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"Die Machine als Symbol ihrer Wirkungsweise": Wittgenstein, Reuleaux, and Kinematics

Sébastien Gandon

In *PI* 193–94, Wittgenstein draws a notorious analogy between the working of a machine and the application of a rule. According to the view of rule-following that Wittgenstein is criticizing, the future applications of a rule are completely determined by the rule itself, as the movements of the machine components are completely determined by the machine configuration. On what conception of the machine is such an analogy based? In this paper, I intend to show that Wittgenstein relied on quite a specific scientific tradition very active at the beginning of the twentieth century: the kinematic or the general science of machines. To explain the fundamental tenets of this line of research and its links with Wittgenstein, I focus on Franz Reuleaux (1829–1905), whose works were known to Wittgenstein.

The first payoff of this investigation is to help distance the functionalist framework from which this passage is often read: Wittgenstein's machines are not (or not primarily) computers. The second payoff is to explain why Wittgenstein talks about machines at this place in his discussion on rule-following: it is not the machine model in itself that is criticized in *PI* 193–94, but the "philosophical" temptation to generalize from it.

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1. Introduction

Philosophical Investigations §§185–202 is the passage where Wittgenstein formulates his notorious "paradox" of rule-following. In *PI* 193–94 (an expanded version of which can be found in *RFM* I 119–25), Wittgenstein takes a step aside to speak about an apparently different issue, namely, the special way movements are generated in a machine. Let me quote the beginning of *PI* 193:

> The machine as symbolizing its action: the action [*Wirkungsweise*] of a machine—I might say at first—seems to be there in it from the start. What does that mean?—If we know the machine, everything else, that is its movement, seems to be already completely determined [*ganz bestimmt*]. We talk as if these parts could only move in this way, as if they could not do anything else. How is this—do we forget the possibility of their bending, breaking off, melting, and so on? Yes; in many cases we don't think of that at all. We use a machine, or the drawing of a machine, to symbolize a particular action [*eine bestimmte Wirkungsweise*] of the machine. For instance, we give someone such a drawing and assume that he will derive the movement of the parts from it. (Just as we can give someone a number by telling him that it is the twenty-fifth in the series 1, 4, 9, 16, . . .).

In this passage, an analogy is drawn between the relation of a machine to its movements on one side and the relation of a rule to its applications on the other. In the same way as the movements in a machine are predetermined by the configuration of the machine, the applications of a rule would be completely controlled by the rule. Thus, the analogy can be displayed as follows:

Machine	Rule
Movements	Application

Table 1

This article aims at providing a historical context to the comparison between rule and machine. I will argue that this analogy is not simply the product of Wittgenstein's imagination, but that its source goes back to a scientific tradition very active at the beginning of the twentieth century and to which Wittgenstein was exposed during his engineering training: the general science of machine or kinematics as it has been developed by Franz Reuleaux (1829–1905).

I am not the first to make this connection—Mark Wilson (1997, 2017) has related Wittgenstein's work to Reuleaux's.[1] However, at the notable exception of Wilson, Wittgenstein's remarks on machines have usually been interpreted in the perspective of the seventies' functionalist comparison between mind and computer. The first goal of this paper is to establish that Wittgenstein's machine has *prima facie* nothing to do with a computer. The second aim of this historical exploration is to explain why Wittgenstein talks about machines in the middle of his rule-following discus-

[1]Wilson places great emphasis on the idea that Reuleaux's work was driven by the research of design improvement in machine building. Reuleaux's sharp delimitation of localized sets of possibilities is then conceived of as a step in this optimization process (Wilson 1997, 293, 300; 2017, 293–301). For Wilson, the disappearance of the Tractarian "logical space", which is a global notion, and the appearence of the later "logical grammar", which is a local notion, would echo Reuleaux's pragmatic approach (2017, 299). Without being incompatible, the story I tell here is different from his. Instead of focusing on a process of optimization, I insist on Reuleaux's "architectonic" project, i.e., his aim to free kinematics from the tutelage of classical mechanics. For more on Wilson's interpretation, see footnote 15, 16, 20, 31, and 41 below.

sion. At the heart of Reuleaux's theory, one finds the idea that kinematics should be made independent from classical mechanics. To this end, Reuleaux changes the status of certain propositions (those concerning the rigidity of machine components), which, while they were regarded as empirical in mechanics, become *a priori* rules in kinematics. Using Wittgenstein's terminology, one could say that Reuleaux, by "hardening" certain devices belonging to a given language-game (classical mechanics), generated a new and distinct language-game (kinematics). In *PI* 193–94, Wittgenstein would not be criticizing Reuleaux's move. He would rather be criticizing the philosophical temptation to develop a general theory of rule-following from what holds in the particular language-game of kinematics.

The article is organized in the following way. In Section 2, I consider two influential interpretations of Wittgenstein's analogy: Kripke (1982) and Baker and Hacker (2009). I agree with these readings that Wittgenstein, in the machine analogy, is targeting a confusion between factual causal laws and normative grammatical rules. I disagree, however, with their interpretations of the term "machine". In Section 3, I give a brief presentation of kinematics (also called the general theory of machines) as it has been developed by Franz Reuleaux. I explain why Reuleaux wanted to make kinematics distinct from classical mechanics and how he succeeded in doing so by assuming that, in a machine, the movements of the components are completely determined by their geometrical shapes and arrangement. I show, in Section 4, how this insight which I call Reuleaux's fundamental insight (RFI) led him to elaborate on a special kinematic symbolism. In Section 5, I come back to *PI* 193–94 to show how Wittgenstein's machine analogy fits into this context. Reuleaux's kinematic approach provides Wittgenstein with a model case (a language-game, as it were) where rules do predetermine future applications. In the conclusive Section 6, I make clear that Wittgenstein does not criticize this model, but only the uniformization induced by its "uncivilized" and illegitimate philosophical generalization.

2. Two Interpretations

Kripke (1982) discusses *PI* 193–94 in the context of his critique of the dispositionalist account to Wittgenstein's rule-following paradox. His claim is that *PI* 193–94 provides dispositionalism with an additional argument: if a machine can be designed to determine in advance the steps to be taken, then, surely, it is quite natural to imagine that some dispositional facts can account for the difference between the *plus* and *quus* hypotheses. Kripke shows, however, that this additional argument does not help: the dispositionalist gives us a causal-descriptive story of how a (dispositional) fact determines the intention to produce a future action while what is needed to meet the skeptical challenge is a normative account explaining why the way I apply a rule is correct.[2] Indeed, Kripke emphasizes that the term "machine" is ambiguous: it can refer either to the machine program, or to the concrete thing, "made of metal and gears (or transistors and wires)" (1982, 34). Concrete machines sometimes malfunction and behave in a way that deviates from the machine's programming. So one cannot conflate what a machine actually does with what the program should make it do. When a full account is taken of the ambiguity of the term "machine", new versions of the rule-following paradox are easy to mount.

