of generalized molecular formulas*. I clarify the problem here by distinguishing two kinds of generalized molecular formula: *incompletely generalized* molecular formulas and *completely generalized* molecular formulas. I next argue that, if empty worlds are logically possible, then the model-theoretic and truth-functional considerations that are usually given address the problem posed by the first kind of formula, but not the problem posed by the second kind. I then show that Russell's commitments in 1918 provide an answer to the problem of completely generalized molecular formulas: some truth-makers will be non-atomic facts that have no constituents. This shows that the neo-logical atomist goal of defending the *principle of atomicity*—the principle that only atomic facts are truth-makers—is not realizable.

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1. Introduction

Russell argued in his 1918 logical atomism lectures that there are no molecular facts, though he held that there are positive and negative atomic facts, and that there are universal and existential facts. However, he posed a problem for anyone wanting to avoid molecular facts: we need truth-makers for generalizations of molecular formulas—formulas such that a molecular connective occurs in the scope of a quantifier. But such truth-makers seem to be have an abominably molecular character because some of the formulas that they would make true are ineliminably molecular. Call this the *problem of generalized molecular formulas*.

What often is missed in discussions of molecular facts is how difficult Russell's problem is. Most scholars working in metaphysics today agree that molecular facts are to be avoided if possible. They usually try to avoid such molecular truth-makers by appealing to truth-functional considerations: once we have truth-makers for the formulas '*Fa*' and '*Gb*,' we can maintain that what makes the formula '*Fa* ∧ *Gb*' true is the non-molecular facts *Fa* and *Gb*.

As I argue here, such reasoning does not avoid the need for molecular truth-makers because it does not apply to generalized molecular formulas. First, I distinguish two different kinds of generalized molecular formula, *incompletely generalized* and *completely generalized* ones. The former might be called material claims because they involve at least one non-variable term. Two

examples are 'all humans are mammals' and 'Being a biological sibling is a symmetric relation'. The latter might be called logical claims because they involve only variable terms. Examples of completely generalized molecular formulas include

$$'\forall x[x = x \rightarrow x = x]', \quad '\forall x \forall F[Fx \rightarrow Fx]', \quad '\exists x \exists F[Fx \rightarrow Fx]',$$

and so on.

This distinction is new—Russell himself does not explicitly make this distinction in the logical atomism lectures—and also necessary because the latter kind of formula poses more serious difficulties. The standard truth-functional reasoning above applies to incompletely generalized molecular formulas. We do not need molecular truth makers for them.

Truth-functional considerations do not, however, apply to the second sort of formula. In arguing for this point, I give some motivation for the following three claims:

(a) An empty world is logically possible.

(b) In all worlds logic is *non-trivial* in that some but not all logical formulas are true.

(c) *Ante rem* realism, or 'platonism', about universals is false.

If these three claims are accepted, then we have true and ineliminably molecular formulas whose truth-makers accordingly must be non-atomic. Such facts must also make true completely generalized molecular formulas even at the empty world. Assuming *ante rem* realism is also false, then such truth makers cannot have any (concrete or abstract) constituents.

There are two main takeaways from this conclusion. One is for truth maker theorists: as we will see, there is a tradition in truth maker theory that identifies non-atomic facts with mereological sums of other facts. Contrary to that tradition, if the empty world is possible, then some non-atomic facts cannot be mereological sums of entities because there are no concreta to sum

in the empty world. Accordingly, the argument will show that some non-atomic facts are *sui generis* entities that would exist even if nothing concrete did. Thus, reconsidering the problem of generalized molecular formulas raised in Russell's 1918 logical atomism lectures indicates unrecognized but severe difficulties in the current accounts of non-atomic facts as compositions, mereological or otherwise, of atomic facts.[1]

The second takeaway is for the history of analytic philosophy: as we will see in Section 2, there is a widespread view that logical atomism is committed to the *principle of atomicity*, the claim that only atomic facts are truth-makers. The argument for non-atomic facts here, developed out of Russell's remarks in the logical atomism lectures, shows that this principle implausible on both metaphysical and historical grounds. Rejecting the principle of atomicity has a long history: past philosophers, like Russell and Armstrong, and present ones, like Barker and Jago, have previously argued for non-atomic truth-makers.[2] The argument here suggests that we can go further: accepting claims (a), (b), and (c), the principle of atomicity cannot be taken as definitive of, or implied by, any plausible form of logical atomism. And on the historical grounds of Russell's own ontological commitments to non-atomic facts in the 1918 logical atomism lectures, the principle of atomicity cannot plausibly be taken as definitive of, or implied by, the Russell brand of logical atomism.

The outline of my argument is as follows. The interpretation of logical atomism as defined by, or at least crucially committed to, the principle of atomicity is presented in Section 2. Then Russell's arguments in the logical atomism lectures are discussed: his

case against molecular facts, argument for general facts, and his worry about generalized molecular formulas are considered in Sections 3–5 respectively. In Section 5 the distinction between incompletely and completely generalized molecular formulas are put to use: it is shown using Tarski-style, model-theoretic and truth-functional considerations that one can provide truth-makers for incompletely generalized molecular formulas using atomic facts alone. In Section 6 it is argued that completely generalized molecular formulas do require non-atomic facts as truth-makers and a valid five-premise argument to that effect is given. In Section 7 the truth of those premises, and so the soundness of the argument, is independently motivated. In Section 8 it is argued that, faced with the choice between positing, as *sui generis* entities, molecular facts without constituents or general facts without constituents, we should posit the latter.

2. Logical Atomism and the Principle of Atomicity

Some scholars characterize logical atomism as a view about truth-makers.[3] Some aspects of modern truth-maker theory are anticipated in Russell's 1918 logical atomism lectures and in Wittgenstein's 1921 *Tractatus*.[4] *Truth-makers* are the really existing stuff that makes formulas true, or in virtue of which formulas are true, when they are true.[5] *Truth-maker theories* are views that

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[1]A good discussion of non-mereological composition is in Barker and Jago (2012, §3).

[2]Armstrong defended general facts (2004, 74; 1997, 135). Russell defended general and negative facts (1918/1986, 190, 207). Perovic (2018, §2) has argued that Russell offers his argument for negative facts tentatively, though he seems confident in positing them. Not to be outdone by the dead, Barker and Jago posit general, molecular, and negative facts (2012, 125–26).

[3]'Logical atomism is a view about the relationship between truths and what make truths true. . . If P is a class of propositions, logical atomism with respect to P is the view that all the true propositions in P are made true by atomic facts' (Simons 1992, 158, 160).

[4]'When I speak of a fact. . . I mean the kind of thing that makes a proposition true or false' (Russell 1918/1986, 163). Wittgenstein seems to view facts as truth-makers without describing them in those words (1922/1971, 2.222, 4.063, 5.101). The notion of correspondence occurs in his early writings, too, including his 1913 *Notes on Logic* (1913/2009, B10).

[5]'The idea of a truthmaker for a particular truth, then, is just some existent, some portion of reality, in virtue of which that truth is true' (Armstrong 2004, 5).

accept the following principle: if a formula ϕ is true, then there is some really existing stuff f such that f makes true ϕ.[6] The exact meaning of the 'makes true' locution is a matter of debate, but we will not need to digress on the various accounts of 'makes true' locution here.[7] Without delving into that issue, we can say that the family of truth-maker theories, disagreeing though they do in the details, agree on the truth-maker principle at the generic level.[8] This is not to say those details are unimportant: truth-maker theory stands or falls with its account of 'makes true' locution, that is, of what making true consists in. That in turn depends on the broader account of truth that is accepted. Here I do not propose to defend a theory of truth. Our main interest is in logical atomism, and especially in the Russellian brand of it. So we can for present purposes assume the *correspondence theory of truth* according to which what it is for a formula to be true is for it to correspond to some really existing stuff. We can then hang the truth-maker theory upon that view by adding that the really existing stuff to which a true formula corresponds is a truth-maker.[9]

Unfortunately, it is quite difficult to determine what really existing stuff is needed to make formulas true. Let us take a

true affirmative claim, like 'Bertrand Russell is human'. For the sake of argument, let us suppose that it is an atomic fact that makes this true, that is, an atomic fact Hb, which is an instance of the general form $R^n(a_1, \ldots, a_n)$. Atomic facts consist of some property holding of one or more things. They are, as Russell says, 'as facts go very simple' (Russell 1918/1986, 177). Most everyone who buys into truth-maker theory and an ontology of facts, or states of affairs, posits such really existing stuff like atomic facts.

Now take any negative true claim, like 'Bertrand Russell is not alive', which for the sake of argument we will suppose is similarly made true by an atomic fact. But now truth-maker theorists may have a problem. This fact appears to have a rather different form from whatever is a truth-maker for a positive true claim, for it is precisely the property of *being alive* failing to hold of Russell that makes it the case that he is not alive. Yet this places us in the uncomfortable position of saying that the really existing stuff, which makes it true that 'Bertrand Russell is not alive', is somehow not existing, uncombined, or combined differently.

Enter logical atomism. According to some truth-maker theorists, *logical atomism* is the view that accepts the following principle: only positive atomic facts are truth-makers (Russell's own 1918 view notwithstanding).[10] Different scholars assess this thesis with quite varying degrees of enthusiasm, but a fair number of them seem united in viewing logical atomism as the thesis in truth-maker theory that only atomic facts are truth-makers:

> The glory of logical atomism was that it showed that not every kind of sentence needs its own characteristic truth-maker. Provided we can account for the truth and falsehood of atomic sentences, we can

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[6]'When there is truth, there must be some thing (or things) to account for that truth: some thing(s) couldn't exist and the true proposition fail to be true. That is the truthmaker principle. True propositions are made true by entities in the mind-independently existing external world' (Cameron 2008, 412).

[7]See MacBride (2016, §1) for an overview and references.

[8]Hence Cameron (2005, 43) could over fifteen years ago call truth-maker theory a 'familiar thought' despite the debate, which was ongoing then and continues to the present day, over how to understand 'makes true' and over the nature of both whatever is made true and the really existing stuff that makes true.

[9]The relationship between truth-maker theory and the correspondence theory of truth is a bit more complex than this lets on. Truth-maker theory can in fact be viewed as either an alternative or a version of the correspondence theory of truth (David 2016, §8.5). The correspondence theory also arguably can motivate truth-maker theory, as Armstrong (1997, 14) has noted; for discussion, see MacBride (2016, §3.3).

[10]As mentioned above in footnote 2, Russell defended negative atomic facts, however tentatively. So there would seem to be some room for debate on this score amongst logical atomists. Still, modern scholars who interpret logical atomism as the view that accepts the principle of atomicity generally hold that logical atomism admits only positive atomic facts. Since general and molecular formulas will be our concern below, we can set aside the issue of negated atomic formulas and whether they have 'negative' truth-makers when they are true.

dispense with special truth-makers for, e.g., negative, conjunctive, disjunctive, and identity sentences (Mulligan, Simons and Smith 1984, 289).

The idea that the complex natural world is reducible to ontologically simple objects and atomic states of affairs is a difficult, if not impossible, thesis to defend (Cocchiarella 2007, 141).

Logical atomism is designed to go with the ontological view that the world is the totality of atomic facts. . . doing without funny facts: atomic facts are all the facts there are. . . Logical complexity, so the idea goes, belongs to the structure of language and/or thought; it is not a feature of the world (David 2016, §7.1).

Following Russell, who himself attributes a version of it to Wittgenstein, let us name the claim that only positive atomic facts are truth-makers the *principle of atomicity*.[11] Note that this claim does not imply that all truth-bearers are atomic: the *atomistic hierarchy of sentences*, as Russell calls it, is the collection of all formulas closed under substitution, truth-functional combination, and generalization.[12] It is the collection of all formulas that can be constructed out of elementary formulas from these specific operations.[13] The chief point is that only positive atomic facts are truth-makers, even though not all formulas are atomic. This at least is the reading of modern scholars who prefer Russell's later logical atomism on which the principle of atomicity

is embraced, so that there are no negative atomic, molecular, or general facts. (A note on terminology: when I use the phrases 'molecular facts' or 'molecular formulas' below, I will not use them to include negative atomic facts and negated atomic formulas.) An upshot of the view that embraces the principle of atomicity is a sparse ontology: the world consists of only positive atomic facts and their constituents—one logical kind of fact, plus whatever is a constituent of them.

The argument below shows that the principle of atomicity is false. Consideration of a certain kind of formula—generalized molecular ones—will show that we cannot get by with only atomic facts. A corollary of this is that if logical atomism is critically committed to the principle of atomicity, then logical atomism is not viable. Russell saw this and argued the point in his logical atomism lectures: he first dispenses with molecular facts, then suggests we need general facts, and finally points to generalized molecular formulas as posing a difficulty for his rejection of molecular facts. We will develop this point further than Russell did, but first we review Russell's arguments in Sections 3–5.[14]

3. Russell's Case for Dispensing with Molecular Facts

Russell dispenses with molecular facts by assuming an ontology of atomic facts $R^n(a_1, \ldots, a_n)$ as truth-makers for atomic formulas.[15] This is unproblematic from the modern point of view

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[11]'. . . [the principle of atomicity] states that everything we wish to say can be said in sentences belonging to the "atomistic hierarchy" which will be defined in section C of Chapter 13' (Russell 1940/1973, 160). See footnote 12.

[12]'. . . I shall call the assemblage of sentences obtained from atomic judgments of perception by the three operations of substitution, combination, and generalization, the *atomistic* hierarchy of sentences' (Russell 1940/1973, 187).

[13]'Suppose that I am given *all* elementary propositions: then I can simply ask what propositions I can construct out of them. And there I have *all* propositions, and *that* fixes their limits. Propositions comprise all that follows from the totality of all elementary propositions (and, of course, from its being the *totality* of them *all*). (Thus, in a certain sense, it could be said that *all* propositions were generalizations of elementary propositions)' (Wittgenstein 1922/1971, 4.51–4.52).

[14]Note that logical atomism is arguably not committed to the principle of atomicity for the additional reason that logic, not any specific metaphysics, is fundamental to Russell's logical atomism; for discussion, see Elkind (2018, 29) and Maclean (2018, 87).

[15]'There you have a whole hierarchy of facts—facts in which you have a thing and a quality, two things and a relation, three things and a relation, four things and a relation, and so on. That whole hierarchy constitutes what I call *atomic* facts, and they are the simplest sort of fact. . . The propositions expressing them are what I call atomic propositions' (Russell 1918/1986, 177).

in that it is standard among metaphysicists to posit something analogous to atomic facts.

With these raw materials, Russell argues that molecular facts are dispensable. He focuses on molecular formulas whose constituent formulas are atomic.[16] He then argues that their truth-conditions are given entirely by the those of their constituent atomic formulas, plus the truth-tables for each connective. For example, the disjunctive claim '$p \lor q$' is true if either there is an atomic fact corresponding to the claim 'p' or one corresponding to the claim 'q'.[17] So only atomic facts are needed to make disjunctive claims true.

Russell's truth-functional argument generalizes to the other binary truth-functional connectives (Russell 1918/1986, 185–86). In each case, we already have truth-makers for molecular formulas given an ontology of atomic facts plus the truth-functions' definitions.

Many truth-maker theorists disagree with Russell's 1918 arguments for positing other kinds of facts like general, existence, or negative facts. In contrast, Russell's argument for dispensing with molecular facts as truth-makers for molecular formulas is widely seen as persuasive among truth-maker theorists today:

Disjunctive sentences raise no special problems for the theory, since a disjunctive sentence is true only to the extent that one or other of its disjuncts is true. . . (Mulligan, Simons and Smith 1984, 314).

He [Russell] draws the line at disjunctive facts, for obvious reasons. All that is required for them is a truthmaker for at least one disjunct, and then there seems no need to postulate disjunctive facts in addition (Armstrong 2004, 54).