As this brief presentation of Kripke's interpretation shows, the distinction between the abstract notion of a program and its implementation in a concrete physical machine—a distinction coming from computer science and Turing's "modern theory of automata" (Kripke 1982, 35–36)—is, according to Kripke, what Wittgenstein had in mind in *PI* 193–94. In a footnote, Kripke explains that:

[2]The dispositionalist gives us a causal-descriptive story explaining how the dispositional fact determines the intention to future action, while what is needed to meet the skeptical challenge is a normative account: "A candidate for what constitutes the state of my meaning one function, rather than another, by a given function sign, ought to be such that, whatever in fact I (am disposed to) do, there is a unique thing that I should do. Is not the dispositional view simply an equation of performance and correctness?" (Kripke 1982, 24).

Wittgenstein discusses machines explicitly in §§193–5. . . . [My criticisms of] the dispositional analysis and of the use of machines to solve the problem are inspired by these sections. In particular, Wittgenstein himself draws the distinction between the machine as an abstract program ("der Maschine, als Symbol" §193) and the actual physical machine, which is subject to breakdown ("do we forget the possibility of their bending, breaking off, melting, and so on?" (§193)). The dispositional theory views the subject himself as a kind of machine, whose potential actions embody the function. So in this sense the dispositional theory and the idea of the machine-as-embodying-the-function are really one. Wittgenstein's attitude toward both is the same: they confuse the 'hardness of a rule' with the 'hardness of a material' (*RFM*, II [III], §87).
(Kripke 1982, 35 n. 24)

The phrase "the machine as symbol" would then correspond to the notion of an abstract program while the actual physical machine would correspond to its physical implementation. Is this reading legitimate?

Kripke gives no textual evidence in favor of what he advances. Recently, however, some scholars have pointed out that, when writing the first versions of *PI* 193–94 in 1937, Wittgenstein was deeply influenced by Turing (1936).[3] This observation might seem to give credit to Kripke's assumption. Let me give two elements to disqualify this hasty conclusion. First, the connection between machines and rules does not date back to 1937. One finds many passages which antedate Wittgenstein's meeting with Turing.[4] Second, the machines Wittgenstein is speaking about in *PI* 193–94 are not computers or Turing machines nor even computing machines. The machines are mechanisms, or parts of ordinary artefacts. In *PI* 194, Wittgenstein takes the example of the pin and the socket and in *RFM* (I, 119) he refers to the crank and slider mechanism. Of course, Kripke's interpretation of Wittgenstein's philosophical point might also apply to mechanical machines.[5] But in this perspective, how can he explain that Wittgenstein used so convoluted a means to speak of a model (the Turing machine) to which he could refer directly? I don't want to suggest that nothing important in *PI* is coming from Turing and that the interpretations based on the assimilation of Wittgenstein's machines to computers are false. My point is only that, in *PI* 193–94 and the related passages, the term "machine" does not seem to refer to Turing's machine and computer.

Unlike Kripke, Baker and Hacker (Baker and Hacker 2009) do not project on *PI* 193–94 a terminology coming from functionalism and computer science. They take Wittgenstein's mechanical examples at face value. However, Baker and Hacker's interpretation is close to Kripke's since they take the machine analogy to show that Wittgenstein is targeting the confusion between the causal (descriptive) and grammatical (normative) explanation. More precisely, they consider that in the sentence "a machine determines in advance the movements to be taken", the word "determination" is ambiguous: one must distinguish between "a causal sense in which future actions are determined, and a grammatical sense in which the applicability of a description is determined" (2009, 108). According to Baker and Hacker, such a confusion "commonly" occurs in mechanics textbooks:

When we explain a machine design, for example, we sometimes say that a part of a mechanism *must* (not just *will*) move thus-and-so if

[3] See Wagner (2005), Floyd (2016, 2018). Floyd convincingly argues that 1937 is a major turning point in Wittgenstein's evolution since it is at this time that he abandons the revision of the *Brown Book* to write in his mature dialogical way a first sketch of *PI*. According to Floyd, the interactions between Turing and Wittgenstein are responsible for the rapid changes in Wittgenstein's mind at this time.

[4] Let me quote a remark dated from May 1930: "We look at a machine as expression of a rule: e.g., drawing of a piston. We look on it as a rule of possible motion. The machine has not committed itself to anything. Our interpretation of it is the way it ought to work: what we see is the intention" (*WLC* 1979, 66). See also *PR*, 64; *BB*, 191–92.

[5] As Turing emphasizes, "the idea of a digital computer is an old one", that can be traced back at least to Babbage, and the idea that digital computers must be electrical are nothing else than a "superstition" (Turing 1950, 468).

another part moves in such-and-such a way. Whence the 'must'? We use parts of machines (and *drawings* of machines) to *symbolize*. In particular, we use them to symbolize laws of kinematics. A pair of cog-wheels, for example, is commonly used to demonstrate the principle or law that one revolves clockwise if the other turns anticlockwise. In such cases, a simple mechanism is used to demonstrate a law of kinematics and this law has the status akin to a theorem of geometry, and is not akin to a generalization in a manual of practical mechanics. (Baker and Hacker 2009, 107)

For Baker and Hacker, Wittgenstein is here criticizing a confusion between *a priori* and *a posteriori* laws: while kinematics is a part of general mechanics, and as such a *a posteriori* science, textbooks "commonly" use mechanisms in order to demonstrate kinematic laws, thereby giving them a status akin to *a priori* geometrical theorems. Baker and Hacker do not say much on who is supposed to make this "common" mistake, however. Who "use[s] parts of machines to symbolize"? Which books commit this fallacy?

There is no element in Baker and Hacker (2009) to answer this question. One might argue that, in *PI* 193–94, Wittgenstein wanted to illustrate a misleading conception of rule-following rather than refer to a specific scientific practice. But this line is contradicted by passages which show that Wittgenstein did have a specific target in mind. Let me quote *WLFM*:

> When we think of a logical machinery explaining logical necessity, then we have a peculiar idea of the parts of the logical machinery—an idea which makes logical necessity much more necessary than other kinds of necessity. If we were comparing the logical machinery with the machinery of a watch, one might say that the logical machinery is made of parts which cannot be bent . . . How can we justify this sort of idea? One has in mind that branch of mathematics which is called kinematics (though the word "kinematics" may be used also in other senses). Kinematics is really a branch of geometry; in it one works out how pistons will move if one moves the crankshaft in such-and-such a way, and so on. One always assumes that the parts are perfectly rigid. (*WLFM* XX, 196)

Wittgenstein clearly refers to something quite specific: to a certain science, which he calls kinematics and which is one he characterizes as "really a branch of geometry".[6] One cannot then content oneself with the too vague characterization given by Baker and Hacker. What is kinematics then?

Before turning to this issue, let me make clear that I do not reject the basic features of Kripke's and Hacker and Baker's interpretative framework. I agree with them when they consider that Wittgenstein, in his discussion of dispositionalism and rule-following, makes a contrast between the *a priori*, geometrical, necessary way in which a machine is supposed to determine the movement of its parts and the *a posteriori*, physical, contingent way in which movements are determined in reality. To use a terminology coming from the *TLP*, the relation between the machine and its movements are internal (where an internal relation "exists as soon as, and by the very fact that, the [terms in relation] exist" (5.131)), whereas the causal relation which determines the movements of the bodies in physical reality is external. The analogy displayed in Table 1 can be extended to another analogy that can be summarized in this way:

Movements in the machine as symbol	Necessity	Grammatical rule	Internal relation
Movements in the real machine	Contingence	Factual description	External relation

Table 2

In *PI* 193–94, the oppositions between internal and external relations, between grammar and fact, and between necessity and contingence are illustrated by a distinction between two notions of a machine: the ideal, symbolic machine, which is not subject

[6] Note that this last comment goes against Baker and Hacker's interpretation: if kinematics is really a branch of geometry, it can't be a mistake to give to the "kinematic law" (whatever that means) a status akin to geometry. For more on this, see section 6 below.

to any deformation, and the real, material machine, which is not super-rigid. My task is to explain why Wittgenstein considered that this last contrast (see the left column in Table 2 (above)) is a particularly vivid illustration of the former, more abstract oppositions (see the right columns of Table 2). Kripke's interpretation has at least the merit to provide a plausible answer to this worry: since the distinction between a program and its implementation is now a familiar one, it can be used to give flesh to more abstract distinctions. But if Wittgenstein did not have a computer in mind, why did he suddenly refer to machines, to their movements, and to "kinematics"?

3. Kinematics and the Theory of Machines in Reuleaux

When Ampère (1775–1836), continuing Monge's pioneering research,[7] coined the term kinematics for the first time in 1834, the term designated both the pure science of motions[8] and the general theory of machines.[9] These two meanings were related. As Moon (2007) explains,[10] the domestication of steam energy was a decisive moment in the evolution of machine. Before the eighteenth century, where animal or human forces were the sole source of power, machines were mainly seen as devices aimed at enhancing muscular strength. The rise of the steam industry drastically changed the situation: the wish to enhance human power was progressively replaced by the project to control and regulate steam energy. That is, once the question of the source of the movement was definitively resolved and the issue of the source of the movement was settled, the main issue became how one type of movement could be converted into another. A machine was accordingly redefined by Monge and his followers "as a device that transformed motions" (Moon 2007, 53).[11] Ampère's terminological innovation should be reinserted into this context. It is because the machine is no longer defined "as an instrument by the help of which the direction and intensity of a given force can be altered, but as an instrument by the help of which the direction and velocity of a given motion can be altered" (Ampère 1836, 51) that Ampère equates the science of machines with the science of pure movement.

This insight is at the basis of a scientific tradition (first developed in France) aimed at basing the classification of machines on rational principles coming from the science of pure motion: "Monge and his contemporaries,. . . grouped machines according to how they changed motion, from say circular to rectilinear or from rectilinear to alternating motion" (Moon 2007, 79). But the science of pure movement quickly proved to be incapable of providing clear and comprehensive classification methods. As it happened at the beginning of natural history and chemistry, different kinematical classifications based on different principles and methodologies were proposed during the nineteenth centuries.[12] Faced with this proliferation of classifications and methodologies, scientists who argued that empirical investiga-

[7]Monge did not publish much on this topic. Lanz and Betancourt (1808) is considered the best development of Monge's ideas.

[8]In this sense, kinematics deals with the motion of points, bodies, and systems of bodies without considering the forces that caused the motion; the other branches of mechanics were statics (the study of equilibrium and its relation to forces) and kinetics (the study of motion in its relation to forces). On this traditional presentation, see Wright (1896).

[9]". . . to this science, in which motions are considered by themselves as observed in the bodies surrounding us, and specially in those systems of apparatus which we call machines, I have given the name Kinematics (Cinématique), from *κιημα*, motion" (Ampère 1836, 52).

[10]See also the introductions of Willis (1841), Laboulaye (1854), Reuleaux (1875).

[11]See for instance the discussion of the so-called Watt four bar linkage (a notorious mechanism invented by James Watt in 1784 which converted rotary into linear motion) in Reuleaux (1875, 3–5).

[12]For an overview of these different classifications, see the historical introduction of Willis (1841), Laboulaye (1854), and Reuleaux (1875). National differences also explain the diversity of classification systems—German, English, French, Italian, Russian, and American engineers did not have exactly the same needs and backgrounds.

tions should prevail abandoned the idea of deriving the classification of machines from scientific principles.[13]

The issue at the heart of Reuleaux (1875)[14] is precisely the following one: should one consider the science of machines as a scientific field on its own, endowed with its own objects, methods and problems, which is distinct from classical mechanics?[15] Should one see it as a mere application of mechanics? Or should one abandon any scientific ambition and espouse the view that the theory of machines is merely a heterogeneous mix of recipes and empirical results? In the introduction of his (1875), Reuleaux criticizes two opposing views: the "empiricist" approach, which contends that the classification of machines and machine elements should proceed on a case-by-case basis; the "mathematical" approach, which reduces the science of machines to the science of pure motion. Against the "empiricists", Reuleaux contends that one can find theoretical principles governing the classification of machines; against the "reductionists", he contends that machinic movements have features that are not shared by natural movements. In other words, Reuleaux based his defense of kinematics as an independent theoretical science on the identification of features that are specific to machinic phenomena.[16] Let me explain his reasoning.