There is consensus in the literature that not every proposition has its own distinctive truthmaker. For instance, disjunctions are thought to be made true, *separately*, by the truthmakers for their disjuncts...Thus there is no need to postulate a distinctive kind of entity, like *disjunctive states of affairs*, that is supposed to make disjunctions true. . . (Rodriguez-Pereyra 2006, 193).

It's not unreasonable to think that no further truthmaker is needed for a conjunction than the truthmakers for each of its conjuncts, or that once you make a proposition true you thereby make true any disjunction of which that proposition is a disjunct. . . (Cameron 2008, 411).

. . . once truth-makers have been supplied for the atomic truths, there is simply *no need* to posit further truth-makers for the molecular ones. All we need to recognise is that an atomic statement is true whenever a truth-maker for P exists, that P is false if and only if no truth-maker for P exists. Once the existence of and non-existence of the truth-makers has settled the truth-values of all atomic statements, the logical operations described by the truth-tables then settle the truth and falsity of all molecular statements. . . (MacBride 2016, 36; see 25–26).

This consensus is, so far as the literature shows, widely-accepted. Now if we accept that, for any two facts, there is a fusion of them, then we have fusions of facts that look 'conjunctive' and 'disjunctive' (Jago 2011, 44). But even fans of fused facts reject that there are facts having a conjunctive or disjunctive structure (Barker and Jago 2012, 126).

At this juncture, the principle of atomicity is still viable: so far, only positive atomic facts are truth-makers.[18] Russell's truth-functional argument is persuasive, and welcomed by those who

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[16]'But I am talking today about molecular propositions, and you will understand that you can make propositions with "or" and "and" and so forth, where the constituent propositions are not atomic, but for the moment we can confine ourselves to the case where the constituent propositions are atomic' (Russell 1918/1986, 184–85).

[17]'That is to say, the truth or falsehood of this proposition "p or q" depends upon two facts, and not upon one, as p does and as q does' (Russell 1918/1986, 185).

[18]Again, this sets aside whether we need negative facts as truth-makers for negated atomic formulas. If Russell was right and we do need negative facts, then we would need to say 'positive and negative atomic facts' instead, and

find non-atomic facts uncomfortable. But note that Russell's argument only dispenses with molecular facts because it only deals with molecular formulas whose constituent formulas are atomic like '*Fa* ∨ *Ga*'. To show that all non-atomic facts are dispensable, we must consider other sorts of formulas, including those whose constituent formulas are not atomic. Russell does consider such kinds of formula in the logical atomism lectures and reasons that some non-atomic facts—general and existence ones—are unavoidable.

4. The Argument for Positing General Facts

Russell next argues that general and existential formulas are made true by a special kind of fact, namely, *general facts* and *existence facts*, respectively. Here I am mainly concerned with general facts, so I will concentrate on his argument for those. But a brief detour into Russell's argument for existence facts will facilitate discussion of his case for general facts.

 Perhaps surprisingly, Russell says that positing existence facts will be readily agreed to:

> Of course, it is not so difficult to admit what I might call existence-facts—such facts as 'There are men', 'There are sheep', and so on. Those, I think, you will readily admit as separate and distinct facts over and above the atomic facts I spoke of before. Those facts have got to come into the inventory of the world, and in that way propositional functions come in as involved in the study of general facts (Russell 1918/1986, 207).

Russell likely thinks that his interlocutors will 'readily admit' existence facts because, as a point of *Principia*-logic, there will be an existence fact corresponding to a formula $(\exists x) . R(\dots, x, \dots)$ if and only if there are one or more atomic facts corresponding

to a formula $R(\dots, a, \dots)$.[19] Certainly *Principia*'s *9·1, which is ⊢: $\phi x . \supset . (\exists z) . \phi z$, supports Russell's reasoning from atomic formulas to their existential generalizations. And it may also be, although *Principia* does not hint at this issue, that, as Klement (2004, 28) suggests, Russell in these lectures found problematic the fact that the semantics for a generalized formula is not reducible to the truth-value of some or all of its instances. A key phrase here seems to be 'and in that way propositional functions come in as involved in the study of general [universal or existence] facts'. It seems that Russell's confidence in the case for existence facts partly lies in his view that the semantic clauses for propositional functions is not eliminable through the semantics for atomic and molecular formulas that are their instances. As Russell (1918/1986, 204) sees it, to say that '$(\exists x) . R(\dots, x, \dots)$' is true is not tantamount to saying $R(\dots, a, \dots)$; rather, one is saying that there is a value of x for which '$(\exists x) . R(\dots, x, \dots)$' is true. Russell seems to be leaning on this semantic point in briefly arguing for positing existence facts.[20]

In contrast, according to Russell's argument discussed below, general facts may exist even when there are no atomic facts. Indeed, *Principia*'s primitive proposition *9·13 is a meta-theoretic inference rule allowing inferences to $(x) . \phi x$ from the truth of ϕy however the value of y is chosen. This logical point, that when no value of y is possible $(x) . \phi x$ is satisfied because the 'however the value of y is chosen' clause becomes vacuously satisfied, resurfaces in Russell's reasoning for general facts, as we will see.

Turning now to Russell's case for general facts, he first points out that we logically cannot infer any general formula merely on the basis of enumerating all its instances: for the general formula

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would accordingly have to embrace at best a modified version of the principle of atomicity.

[19]Put a little more precisely, it will be an existence fact that a propositional function $(\exists x) . R(\dots, x, \dots)$ is possible, that is, it is true for some value. See Russell (1918/1986, 204–5; 1919, 159–60).

[20]I thank a reviewer for inviting a brief discussion of Russell's case for existence facts.

to follow, one needs the further claim that the enumerated instances are all of them. So no general formula is logically implied by any collection of only atomic formulas:

> You can never arrive at a general proposition by inference from particular propositions alone. You will always have to have at least one general proposition in your premises. That illustrates, I think, various points. One, which is epistemological, is that if there is, as there seems to be, knowledge of general propositions, then there must be *primitive* knowledge of general propositions (I mean by that, knowledge of general propositions which is not obtained by inference), because if you can never infer a general proposition except from premises of which one at least is general, it is clear that you can never have knowledge of such propositions by inference unless there is knowledge of some general propositions which is not by inference (Russell 1918/1986, 206).

He then suggests that this point about inference supports positing general facts:

> . . . I do not think one can doubt that there are general facts. It is perfectly clear, I think, that when you have enumerated all the atomic facts in the world, it is a further fact about the world that those are all the atomic facts there are about the world, and that is just as much an objective fact about the world as any of them are. It is clear, I think, that you must admit general facts as distinct from and over and above particular facts. The same thing applies to 'All men are mortal.' When you have taken all the particular men that there are, and found each one of them severally to be mortal, it is definitely a new fact that all men are mortal; how new a fact, appears from what I said a moment ago, that it could not be inferred from the mortality of the several men that there are in the world (Russell 1918/1986, 207).

Russell thus takes a point about the logical relationship between claims to suggest something about what sort of facts make them true. General claims like 'those are all the atomic facts in the world' and 'all humans are mortal' are not logically implied by the conjunction of their instances. As such, all the atomic facts

alone—the facts corresponding to all the atomic claims that are instances of the general claims—do not make the general claim true.

Russell's argument for general facts may seem to rely on a key principle that most modern truth maker theorists accept, *Truth-Maker Necessitarianism* (Rodriguez-Pereyra 2006, 188):

TMN Necessarily, if f is a truth-maker for ϕ, then that f exists implies ϕ.

The word 'implies' here requires some qualification: Greg Restall in his critical discussion of **TMN** shows that the logical entailment cannot be right reading of 'implies' here (2008, 89).[21] We can set them aside here: whatever the right reading of 'implies' in **TMN** is, Russell does not appeal to this principle. **TMN** concerns some kind of 'implies' relationship between facts and formulas.