Reuleaux (1875) introduces his approach by contrasting it from Monge's tradition:

> The real cause of the insufficiency of [the French classifications] is not, however, the classification itself; it must be looked for deeper. It lies . . . in the circumstance that . . . classification has been attempted without any real comprehension being obtained of the objects to be classified. In the old classification a commencement was made very commonly with the changing of one rectilinear motion into another; but no one asked whence the first rectilinear motion came, why it existed, how it had been created [*wie man sie erzeugte*]. To take a special case, Hachette and Lanz choose for their first mechanism the so-called "fixed pulley." In this case it is the rectilinear motion of the cord as it runs off the pulley which is changed into another such motion in the part of the cord running on in the opposite direction. Why, however, the first motion is rectilinear we do not understand. (Reuleaux 1875, 18)

Before worrying about how movements are transmitted, one needs to understand how they are generated in the first place. What determines the movement of the cord in a pulley to be rectilinear?

In section 1 of his *Kinematics*, Reuleaux distinguishes two kinds of physical systems according to the ways they react to external disturbing forces. In what he calls the "kosmical systems" (i.e., the systems one encounters in nature), maintaining equilibrium requires that a similar external force, opposite to the disturbing one, is brought into action. In what he calls the "machine systems", pieces are arranged so that they could resist the disturbing force and exclude any not wished-for motion. To explain what he has in mind, Reuleaux takes the example of the movement

[13] See for instance Reuleaux's portrait of the work of his teacher, Ferdinand Redtenbacher (1809–1863) in Reuleaux (1875, 14–15). For more on the evolution of kinematics from Monge to the present time, one finds a lot of material in the Springer series *International Symposium on History of Machines and Mechanisms Proceedings* and in Moon (2007). One finds also historical presentations of kinematics in the introductions of Ampère (1836), Willis (1841), Laboulaye (1854), Reuleaux (1875), and Burmaster (1888).

[14] Franz Reuleaux (1829–1905) was a Professor of mechanical engineering, first in Zürich (1856–1864), then at the Berlin Technische Hochschule (where Wittgenstein was trained as engineer from 1906 to 1908), where he was appointed chancellor in 1890. Then a central figure on the Prussian intellectual and scientific landscape (he was the friend of the most important German industrials, ambassador to international expositions, was referenced in dozens of books and papers and memorialized in Berlin with a monument and a named street). It is for his masterpiece *Kinematics of Machinery: Outlines of a Theory of Machines*, published in German in 1875 (translated in English one year later), that Reuleaux is still known today as one of the most important forerunners of the modern mechanism and machine science. See Moon (2007, 47ff).

[15] I differ from Wilson (1997, 2017) in that I attach central importance to the "architectonic" issue concerning the relation between kinematics and mechanics.

[16] On this "essentialism", see Wilson (1997).

of a body T (for instance, a satellite) rotating around another body P (for instance, a planet). So long as the conditions remain unaltered, the movement continues the same. However, as soon "as any external disturbing force Q_1 (. . . perpendicular to the plane of motion), begins to act upon one side of T, T alters its path; . . . if this is to be prevented, another external force Q_2, equal and opposite to Q_1, must be brought simultaneously into action" (1875, 31). This is what happens in nature according to Newton's classical mechanics. In a machine, the situation is different. Imagine that T is fixed on a wheel which rotates around an axis passing through P. Now,

> if any disturbing force Q acts sideways upon the wheel, then (if we suppose the material of the wheel, shaft, and bearings to be completely rigid) no alteration of the circular motion occurs; and this is true equally whether Q be great or small, continuous or intermittent, constant or changing in direction. (Reuleaux 1875, 31)

For Reuleaux, unlike Monge, in machine as in nature, movements are guided by forces. But, whereas in nature the occurrence of a disturbing force Q_1 does not generate by itself any other forces Q_2 which opposes to its action, in a machine, when a force acts on a body in a non "required manner", others forces appear, which prevent any movement. Let me quote how Reuleaux summarizes the fundamental distinction between natural and machinic phenomena:

> Whilst in the first system, which we may call kosmical, the external measurable mechanical forces are opposed by similar external forces, in the second, the machine system, there are opposed to all external forces others concealed in the interior of the bodies forming the system, and appearing there, and acting in exactly the required manner [*erforderlichen Weise zu wirken*], in consequence of the action of the external forces . . . The difference between the two systems is therefore that sensible forces are in the one case opposed by other and independent sensible forces, and in the other case by dependent latent forces [*abhängige latente Kräfte*].
> (Reuleaux 1875, 33–34)

In a machine, "latent forces" (generated by the rigidity of the mechanism controlling the movements) automatically compensate for any disturbing external forces.

Reuleaux acknowledges that his distinction between machine and kosmos is an idealization: "the two systems are not divided by a hair-line" (1875, 34), they are ideal-types that are rarely found in pure form in reality.[17] He also acknowledges that theoretical kinematics (which is about a pure machine) should be supplemented by applied kinematics (taking into account the deformation of the machine components) in order to apply itself to the real machines that surround us.[18] But the distinction nevertheless helps to distinguish two important ways of generating movements, and Reuleaux steadily maintains that "the balancing [*das Wirken*] of sensible by latent forces is [the] principal characteristic [*Hauptkennzeichen*] of the machine like or machinal as distinguished from the kosmical," and that it "must be kept distinctly in view in endeavouring to understand the exact idea conveyed by the word machine" (Reuleaux 1875, 34–35).

Now, if latent forces are the principal characteristic of machines, what are these forces exactly? How do they act? Reuleaux does not engage in an ontological analysis of latent forces.[19] Instead, he emphasizes that the action of the latent forces is determined by the form and arrangement of the rigid bodies in which they are concealed. Let me quote an important passage which summarizes his position:

> When a machine is constructed it is meant to be an arrangement for carrying on some definite mechanical work . . . For such a purpose

[17]There are latent forces in nature (Reuleaux mentions the motion of sap in plants) and external forces are sometimes required in machines as well (Reuleaux mentions machine with flectional elements or strings and machine with force-closure—see chapter IV of Reuleaux 1875).

[18]Reuleaux (1875), introduction and section 2. See also the introduction of Reuleaux (1893).