Russell would not have endorsed **TMN** because it is difficult to see how it could be explicated without involving some metaphysical necessity between facts and formulas that is clearly not logical. Indeed, in symbols **TMN** looks like

$$\Box\{(f \rightarrow_{TM} \phi) \rightarrow_L (\mathbf{E!}f \rightarrow_L \phi)\},$$

where '$\rightarrow_{TM}$' stands for the truth-making relation between a truth-maker f and a truth-bearer ϕ, and '$\rightarrow_L$' stands for logical entailment between truth-bearers $\mathbf{E!}f$ and ϕ.[22] In many places, Russell (1918/1986, 205–6; 1919, 165–66) rejects all such nonlogical necessary relations and modalities such as '$\rightarrow_{TM}$' would

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[21]Lewis proposes **TMD**: for any two worlds w and v, if ϕ is true in w but not true in v, then some f exists in w but not in v (2001, 606). See also Armstrong (2004, 69) and Rodriguez-Pereyra (2006, §2).

[22]Note that we can eliminate the apparent term 'f' for a truth-maker, which f would presumably be logically complex, with $\exists R^n \exists x_1 \ldots \exists x_n [R^n(x_1, \ldots, x_n) \rightarrow_{TM} \phi]$. **TMN**, in symbols, would then be: $\Box\{\exists R^n \exists x_1 \ldots \exists x_n [R^n(x_1, \ldots, x_n) \rightarrow_{TM} \phi] \rightarrow_L (\exists R^n \exists x_1 \ldots \exists x_n [R^x(x_1, \ldots, x_n)] \rightarrow_L \phi)\}$.

indicate.[23] So **TMN** cannot be the principle Russell is using in his argument for general facts.[24]

Russell's argument rather appeals to the principle that whenever one cannot validly infer one formula ϕ from others $\phi_1, \phi_2, \ldots$, that is, whenever implication from $\phi_1, \phi_2, \ldots$ to ϕ does not obtain, then their corresponding facts must differ.[25] Call this principle *Implication under Truth-Making*:[26]

ITM If all the formulas $\phi_1, \phi_2, \ldots$ jointly do not logically imply the formula ϕ, then the facts $f_1, f_2, \ldots$ that make true $\phi_1, \phi_2, \ldots$ are such that their existence does not imply ϕ.

Let '$(\imath f_1), (\imath f_2), \ldots$' abbreviate 'the facts that make true $\phi_1, \phi_2, \ldots$,' taking for granted that we can generally identify, as we can readily with atomic formulas, what facts make true a given ϕ through the notion of a fact corresponding to a formula. Then **ITM** in symbols is:

$$\neg[(\phi_1 \land \phi_2 \land \ldots) \to_L \phi] \to_L \neg[(\imath f_1), (\imath f_2), \ldots][(\mathbf{E!}f_1 \land \mathbf{E!}f_2 \land \ldots) \to_L \phi].$$

Taking the contrapositive gives us that if some of the facts that make true the formulas $\phi_1, \phi_2, \ldots$ imply the formula ϕ, then some of $\phi_1, \phi_2, \ldots$ together imply ϕ. Hence the name 'implication under truth-making'.

No modalities untoward to Russell are implicit in **ITM**: there are no cross-categorical ties of necessitation like $\to_{TM}$, nor necessary relations between entities—at least assuming, as Russell did,

that logically implications between truth-bearers do not presuppose such modalities.[27] And we saw above that Russell explicitly asserts **ITM** in the text.

Russell's point is that the atomic claims '$Ha \to Ma$', '$Hb \to Mb$', and so on, do not logically entail the general claim '$\forall x[Hx \to Mx]$'. Thus, by **ITM**, the atomic facts making true the molecular claims like '$Ha \to Ma$' cannot make true the general claim. So we have to posit at a truth-maker of a non-atomic and general character, namely, a general fact.

Utilizing **ITM**, Russell's argument can be rendered as follows:

Russell's Argument for General Facts

(1) **ITM** and no collection of atomic formulas   Premise
     of the form '$Ha_i \to Ma_i$' logically implies
     '$\forall x[Hx \to Mx]$'.

(2) If Premise (1), then there are non-atomic facts   Premise
     as truth-makers.

(3) So, there are non-atomic facts as truth-makers.   By 1–2

I expect that many truth-maker theorists would find Russell's century-old argument compelling. It avoids problems plaguing **TMN** and appeals to a plausible truth-making thesis.

Russell's argument, if accepted, requires that we abandon the principle of atomicity: we have non-atomic facts as truth-makers. Such facts presumably have a general character or logical form, where 'general fact' here is shorthand for either a universal or existential fact.

As Russell noted in the logical atomism lectures, the issues of whether there are general facts or molecular facts are quite closely interrelated: Russell worried that if we accept general facts as truth-makers for general formulas, then we may have to posit molecular facts after all, as truth-makers for generalized

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[23]Russell (1919, 153–54) also rejects the restriction that inferences involving molecular formulas, like disjunctive syllogism, are based on strict entailment; Russell holds that material implication—a truth-functional connection—suffices for validity in all cases where logical inferences involving molecular formulas are made.

[24]I thank Gregory Landini for bringing up this point in conversation.

[25]Following early Russell (1937, §37), I am reading Russell's talk of inference on page 206 as parasitic on logical implication between formulas, although without early Russell's metaphysic of propositions.

[26]Notice that the converse of **ITM** and other principles like it are unacceptable: they imply the paradoxical claims like *truth-maker monism*, that every truthmaker makes true every truth (Restall 2008, 89–90).

[27]Russell claims that the notion of logical implication, whether material or formal, does not involve any modal notions, even of necessity or possibility in a logical sense, but is explicable using truth-functional connections. See Russell (1905/1994, 508, 511; 1908/2014, 583).

molecular formulas. Happily, once we distinguish between the kinds of generalized molecular formula, we can address Russell's worry.

5. Incompletely Generalized Molecular Formulas

Russell was among the first to give the now widely-accepted argument for dispensing with molecular facts discussed in Section 4. He also noticed that the reasoning given in Section 3 above is difficult to extend in some cases where the molecular formulas occurs within a quantifier's scope:

> There is one point about whether there are molecular facts. I think I mentioned, when I was saying that I did not think there were disjunctive facts, that a certain difficulty does arise in regard to general facts. Take 'All men are mortal.' That means: 'x is a man' implies 'x is a mortal' whatever x may be... It is perhaps a little difficult to see how that can be true if one is going to say that "'Socrates is a man" implies "Socrates is mortal"' is not itself a fact, which is what I suggested when I was discussing disjunctive facts. I do not feel sure that you could not get round that difficulty. I only suggest it as a point which should be considered when one is denying that there are molecular facts, since, if it cannot be got round, we shall have to admit molecular facts (Russell 1918/1986, 208).

Russell's concern is that the truth-conditions of a formula such as 'for all x, if x is human, then x is mortal' cannot be adequately explained by appealing only to atomic facts. This is because, unlike in the case of 'Hs implies Ms'—'Socrates is human implies Socrates is mortal'—the molecular formula occurs within the scope of a formula, and its terms are bound by an initial quantifier. That is, while the truth-conditions of 'Hs implies Ms' are given by the truth-conditions of its atomic constituents, the formula '$\forall x[Hx \rightarrow Mx]$' is not: it is not semantically equivalent either to '$\forall x[Hx]$ implies $\forall x[Mx]$' or to any collection of instances of 'Ha implies Ma', 'Hb implies Mb', and so on. So one

might be tempted to think that the 'molecularity' of the formula '$\forall x[Hx \rightarrow Mx]$' is in a sense trapped by the universal quantifier, so that it seems to have a molecular truth-maker.

One can see how Russell in 1918 was concerned about this. But today we are blessed with model-theoretic semantics: in 1933, Tarski showed how to recursively define the truth of quantified formulas through their instances. One simply says that a universally quantified formula $\phi(x)$ is satisfied if and only if every assignment of the variable x to some object (in the domain) results in a true formula (Mendelson 1997, 59–60). Supposing that the truth-conditions of the formula '$\forall x[Hx \rightarrow Mx]$' are completely given by model-theoretic account of its truth as satisfaction, the generalized molecular formula's truth is given by the truth-conditions of all its instances, which in turn are given by the truth-table for implication. So we once again have all and only atomic formulas as truth-makers for molecular formulas.