[19]Reuleaux only notes that latent forces might be explained by the microstructure of the body in which they are concealed—contenting himself to point out an analogy with the theory of heat. See Reuleaux (1875, 33).

we require that so soon as motion is caused [*Bewegung erzeugen*] by any effort in any part of the machine that motion shall be of an absolutely defined nature [*ganz bestimmte Bewegungen*] . . . Every motion then which varies from the one intended will be a disturbing motion, and we therefore give beforehand to the parts which bear the latent forces the bodies, that is, of which the machine is constructed, such arrangement [*Anordnung*], form [*Form*] and rigidity [*widerstandsfähig*] that they permit each moving part to have one motion only, the required one [*nur eine einzige Bewegung, und zwar die bezweckte gestattet*]. This having been done, so soon as the external natural forces which it is intended to employ are allowed to act, the desired motion [*bezweckte Bewegung*] occurs. Our procedure is therefore twofold; negative first—the exclusion of the possibility [*Ausschliessung der Möglichkeit*] of any other than the wished-for motion; and then positive—the introduction of motion.
(Reuleaux 1875, 35).

Because the parts are rigid and then conceal latent forces, their forms and arrangements exclude the possibility of any other motion than the wished-for one. The reference to latent forces is thus, for Reuleaux, a way of saying that it is the geometric connection between the rigid parts of the machine (their shapes and arrangements) which explains how motion is determined.

In section 3, Reuleaux gives more details about how movements are geometrical determined:

In the machine, . . . the moving bodies are prevented, by bodies in contact with them, from making any other than the required motions. This contact also, if the problem is to be entirely solved, must take place continually, which presupposes the possession of certain properties by the bodies in contact. In proceeding to examine these properties more closely, we shall assume in the first instance that the bodies possess complete rigidity, . . . so that only geometrical properties remain for us to consider. (Reuleaux 1875, 41–42)

To go back to our example of the rotating body, it is the contact between the rigid axis and the rigid wheel which makes it the case that the latent force will counterbalance any external "disturbing" one. The wheel and the axis are related by what Reuleaux

Figure 1: Revolute or Pin Kinematic Pair.
From Reuleaux Collection of Mechanisms and Machines at Cornell University.
Photo by Jon Reis. Reproduced by kind permission.

calls a revolute pair. (For more on the notion of a pair, see Section 4 below.) Such a pair involves two parts connected through reciprocal cylindrical envelopes, the hollow element (the wheel) being wrapped around the full one (the axis). Any external force applied in the plane of rotation tangentially to the wheel will move it. But any other move is made impossible by the form of the contact surface.

Let me summarize what we have seen so far. Reuleaux considers that there is a genuine difference between the way movements are produced in nature and the way they are generated in a machine. Whereas, in nature, the motions are caused by sensible forces and should then be investigated with the aid of Newtonian mechanics, in a machine system, motions are controlled by latent forces, that is, by the geometrical shapes of the contact surfaces between the rigid parts of the machines. Kinematics, defined as the theory which studies how motions are generated by geometrical arrangements of the machine components, is then distinct from classical mechanics.[20] The idea that, in an idealized machine, movements are determined by the geometrical way components are arranged, is one that plays a central role in Reuleaux (1875). Let me call it Reuleaux's fundamental insight (RFI):

(RFI) In a machine (as opposed to a kosmos), movements are completely determined by the geometrical forms and arrangement of the machine components.

(RFI) is not present in the kinematic tradition before Reuleaux. One does not find in Ampère, Willis, Laboulaye, etc. such an emphasis on latent forces, rigid bodies, contact between surfaces, or geometrical guidance. Conversely, (RFI) will be, after Reuleaux, considered a fundamental ingredient of the kinematic toolkit—an ingredient that still remains relevant today.[21]

4. Kinematic Grammar

In light of a comparison with Wittgenstein, I need to say a little more on Reuleaux (1875), especially on his effort, in chapter 7, to design a kinematic notation. The basic elements of a machine are called "links" (*Glieder*) by Reuleaux. He explains, however, that the genuine units in a machine are pairs of links.[22] This emphasis on pairs is a direct consequence of (RFI), which says that the movement of an element with respect to another is completely controlled by the shape of the mating surfaces connecting the two. Thus, the geometrical features of the pair entirely determine the kinematic properties of the links it connects. Chapter 3 of Reuleaux (1875) is a systematic investigation of the most elementary types of pair, called "closed pairs".[23] When several links are connected two-by-two by pairs, they are said to form a kinematic chain (*Kette*). A chain can be open or closed (it is closed if all the links are two-by-two connected, open if the chain ends by two unconnected links). A closed chain with one link fixed (the frame link) is called a mechanism,[24] and the type of a mechanism is partially[25] determined by the type of pairs which relates its links. In the simple example of the four-bar linkage (of

[20] I agree with Wilson that optimization in machine building is a running theme in Reuleaux (1875). But I don't see how the stress put on optimization can explain, for instance, why ideal machine components must be rigid while the "architectonic" project to make kinematics independent of mechanics explains it.

[21] For example, let me quote the beginning of a representative modern textbook on the topic (Uicker et al. 2003, 7): "[In a mechanism], the controlling factor that determines the relative motions allowed by a given joint is the shapes of the mating surfaces or elements. Each type of joint has its own characteristic shapes for the elements, and each allows a given type of motion, which is determined by the possible ways in which these elemental surfaces can move with respect to each other . . . These shapes restrict the totally arbitrary motion of two unconnected links to some prescribed type of relative motion and form the constraining conditions or constraints on the mechanism's motion." Burmaster's textbook (1888) played an important role in the diffusion of Reuleaux's work in Germany. On this reception, see Luck (2000).

[22] "A machine consists solely of bodies which thus correspond, pair-wise, reciprocally. These form the kinematic or mechanismal elements of the machine. The shaft and the bearing, the screw and the nut, are examples of such pairs of elements. We see here that the kinematic elements of a machine are not employed singly but always in pairs; or in other words, that the machine cannot so well be said to consist of elements as of pairs of elements. This particular manner of constitution forms a distinguishing characteristic of the machine" (Reuleaux 1875, 43).

[23] A pair is closed if its mating surfaces have the same forms. Not all kinematical pairs are closed (on this, see Reuleaux 1875, 129ff). According to Reuleaux, there are three kinds of closed pairs: the prismatic, the revolute, and the helicoïdal. In a prismatic pair, the link *A* can only slide with respect to the link *B*; in a revolute pair, *A* can only rotate around *B*; in a helicoïdal pair, the movement of *A* with respect to *B* is one of "simple sliding combined with simple rotation proportional to the sliding". For more on this, see Reuleaux (1875, 96ff).