Yet this Tarskian reasoning, as we will see in Section 6, only applies to one of two kinds of generalized molecular formula. Let us define a *generalized molecular formula* to be a quantified formula in which some bound variable term occurs on both sides of the main truth-functional connective within the scope of the quantifier. For example in '$\forall x[\phi(x) \rightarrow \psi(x)]$' the bound variable '$x$' occurs in both '$\phi(x)$' and in '$\psi(x)$'. So a generalized molecular formula has the form $\forall x[\phi(x) \rightarrow \psi(x)]$.[28] This definition is meant to capture the kinds of formulas that worried Russell: generalized molecular formulas have the feature that they appear to be ineliminably molecular because the quantifier binds a variable on each side of the truth-functional main connective in the quantifier's scope. This traps the main truth-functional connective in secondary scope and the quantifier in primary scope. A generalized molecular formula thus has no equivalent formula wherein the truth-functional main connective has the primary or widest

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[28]A similar definition can be given for the other binary truth-functional connectives. We list them for the most common connectives here: $\forall x[\phi(x) \vee \psi(x)]$, $\exists x[\phi(x) \wedge \psi(x)]$, and $\forall x[\phi(x) \leftrightarrow \psi(x)]$.

scope over the quantifier. This matters because if we could bring the quantifier into secondary scope, as in '$\forall x[\phi(x)] \lor \forall x[\psi(x)]$', then we do not have an ineliminably molecular formula. This formula's truth-makers will just be whatever makes true either of its constituent formulas '$\forall x[\phi(x)]$' or '$\forall x[\psi(x)]$' true, namely, general facts, which Russell already has. So our focus must be directed towards ineliminably molecular formulas.

For a formula to be ineliminably molecular, the quantifier has to bind a variable occurring in both parts of the molecular formula. An example is '$\forall x[Hx \to Mx]$'. If instead a quantifier only bound a variable term in one half of the molecular formula, as in '$\forall x[Hs \to Mx]$', then we get a formula equivalent to '$Hs \to \forall x[Mx]$', which is not the sort of formula that worried Russell. However, this necessary condition is not sufficient: '$\exists x[Hx \to Mx]$' is not ineliminably molecular because this is equivalent to '$\exists x[\neg Hx] \lor \exists x[Mx]$'.

Informally, we can say that the quantifier Q in primary scope may be existential or universal, and the binary truth-functional connective in secondary scope may be '$\lor$', '$\to$', '$\land$', or '$\leftrightarrow$', so long as the generalized molecular formula in question is not equivalent to any formula in which the truth-functional connective has primary scope over the quantifier's scope. Any such generalized molecular formula we will call *ineliminably molecular*.

Focusing on the ineliminably molecular generalized formulas that worried Russell, we next distinguish two kinds of completely generalized molecular formulas. An *incompletely generalized* molecular formula is a generalized molecular formula in which some non-variable term occurs. Non-variable terms include the singular term 'Socrates' or 's', the predicate 'human' or 'H', the predicate 'mortal' or 'M', and so on. Russell considers this kind of case in his logical atomism lectures. Happily, this case was resolved by Tarski in the 1930s. The model-theoretic account of truth as satisfaction supplies truth-makers for incompletely generalized molecular formulas: these will be the truth-makers for each molecular instance of the formula, which are

all and only atomic facts, plus the non-atomic fact—a general fact—that these are all the instances of the generalized formula in question.

However, invoking model-theoretic semantics will not help with completely generalized molecular formulas. Russell does discuss completely generalized atomic formulas like 'xRy' and even the completely generalized molecular formula 'xRy implies that x belongs to the domain of R' (1918/1986, 209). Yet Russell never applied the completely generalized—incompletely generalized distinction to the problem of generalized molecular formulas. As we will see in Section 6, completely generalized molecular formulas are a different beast.

Before turning to those formulas, it should be mentioned that Russell has a solution to the problem of generalized molecular formulas independent of Tarski. As Maclean (2018, §7) discusses in detail, Russell's later and non-Tarskian account of generalized formulas in later works aims to avoid general facts entirely. In these works, Russell still accepts that we have some primitive knowledge of general claims.[29] However, he rejects **ITM**: he must reject it because he denies that generalized formulas are made true by non-atomic facts. Quantifiers are used merely to express that a description of the world is complete or to mention a mental fact and not to describe an additional worldly fact:

> The non-mental world can be completely described without the use of any logical word, though we cannot without the word 'all', *state* that the description is complete; but when we come to the mental world, there are facts which cannot be mentioned without the use of logical words (Russell 1940/1973, 88).

> Let us give the name 'first-order omniscience' to knowledge of the truth or falsehood of every sentence not containing general words... Can we say that the only thing he does not know is that his knowledge has first-order completeness? If so, this is a fact about his knowledge, not about facts independent of his knowledge. It

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[29]See Russell (1940/1973, 86; 1948/1967, 504).

might be said that he knows everything except that there is nothing more to know; it would seem that no fact independent of knowing is unknown to him (Russell 1948/1967, 133–34).

Thus, as Maclean puts it, "'all" is needed only for the description of our knowledge' (2018, 83). So no general facts are needed. However, Russell's solution, like the Tarskian one, fails against the argument for non-atomic facts given in the next section.

6. Completely Generalized Molecular Formulas

A *completely generalized* molecular formula is a generalized molecular formula in which only variable terms occur, like '$\forall F \exists G \forall x[Fx \rightarrow Gx]$' or '$\forall F \forall x[Fx \rightarrow Fx]$'.[30] In this section I argue that if we want to avoid molecular facts, we will need non-atomic truth-makers for such formulas: considerations of such formulas will show that we need a new kind of fact—'a new beast for our Zoo' (Russell 1918/1986, 199)—as a truth-maker for such formulas.

The problem raised by completely generalized molecular formulas is that, in some cases, their truth is necessitated without any instances. Russell implicitly raises this kind of case:

> I want to say emphatically that general propositions are to be interpreted as not involving existence. When I say, for instance, 'All Greeks are men', I do not want you to suppose that this implies that there are Greeks. It is to be considered emphatically as not implying that. . . If it happened that there were no Greeks, both the proposition that 'All Greeks are men' and the proposition that 'No Greeks are men' would be true (Russell 1918/1986, 201–2).

This goes hand-in-hand with Russell's immediately preceding remark, 'All general propositions deny the existence of something or other. If you say "All men are mortal", that denies the

existence of an immortal man, and so on' (1918/1986, 201). Russell is raising the possibility of general claims being true despite having no instances.

This point is critical because truth-maker theorists have seemingly taken the view that one general fact—a sum or fusion of (or perhaps over) all the atomic facts that there are—was sufficient to make true generalized molecular truths: this strategy remains common among truth-maker theorists, and has been so at least since Armstrong wrote that 'the general fact that all the facts (states of affairs) of lower order *are* all such facts' (Armstrong 2004, 74). This account seems plausible for incompletely generalized molecular formulas: it seems at least defensible that 'all humans are mortals', like 'no humans are mortals', is vacuously true in a world without humans: in a world without any humans, there are no instances, so both contrary claims are made true by the mereological sum that is the second-order general fact. This suggestion is attractive for truth-maker theorists keen to avoid non-atomic facts: we only get general facts as parasitic on atomic facts.

But this suggestion, which appeals to the existence of atomic facts, is untenable when the world is empty of concrete entities, since in such a world there are no atomic facts—and thus nothing for Armstrongian general facts to mereologically parasitize. Similarly, Russell's appeal to atomic facts and first-order omniscience fails for empty worlds: in an empty world, there are neither atomic facts nor mereological sums of them. There we have no truth-makers for true completely generalized molecular formulas save through positing non-atomic (molecular or general) facts as *sui generis* entities.

So we seem stuck again with ineliminably molecular facts, or at least non-atomic ones, as truth-makers for completely generalized molecular formulas. Here is my argument:[31]

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[30]'Now I want to come to the subject of *completely general* propositions and propositional functions. By those I mean propositions and propositional functions that contain only variables and nothing else at all. This covers the whole of logic' (Russell 1918/1986, 208).

[31]I thank Peter Simons for some helpful comments on an earlier version of my argument.

An Argument for Non-Atomic Facts

(1)	An empty world is logically or metaphysically possible.	Premise
(2)	Some completely generalized molecular formulas are true at an empty world ('$\forall F \exists G \forall x[Fx \rightarrow Gx]$') and some others are false at an empty world ('$\exists F \forall G \forall x[Fx \rightarrow Gx]$').	Premise
(3)	There are neither atomic facts nor sums of them in an empty world.	Premise
(4)	True completely generalized molecular formulas need truth-makers.	Premise
(5)	If (1), (2), (3), and (4), then true completely generalized molecular formulas have non-atomic (molecular or general) truth-makers.	Premise
(6)	So, true completely generalized molecular formulas have non-atomic (molecular or general) truth-makers.	By 1–5

Because molecular truth-makers cause such widespread revulsion, it is worth considering the grounds for accepting these premises. In the next section, I defend the argument's premises.

7. Metaphysical Nihilism and Empty Domains

Premise (5) is manifestly true once (1)-through-(4) are accepted. So one might first try to deny premise (4): completely generalized molecular formulas do not need truth-makers at all. It might be suggested that such formulas are vacuously true.

But such formulas cannot be vacuous. For some of them are theorems of logic, while others are disprovable by logic alone. There is no characterization of these two classes of formulas, consistent with their supposed vacuity, that accounts for why any formula in one class is true while any in the other is false.

For example, suppose that we follow Quine's way of dealing with the empty domain. Let all universally bound formulas be true, and all existentially bound formulas be false (Quine 1954, 177). Now the formulas '$\forall F \exists G \forall x[Fx \rightarrow Gx]$' or '$\forall F \forall x[Fx \rightarrow Fx]$' are both theorems of standard second-order quantification theory. Further, on Quine's approach, they are also theorems in empty domain logics, or *inclusive logics*: they are provable even in a logic consistent with an empty world.

Contrast these formulas with those like '$\exists F \exists G \forall x[Fx \rightarrow Gx]$' and '$\exists F \forall x[Fx \lor \neg Fx]$'. These formulas are just closures of tautologies like '$Pa \rightarrow Pa$' and '$Pa \lor \neg Pa$'. Somehow, these formulas are to be false—but vacuously so—while universal closures of tautologies are true—but also vacuously so. But if, by their vacuity, no truth-maker is needed for the first class of formulas, then why should the other kind not be, by their vacuity, true?

Quine's answer to this question is that vacuous formulas are justified by their logical equivalence to vacuity-free formulas (1954, 178). For example, '$\exists x(\phi)$' is logically equivalent to '$\exists x(\phi \land Fx \rightarrow Fx)$' (where '$\phi$' is a formula in which 'x' does not occur). Thus the vacuous falsehood of '$\exists x(\phi)$' at the empty domain is accounted for by its equivalence to the non-vacuous falsehood of '$\exists x(\phi \land Fx \rightarrow Fx)$'.[32]

But this answer just pushes the problem back. It is widely though not universally agreed among truth-maker theorists that every true formula needs a truth-maker. So some truth-maker is needed at the empty world to account for either the vacuous formula's truth or that of its non-vacuous equivalent. Quine's clever proposal permits us to distinguish vacuously true formulas from vacuously false ones in a principled way. But distinguishing true and false vacuous formulas using their non-vacuous equivalents does not address the underlying problem: indeed, Quine's proposal relies on these vacuity-free formulas being non-vacuously true. So appealing to vacuity to resist premise (4) will not work: completely generalized molecular formulas, if true, are true non-

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[32]The semantics and proof theory for empty domain logics still has no standard semantics and proof theory, though typically Quine's treatment is followed (Williamson 1999, 3–4).

vacuously.[33] As such, a truth-maker theorist needs truth-makers for them. This is just premise (4).

One might object to premise (3), that there are no atomic facts in an empty world, by arguing that this only holds of atomic facts involving concrete entities.[34] But if one embraces universals existing outside space-time, and holds that these exist necessarily, then there could be atomic facts involving relations between properties and properties of properties. That is, if one embraces *ante rem* realism about universals, then it could be that there are atomic facts having the form $R^n(F_1, \ldots, F_n)$ even in an empty world.

I am comfortable conceding that *ante rem* realism about universals is a way out of my argument for facts corresponding to generalized molecular formulas. If one holds that universals necessarily exist even when they have no instances, then one has grounds for rejecting premise (3).[35] But it is still interesting

to establish that it is either *ante rem* realism about universals or accepting that there are non-atomic facts. This is because there are serious worries about positing *ante rem* universals. Such entities belong to a distinct ontological category from particulars, so positing them seems transgresses such parsimony considerations as motivate nominalism and are *prima facie* principles for good metaphysical practice.[36] Additionally, *ante rem* universals are non-causal entities existing outside space-time, so positing them seems to run afoul of such methodological principles as underlie metaphysical naturalism.[37] Further, as we do not causally interact with *ante rem* universals, our knowledge of them is *prima facie* difficult to explain.[38] So premise (3) holds, unless one rejects it by embracing *ante rem* universals, and this alternative has its own serious difficulties.[39]

As for premise (2), the only way to deny this is by asserting that an empty domain logic is *trivial*. There are two senses in which a logic might be trivial. In the usual sense, a trivial logic has that every well-formed formula is a theorem. Now this, as a claim about how empty domain logics must be, is provably false. There are multiple non-trivial, sound, and complete proof systems for inclusive logic.[40] Though there are still outstanding issues in empty domain logic, we know at least that it is non-trivial inasmuch as not every formula is a theorem at the empty world.

Alternatively, a logic might be trivial in the sense that no well-formed formulas are theorems. A motivation for this would be that, at an empty world, every well-formed formula of logic is false. As such, no generalized molecular formulas should be derivable. A truth-maker theorist might find this suggestion at-

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[33]There is a further argument against such formulas being true or false vacuously. There are different and sometimes contradictory logics. We do not wish to say that these logics are all equally true or false. But this seems to be what we are forced to say because they are equally vacuous at the empty world. This has the unhappy corollary that such debates are trivialized into disputes over equally vacuous systems. The view that logicians are rather disagreeing over the facts—over what facts obtain of logical necessity not just when there are concrete things, but in every case—is thus more consonant with logicians' internal point of view.

[34]I thank Mark Textor for pointing this out to me in conversation.

[35]We will see in Section 8 that supplying truth-makers for completely generalized molecular formulas requires accepting abstract entities a different, more plausibly logical sort—completely general facts, which have no constituents. These *sui generis* entities will exist necessarily even when there are no atomic facts falling under them and no concrete objects, as in the empty world. This seems more palatable to me than accepting an ontology of non-logical but necessarily existing universals with no instances. The exact list of them depends on one's other philosophical commitments. But the candidates like *being H₂O*, *being human*, *being siblings*, *being taller than*, *having negative charge*, and so on, all seem to be problematically non-logical entities, as do the atomic facts involving them and involved as truth-makers for general formulas. Facts with no constituents have their own difficulties, but they are not problematically non-logical.

[36]See, for example, Williams (1997, 118–20), but see also Campbell (1997, 133–34).

[37]See, for example, Armstrong (1978b, 22–24, 75; 1978a, 8–11).

[38]See, for example, Benacerraf (1973, 671–73) and Shoemaker (1997, 236–37).

[39]See, for instance, Armstrong (1997, §2.8).

[40]Compare Mendelson (1997, §2.16) and Williamson (1999, 4).

tractive because, after all, there are no truth-makers in empty worlds, so it might seem sensible that all formulas should be false.