[24] This distinction between closed chain and mechanism gives birth to the theory of kinematic inversion: one and the same kinematic chain can generate various mechanisms which differ with respect to the link considered as fixed. On kinematic inversion, see Reuleaux (1875, 92–96), Uicker et al. (2003, 15–19).

[25] There are indeed complications here. Sizes sometimes matter, one can introduce derivative chains, etc. For question of convenience, I oversimplify a development which is much more refined in its detail.

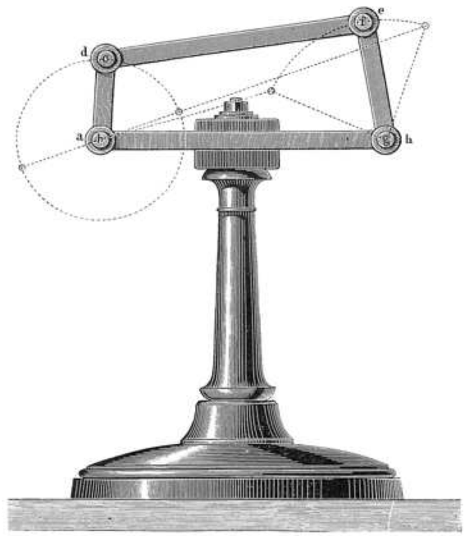

Figure 2: Four bar linkage mechanism
Illustration from Reuleaux (1875, 68).

which a picture is given below), the four links (*bc*, *de*, *fg*, and *ha*) are constrained by four revolute pairs, and when one link is fixed (here *ha*), the motion of any other link determines the motion of the rest, in the way represented in Figure 2.

In Reuleaux (1875), the kinematic classification is based on what is called "kinematic analysis", i.e., the decomposition of any given machine into its elementary mechanisms. Even if Reuleaux's classification is no longer used,[26] many of Reuleaux's basic notions (pairs, closed chains, mechanisms, and kinematic inversion) are still part of the mechanical engineer's toolbox. What I want to emphasize is not so much the success of Reuleaux's project as its coherence: the classification of machines is entirely grounded in the geometrization of movements expressed in (RFI). Indeed, machines are made of mechanisms, i.e., chains of links connected by pairs, which are just links whose (relative) movements are guided by the geometrical form of their rigid mating surfaces. The idea that what is specific to a machine is that the movements of the parts are generated by contact between rigid bodies is then the foundation of Reuleaux's entire theoretical building.

[26]Reuleaux argued that the degree of freedom of any pair should be equal to one. From a contemporary perspective, this limitation unduly restrains the generality of Reuleaux's analysis.

Reuleaux (1875, chap. 7) completes his conceptual construction by developing a specific symbolism made to express, in a concise way, the relation between the composition of a mechanism and the movements of its different parts. There, Reuleaux explains that the immense variety of possible chain-forms makes it difficult, by using ordinary language, or by using picture,[27] to "survey the inner relationships of mechanisms as well as their differences" (1875, 248). He then remarks that, in similar circumstances, mathematics and chemistry have developed special symbolic notations to get around the problem. It is thus not surprising to see that watchmakers first, then, more systematically, Babbage and Willis, tried to design special notation to "express machine combination in some concise form" (248). Reuleaux however, considers that these first attempts could not succeed, owing to the fact that machines were not yet analyzed in a proper way.[28] It is therefore necessary to take up a top-to-bottom approach to this project.

Let me briefly indicate how Reuleaux's notation works. Three types of signs are distinguished: names, form symbols, and relation symbols. Names designate the shapes of the mating surfaces. For instance, a cylinder (i.e., the shape of a mating surface

[27]On the advantage of the formula over the picture, see Reuleaux (1875, 259–60).

[28]On Babbage (1851), Reuleaux (1875, 250) wrote: "What the symbolic memoranda of Babbage express, and were intended to express, is not the essential constitution of the machine, its different parts scientifically defined and recognizably indicated by the stenographic symbols."

in a revolute pair) is denoted by C. Form symbols give extra indication about what is named. For instance, C^+ designates a full cylinder, C^- an open one. Relation symbols are essentially of two kinds: those designating the contact between mating surfaces (for instance, the symbol "=" means that the sizes of the surfaces are equal) and those characterizing the link (the link itself is designated by a sequence of points and the parallelism between links is represented by two vertical bars). Thus, the four-bar linkage drawn in Figure 2 is encoded in this way (the underlining of the last link means that it is fixed):[29]

$$C^+ \ldots \| \ldots C^-_= \; C^+ \ldots \| \ldots C^-_= \; C^+ \ldots \| \ldots C^-_= \; \underline{C^+ \ldots \| \ldots C^-_=}$$

As this example shows, Reuleaux gives prominence in his notation to the representation of pair-types. It is in keeping with his general line since pair-types determine the nature of the relative movements. Note as well how geometric constraints are embedded within the grammatical rules of the new notation. For instance, speaking of the revolute pairs, Reuleaux explains that "we shall not require any sign beyond C^+C^+ to show that the axes of the cylinders are parallel, while C^-C^- is incorrect, for it is impossible to form a kinematic pair from two open cylinders" (1875, 255). Owing to the form of the revolute pair, it is superfluous to indicate that the axes are parallel just as it is superfluous to say that the expression "C^-C^-" does not belong to the symbolism. In other words, in Reuleaux's symbolism, a name contains in itself its possibility of combination with the other names and also the ways in which it can be combined. Reuleaux's geometrization of machinic movement is at the same time a grammaticalization of machinic movement: the geometrical determinations of the machine movements are reflected at the level of the syntax rules of the kinematic symbolism. Of course, in this notation, "the possibility of the bending, breaking off, melting" (*PI* 193) of the machine components are completely forgotten, and what is symbolized is only "a particular action of the machine" (*ibid.*), taken as a highly idealized (geometrized) system.

In light of the comparison I want to make with Wittgenstein, I don't need to expand more on the specific features of Reuleaux's kinematic symbolism. What is important for me is the mere existence of a kinematic symbolism and the fact that grammaticalization and geometrization go hand in hand. Of course, Wittgenstein draws so much from mechanics and engineering in his ruminations on language (in the *TLP* as in *PI*) that a more thorough examination of Reuleaux's kinematic notation could be extremely instructive.[30] But here I merely want to show that in the context of Reuleaux's kinematic, the phrase "the machine as symbol" has an easy interpretation: it simply designates the machine system (that is, what can be encapsulated in a kinematic formula) as opposed to the kosmical system. The machine as symbol is the machine diagram, the idealization of the working of the real machine (the one subjected to external forces which deform its elements and introduce friction) that is represented in kinematic notation. There is thus no need to refer to computer science and to a computer program to give meaning to the enigmatic "*Maschine als Symbol*" occurring in *PI* 193. Such a phrase finds a very clear and precise interpretation in the context of Reuleaux's kinematics.

[29]That the chain is closed is represented by the fact that "=" is appended to the last "C^-". This notation can be expanded in various ways. It can also be abbreviated; see Reuleaux (1875, 263–64).

[30]To take just one example, think of *PI* 12 which addresses the uniform appearance of words which conceals the variety of their uses: "It is like looking into the cabin of a locomotive. We see handles looking more or less alike . . . But one is the handle of a crank which can be moved continuously (it regulates the opening of a valve); another is a handle of a switch, which has two effective positions . . . ; a third is the handle of a brake-lever . . . ; a fourth, the handle of a pump . . . ". Reuleaux explains at the beginning of chapter 7 that the aim of a kinematic notation is to recover the differences between the mechanisms that are difficult to express in the ordinary language and difficult to detect in a picture.

5. Reuleaux, Wittgenstein and the Machine Analogy

To interpret *PI* 193–94, Kripke refers to the distinction between the program and the machine in which it is implemented, while Baker and Hacker refer to the distinction between geometrical theorems and mechanical laws neglected in some (unspecified) books. Our detour through Reuleaux shows us that, to understand Wittgenstein's analogy, we don't need to refer to computer science and we can be more specific than Baker and Hacker. When Wittgenstein says in *WLFM* that kinematics is a "branch of geometry" in which "one works out how pistons will move if one moves the crankshaft in such-and-such a way," it seems he referred to kinematics as developed in Reuleaux (1875).

In the introduction of this paper, I represented in Table 1 the analogy Wittgenstein made between the machine and the rule. In the same way as the movements in a machine are predetermined by the geometric configuration of the machine, the applications of a rule are completely controlled by the rule. The question, then, is as follows: in which conception of the machine does one find the idea that movements are predetermined in the geometric configuration of the machine? Our historical investigation led us to conclude that this idea, which I labelled (RFI) above, lies at the heart of Reuleaux's conceptual reorganization of kinematics and that it distinguishes it from the other ones (in particular, those present in Monge's tradition). The comparison between Reuleaux and Wittgenstein is natural: the insistence on the rigidity of machine parts, on the geometric pre-determination of machinic movement, is found nowhere else but in Reuleaux.[31] In the same vein, note how close Wittgenstein's terminology is to Reuleaux's: phrases such as "*bestimmt Bewegung*", "*Wirkungweise*", "*Ausschliessung der Möglichkeit*", "*starre Körper*", and of course "*Kinematik*", are used by both thinkers. Our historical investigation allows us to clarify the nature of the relationship between machine and movement by adding a new column to Table 1:

Geometrical form of the contact surface between links in a chain	Machine	Rule
Relative movements of the links in a chain	Movements	Application

Table 1′

One might object that one does not find any trace of the phrase "*Die Machine als Symbol*" in Reuleaux (1875). But, as I have shown in Section 4, Reuleaux attached great importance to the design of a special kinematic symbolism. Understanding a kinematic formula which represents a mechanism amounts to knowing how its different parts move. The kinematic grammar incorporates the geometric constraints. One can then complete Table 1 in this alternative way:

Kinematic formula representing a mechanism ("machine as symbol")	Machine	Rule
Movements of the mechanism	Movements	Application

Table 1″

In Section 3 (see Table 2), I explained how the machine analogy is based on the distinction between two notions of a machine: the idealized and the real, material, one. Kripke considers, as Baker and Hacker did, that Wittgenstein is precisely criticizing the conflation of these two notions in *PI* 193–94 and more generally, the conflation between a normative rule and a factual

[31] We are here spelling out the comparison already made by Wilson (1997, 293): "As students, [the apprentice engineers] are trained to grind out endless sequences of pictures . . . that capture the state of the mechanism at various stages of its cycle. It is exactly this sort of *development* of a "rule" . . . that Wittgenstein seems to have in mind in his discussion at *PI* §§193–94."

description. From our perspective, the distinction between the ideal and the real machine is based on Reuleaux's distinction between machine system and kosmical system. More precisely, the real machine is the mixture of machine and kosmical system that is often found in reality, whereas the ideal machine is the ideal-type of the machine system—the one, rarely found in its pure state in reality, in which all the movements are generated by latent forces. In a pure machine system, the action of the machine is, literally speaking, "there in it from the start", concealed in the geometrical shapes of the components. On the contrary, in a kosmical system (or in a mixture between the two systems), external sensible forces are to be taken into account; we can no longer assume that the bodies are rigid and "forget the possibility of bending, breaking off, melting, etc." Note that in Reuleaux, the distinction between machine and kosmos is used to demarcate kinematics from general mechanics: kinematics is "a branch of geometry", as Wittgenstein says in *WLFM*, and thus does not have the same status as general mechanics (Newtonian dynamics) which is considered an *a posteriori* science. All this allows us to complete Table 2 by clarifying the distinction between ideal and real machine:

Kinematics	Machine system (rigidity)	Necessity	Grammatical rule	Internal relation
Classical mechanics	Kosmical system (sensible forces)	Contingence	Factual description	External relation

Table 2′

In kinematics, which studies machine systems, one deals with internal relations, grammatical rule, and necessity; in classical mechanics, which studies kosmical systems, one deals with external relations, factual descriptions, and contingent fact.

Reinserting *PI* 193–94 in the context of Reuleaux's kinematics then allows us to clarify the conception of machine that underlies Wittgenstein's comparison between machine and rules. Kripke's anachronistic reference to computer science is not forced on us: there is another, historically more likely framework that makes it possible to give a precise content to the distinction between ideal and real machines. But despite its attractiveness, our proposal faces an obvious objection: neither in the *TLP* nor in the later passages from the thirties does Wittgenstein mention Reuleaux as an important influence on his thought.[32] If, as I maintain, Wittgenstein had Reuleaux in mind when writing *PI* 193–94, why did he not say so?