The plausibility of this reply depends on denying premise (1). For if the empty world is logically possible, then holding that all formulas are false at the empty world makes any instance of all logical axiom schemata false at the empty world, and so makes them all logically contingent. For example, consider the law of tautology, $(\phi \wedge \phi) \rightarrow \phi$ for any well-formed formula ϕ. Any instance of this is false at the empty world if all formulas are false in that world. Yet the law of tautology is true in all non-empty worlds. This point applies equally to any law of logic. So assuming premise (1) is independently plausible, the reply makes all laws of logic logically contingent. But this is contrary to the view that there is at least one law of logic that is logically necessary.[41] So if premise (1) is independently defensible, then, assuming that there at least one law of logic is logically necessary, which I think readers will agree to, we have premise (2).[42]

Premise (1) is the claim that an empty world is either logically or metaphysically possible. Premise (1) is likely the most controversial premise in the argument. Part of the reason for this is that philosophers disagree over whether logical or metaphys-

ical possibility is the widest, all-encompassing, absolute sense of 'possible' is.[43] Without taking a stance on this debate, I want to argue that the empty world is possible in the widest sense of 'possible' whatever that is—hence the inclusive 'or' in premise (1). So I will first argue that the empty world is metaphysically possible, and then that it is logically possibility.

I begin with the metaphysical possibility of the empty world.[44] It is usual today to understand talk of metaphysical possibility through accounts of how we should understand possible worlds, especially whether talk of worlds is understood in terms of modal operators or, conversely, whether modal operators are understood through talk of possible worlds (Williamson 2013, 333). So the defense of premise (1) in the metaphysical sense of 'possible' can be made by showing that it is possible on various accounts of possible worlds. And the case for premise (1) is strengthened by noting that the standard *ersatz* accounts of metaphysical possibility are such that premise (1) holds.

Consider some accounts of possible worlds: possible worlds might be concrete situations (as Lewis has it), or abstract entities like maximal collections of propositions (as Fine has it), or stipulated situations (as Kripke has it), or combinations of metaphysically simple entities (as Armstrong has it), or fictions (as Rosen has it).[45] These accounts might be broadly categorized as *realist* or *ersatz* according to whether possible worlds are genuine entities or not (Parent 2012). On all the usual *ersatz* accounts, including fictionalist ones, it is metaphysically possible that there should be empty worlds (Coggins 2010, 138).

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[41]One might deny that there are any axiom schemata that are logically necessary (Maddy 2014, 99–100). This is another way out of my argument, but deflating logical and metaphysical necessity is not an attractive strategy for a truth-maker theorist.

[42]There is yet another way of attacking premise (2). One could argue that all formulas at an empty world would be neither true nor false. They might be without truth-values at all or else they might have some truth-value other than true and false. Some free logics allow for developments along these lines (Lambert 2002, 127). However, the argument in this paragraph applies to such systems: the laws of logic would turn out neither true nor false at the empty world, though they would be true in other worlds. Deploying supervaluations will not undercut the argument of this paragraph: even with supervaluations, either all completely generalized molecular formulas will be true or all of them will be false, on pain of admitting premise (2) is true. This seems to make the laws of logic logically contingent.

[43]See Nolan (2011, 315–23) for a nice discussion of this issue.

[44]There has already been substantial debate over premise (1): in the sense of metaphysical possibility, premise (1) is also known as *metaphysical nihilism*. Much discussion has concerned the so-called *subtraction argument* for metaphysical nihilism (Efird and Stoneham 2009, 132–33). I will not add to the subtraction argument here. For detailed discussion, see Coggins (2010, Chapter 6).

[45]A discussion of various accounts is found in Menzel (2017, §2).

For example, to get a maximal set of propositions that would be entailed at the empty world, we can follow Quine in taking the universal closure of all formulas. One may wish to secure a 'more' maximal set of propositions by adding, in the spirit of free logic, the negations of all atomic formulas like '$R(a_1, \ldots, a_n)$' even though 'a_1',..., and 'a_n' do not refer. This can easily be done, giving us a maximal set of propositions. As such, on the *ersatz* view that a possible world is a maximal set of propositions, there is an empty world. As another example, the modal fictionalist would doubtless concede the fictional character of the empty world. Even the Kripkean account of stipulated worlds seems to permit the stipulation of *a world just like this one, except containing no entity at all*.[46]

In contrast, those with realist accounts of metaphysical possibility in terms of really existing, concrete, possible worlds are in a position to reject premise (1) (Lewis 1968, 73–74). But absent an embrace of a realist account of possible worlds, which has its own problems, the usual views of metaphysical possibility using *ersatz* accounts of possible worlds are such that premise (1) in the sense of metaphysical possibility has to be conceded.

Next, consider premise (1) in the sense of logical possibility. Here we must distinguish logical possibility from the existence of a model: there are models of '$Fa \wedge \neg Fa$', but this fact alone does not show that contradictions are logically possible. The guarantee of a model is relative to some logical system. We are concerned with logical possibility in a broadest, non-relative sense.[47]

So taking 'possible' in the broadest, logical sense, there are two reasons to think that empty worlds are possible. First, it is widely though not universally held that logic does not show, and should not have as a theorem, that anything exists.[48] This is why logicians since Russell have hesitated over existence theorems in logic generally, and particularly in regards to what actual concreta exist in the universe.[49] This view of logic as independent of existential claims strongly supports premise (1).

Second, premise (1) is supported by the notion of logical form. Typically, logic is described as being concerned with the form or structure of an argument rather than with its content or its premises' truth.[50] But logical form alone does not show that empty worlds are impossible. It is true that the inference from '$\forall x[\phi(x) \rightarrow \phi(x)]$' to '$\exists x[\phi(x) \rightarrow \phi(x)]$' is valid in classical logic, but this is only valid if the domain is assumed to be non-empty. This assumption is standard, but it is generally maintained that this assumption is not imposed by the argument's form. It is not held that there is a formal feature of the thesis that $\exists x[\phi(x) \rightarrow \phi(x)]$ such that, as a matter of logical form, it follows

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[46]'A possible world is *given by the descriptive conditions we associate with it*... "Possible worlds" are *stipulated*, not *discovered* by powerful telescopes' (Kripke 1980, 44).

[47]It is also established that inclusive logic has a model and is sound and complete with respect to some of its proof systems and formal semantics (Mendelson 1997, §2.16). So if 'logical possibility' just meant something system-relative, like having a model or entailing no contradiction, then premise (1) follows anyway.

[48]If one embraces *necessitism*—the view that, necessarily, any x is such that, necessarily, some y is identical to x—then premise (1) can and must be rejected. But the above argument takes truth-maker theory as a premise, and necessitism is inconsistent with truth-maker theory (Williamson 2013, 391–92). So for truth-maker theorists who already must reject necessitism, this way of resisting premise (1) is closed.

[49]See Russell (1937, §5; 1919, 203).

[50]I cite five entirely typical examples. 'The argument, it may be held, is valid from its form alone, independently of the matter, and independently in particular of the question whether the premises and the conclusion are in themselves right or wrong' (Church 1956, 2). 'The truth or falsity of the particular premises and conclusions is of no concern to logicians. They want to know only whether the premises imply the conclusion' (Mendelson 1997, 1). 'To sum up, formal logic is fundamentally concerned with the form and structure of arguments and not, primarily, with their content' (Tomassi 1999, 17). '[A] formal derivation exploits only the shape of formulas, not any consideration of their truth or falsity' (Goldrei 2005, 87). 'It is important to remember that when we evaluate arguments, we must always distinguish truth value analysis from the logical analysis' (Baronett 2013, 30).

that $\forall x[\phi(x) \rightarrow \phi(x)]$. Rather, non-empty domains are usually justified, and have been since Quine, as a matter of technical convenience.[51] However, on the usual understanding of logical form, there is no reason to believe that '$\forall x[\phi(x) \rightarrow \phi(x)]$' requires for its truth that there be some a that satisfies '$\phi(a) \rightarrow \phi(a)$'.[52] Indeed, the possibility of empty worlds is a natural extension of the notion that purely formal matters are independent of which specific non-logical entities exist, and which specific non-logical features are exemplified, at or in a world.[53] The natural extension is that purely formal matters are independent of which specific non-logical entities exist, *if any*, and which specific non-logical features are exemplified, *if any*.[54]

So for these two reasons, we must admit that empty worlds are logically possible. This gives us premise (1) in the sense of logical possibility, which was our final premise. So the argument is sound.

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[51]Quine, for example, argues against including the empty domain because doing so 'would mean surrendering some formulas which are valid everywhere else and thus generally useful.' (Quine 1954, 177) See also Hunter (1973, 255), Mendelson (1997, 147), and Tomassi (1999, 291–92). It should be noted that there are some 'subtleties' involved in the semantics for empty domains (Williamson 1999, 3).

[52]Indeed, to say otherwise jars with the standard history that logic since Boole has made an advance in rejecting the traditional view that 'Some s are P' follows from 'All s are P'.

[53]'Thus the absence of all mention of particular things or properties in logic or pure mathematics is a necessary result of the fact that this study is, as we say, "purely formal"' (Russell 1919, 198).

[54]This could be true, that is, a world could be empty, and the truth-makers for completely general formulas might nonetheless be really existing stuff like facts. And the argument that we would need truth-makers for such truths, even for theorems of logic, stems from the truth-maker principle outlined in Section 2. It is true that some philosophers deny that purely formal truths like theorems of logic need or have truth-makers, holding instead that such truths are somehow related to formal systems rather than to really existing stuff, but this view if anything would further support the view that logical form does not preclude an empty world. I thank a reviewer for raising some worries centered on a more formal understanding of logical form.

To sum up: the argument is decisive for any truth-maker theorists who (a) admit the logical or metaphysical possibility of an empty world, (b) accept that not every completely generalized molecular formula is a theorem of inclusive logic, and (c) deny *ante rem* realism about non-logical universals. Any such truth-maker theorists are thereby committed to positing non-atomic facts as truth-makers for completely generalized molecular formulas.

8. Back to 1918? Completely General Facts

We saw in the previous two sections that there are non-atomic facts as truth-makers for completely generalized molecular formulas. Such formulas seem more intractable for a truth-maker theorist than has been generally acknowledged: as we saw, they are not reducible to the case of incompletely generalized molecular formulas if empty worlds are possible. Some facts must be supplied for such formulas.

What sort of facts are these? We noted that there are no atomic facts in an empty world. So the truth-makers for true completely generalized molecular formulas apparently are non-atomic. Indeed, they seem to be molecular facts. *Pace* Barker and Jago (2012), positing such facts is an unappealing move. But can we avoid positing molecular facts?

The natural solution to this problem is to get rid of completely generalized molecular formulas. One would define formulas like '$\forall F \exists G \forall x[Fx \rightarrow Gx]$' in such a way that no molecular expressions occur within the scope of a quantifier. Then truth-functional considerations would apply so no molecular facts would be needed to account the truth of completely generalized molecular formulas. This would be a symbolic dissolution to the problem of providing truth-makers for completely generalized molecular formulas.[55]

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[55]There are some quantifier-like notions that are not covered by the proposals given below. For example, the natural language notion of *most* seemingly is not

Such a symbolic dissolution is not possible. Quantifiers at the front of a formula cannot in general be brought into the scope of molecular connectives occurring in their scope.[56] For a counterexample, there is no formula equivalent to $\forall F \forall x [Fx \lor \neg Fx]$ which is such that no molecular connective occurs in the scope of a quantifier by the usual quantifier distribution laws.[57] Since we cannot get rid of the molecularity in completely generalized molecular formulas by defining such formulas away, the truth-maker theorist must address them: some non-atomic truth-makers must be supplied in light of the argument in Section 6.

One way of addressing the problem would be to supply truth-makers for completely generalized molecular formulas that are non-molecular. The natural candidate truth-makers are general facts. But these will be an unusual sort of fact. They will need to be facts that make true formulas with only variable terms and logical constants. Since some such formulas will be true at an empty world, these general facts will need to exist at an empty world. As such, these truth-makers will seemingly be facts—complexes—with no constituents.

Furthermore, as Lewis notes, there are no mereological sums of anything in empty worlds. So such general facts will not be mereological sums as Armstrong has it. Such general facts are *sui generis* entities and are not amenable to definition. They are

the non-molecular facts that are, by the argument above, needed as truth-makers for the *definiens* of completely generalized formulas.[58] Call these *completely general facts*.[59]

Crucially, completely general facts do not have a structure: indeed, they have no constituents or parts in any sense, even though the formulas that pick them out have linguistic components. So they can serve as truth-makers for completely generalized molecular formulas without leading one to embrace molecular facts. In this way, completely general facts allow one to solve the problem of providing non-molecular truth-makers for completely generalized molecular formulas: although such facts are non-atomic, they are not molecular because they do not have a structure at all: rather, such facts *are* logical structures.[60] Molecular facts in contrast are parasitic on, or at least presuppose, atomic facts of which they are sums or fusions. Not so with completely general facts: indeed, they are introduced precisely because, absent ante rem realism about universals or necessitism, there is a logical need for facts that are independent of whether there are atomic facts or not, as shown by the possibility of the empty world.

Now Russell (1918/1986, 208) already commits to completely general facts in the 1918 lectures on logical atomism: such facts are picked out with formulas containing only variable terms and logical constants according to Russell. Their complete generality—their avoidance of mentioning any particular

translatable into first-order language using only '∀', '∃', and the usual truth-functional connectives (Stevens 2011, 112–13). This raises a question: could one such as Russell, with the higher-order language of *Principia* and class talk in hand, express the truth conditions of natural language sentences involving quantifier-like notions, such as *most philosophers are wise*? I do not address that question here, but if the answer is 'yes', then what I say here will extend to quantifier-like notions like *most*. If the answer is 'no', then additional steps will be needed for other quantifier-like notions.

[56]This is possible in some cases, as in $\forall F \forall G \forall x [Fx \land Gx]$ because ∀ distributes over ∧. Since this formula is logically equivalent to $\forall F \forall x [Fx] \land \forall G \forall x [Gx]$, we can define the former using the latter.

[57]It is true that any formula in a classical logic is equivalent to some formula with all its quantifiers occurring at the front of the formula; such a formula is said to be in *prenex normal form* (Mendelson 1997, §2.10).

[58]An alternative characterization would be that they are facts such that they make true some formula with only variable terms.

[59]Putting all formulas into their prenex normal form, completely general facts will be of different sorts according to the initial binding quantifier. For example, *completely general universal facts* are the would-be truth-makers for formulas with an initial universal quantifier. *Completely general existence facts* are the would-be truth-makers for formulas prefixed by an existential quantifier.

[60]See Russell (1913/1984, 114). But the comparison between Russell's notion of logical form and his posit of completely general facts should not be overemphasized: his discussion of logical form is brief and highly tentative (Griffin 1980, 117). See Griffin (1980, 144, 152) and Klement (2015, 216).

things in the world—gives general facts a distinctly logical character that Russell exploits in accounting for the truth of logical principles. Russell says, 'All the statements of logic are of that sort' (Russell 1918/1986, 209). So Russell has the materials required to solve the problem of completely generalized molecular formulas.[61]

So if one has independent reasons to posit completely general facts, like the argument of Section 5, then it makes sense to put these posits to work. Completely general facts, besides being the facts of logic itself, will be truth-makers for completely generalized molecular formulas. Further, the discovery and categorization of such facts is an explicit aim of logical atomism:

> In logic you are concerned with the forms of facts, with getting hold of the different sorts of facts, different *logical* sorts of facts, that there are in the world (Russell 1918/1986, 191).

In contrast, the principle of atomicity is implausible absent *ante rem* universals, realism about possible worlds, or trivialization of inclusive logic, given the severity of Russell's problem of completely generalized molecular formulas. Furthermore, the principle of atomicity is not crucial to logical atomism: it is rather one among many views as to the results of inquiry into what logical kinds of facts there are, and it is a view that Russell himself did not hold in 1918. Now if non-atomic sorts of facts must be admitted in light of Russell's problem of generalized molecular formulas, then with completely general facts in our ontology, we avoid molecular facts while solving the problem of completely general formulas. More ambitiously, logic may be naturally identified, as Russell explicitly does, with the study of such facts. Enter logical atomism.

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[61]It is reasonable to ask why Russell did not solve this problem if he had all the materials for the solution available to him. But Russell did not cleanly separate the two problems as I have done here, and completely general facts would not assist in solving the problem of incompletely generalized molecular formulas—and this is the only side of the problem that Russell explicitly discusses in 1918.

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