There is evidence that Wittgenstein read Reuleaux (1875): a copy of the book was in Karl Wittgenstein's library and some of Reuleaux's drawings are copied in Wittgenstein's *Nachlass*.[33] Moreover, it seems natural to assume that, even if Reuleaux was retired when Wittgenstein came to Berlin, the program of the Technische Hochschule bore the mark of his work.[34] (For instance, the collection of 800 models built by Reuleaux for expressing mathematical and kinematic ideas was kept in Charlottenburg and remained heavily used during Wittgenstein's formative years.)[35] As we have seen, Reuleaux's fundamental concepts and techniques, those which are important in our story, rapidly became an integral part of the science of machines (and are still used today). We don't see how they could have escaped the engineering student that Wittgenstein was at the time.[36] Even if he did not mention Reuleaux's name, one can be sure that

[32] In a well-known passage from *Culture and Value* (1980, 19) dating from 1931, Wittgenstein only lists Boltzmann, Hertz, Schopenhauer, Frege, Russell, Kraus, Loos, Weininger, Spengler, Sraffa, as authors who influenced him. Reuleaux does not appear there.

[33] See Seekircher (2003, 325).

[34] Things are more complicated, however, since, in 1896, when Wittgenstein arrived, the university was led by Aloïs Riedler, a proponent of practically-oriented engineering education who fiercely opposed Reuleaux's theoretical leaning. On this, see Hamilton (2001) and König (2007).

[35] On this, see Moon (2004) and König (2007). For a survey of the literature on Wittgenstein's scientific training, see Nordmann (2002) and Abel, Kroß and Nedo (2007).

[36] Note also that Wittgenstein had an extraordinary manual dexterity and that his interest in machines and their workings never wavered; see McGuinness (1982) and (1988).

Wittgenstein had been exposed at length to Reuleaux through his reading and through his training in Berlin.

Let me emphasize the importance of this second element. In the introduction of the 1963 edition of the English translation of Reuleaux (1875), the historian of technology Eugene Ferguson explains why the figure of Reuleaux gradually faded away:

> Many of the ideas and concepts introduced in this book have become so familiar to us that we are likely to underestimate Reuleaux's originality and consider him merely a recorder of the obvious. We may feel that there is indeed no other way of approaching kinematics. But that is perhaps the hallmark of genius: to state a new idea in such convincing and uncompromising terms that it becomes immediately obvious and soon a truism. (Ferguson 1963, v–vi)

For Ferguson, what makes Reuleaux important is at the same time what made him invisible later on: his capacity to give birth to a new scientific practice is at the same time a capacity to be completely absorbed into it and to disappear from the scene. My suggestion is then that, when speaking about machine in *PI* 193–94, what Wittgenstein had in mind is less Reuleaux's book than the intellectual techniques and routines, kinds of problems, explanations, and exercises that formed his intellectual daily life during his learning years. Thus, in the story I told, Reuleaux does not appear as an original author, like Boltzmann, Hertz, or Russell, for instance, whom Wittgenstein discovered through books. Reuleaux is rather a teacher, or perhaps not even that. Reuleaux may just be a source which allowed Wittgenstein to recover and articulate the "picture" which governed the scientific practice that Wittgenstein was immersed in (and that he liked). In my opinion, the lack of explicit reference to Reuleaux could then be explained by the fact that the impact of Reuleaux's thought was spread through primarily practical channels.

6. Conclusion: Why Machines?

Assuming that our story about Wittgenstein's machine analogy is true, is there anything to learn from it about the development of the argument in *PI* 193–94? In *PI* 191–92, Wittgenstein asks for a model for dispositionalism, i.e., for explaining how a rule ("grasped in a flash") can contain its (future) application.[37] The machine provides Wittgenstein with just this. The trouble with Kripke's and Baker and Hacker's interpretations is that they do not explain why Wittgenstein takes the machine (rather than anything else) as a model. Kripke's reference to the now familiar computer is anachronistic, and Baker and Hacker's reference to some scientists' confusion is rather vague.

Our recontextualization can explain why Wittgenstein speaks about a machine at this place in *PI*. As we saw in detail, to guarantee the unity and independence of the general theory of machines, Reuleaux changed the status of some sentences (those describing the behavior of bodies in contact with each other). By assuming that bodies are super-rigid, certain descriptions, which are regarded as factual in general mechanics, are deliberately granted a "status akin to a theorem of geometry" (as Baker and Hacker (2009, 107) wrote). This "hardening" of certain (*prima facie* empirical) propositions is not an insignificant step, which Reuleaux could do without in his reasoning. (RFI) is a decisive stage in Reuleaux, since it guarantees the independence of kinematics *vis-à-vis* classical mechanics. In Wittgenstein's terms, this move is what makes kinematics a new language-game, distinct from the language-game of mechanics.[38] Kinematics is therefore somehow linked to the notion of a rule; it becomes a language-game on its own by fixing some propositions that were usually taken as empirical.[39] In my perspective, if Wittgenstein speaks

[37] *PI* 191: "It is as if we could grasp the whole use of the word in a flash . . . — But have you a model for this? No."

[38] There is an interesting passage in *WLFM* (196) where Wisdom suggests that we consider rigidity as a conditional clause since, "if we put in the clause "assuming of course that the parts are rigid", aren't we explaining the part which rigidity plays in the calculus?". Wittgenstein rejects the proposal: "rigidity does not come into the calculus at all". Reinserted into the context just described, the answer amounts to refusing to turn the kinematic calculus into the mechanical one.

[39] In Table 2, kinematics is put in the first line with the notion of rule.

about a machine in *PI* 193–94, it is because kinematics presents us with a paradigmatic case where the "grammaticalization" of some empirical laws gave birth to a new language-game. Let me explain further.

What does Wittgenstein say about the kinematic model? Does he consider (RFI) as a clear case of confusion between rule and fact? One might think so. After all, in kinematics, scientists deal with idealized rigid machines as if they were concrete things—and in so doing, it might seem, they conflate grammatical rules and factual propositions. On this reading, however, one forgets that kinematics is a particular kind of language-game which has its own relevance and field of application. Kinematicians do not want to apply their rules directly to the behavior of real bodies. Their problems are different. They want, for instance, to classify existing machines, and (RFI) has proven useful in this regard. When doing kinematics, engineers and scientists are not mixing up two different notions; they do not, in particular, conflate grammar and fact. When Wittgenstein speaks about the machine model as pre-determining its action, I therefore do not think that he is criticizing the model. The model is all right; it is its philosophical use that is at issue.

As a matter of fact, the distinction between rules and fact in *PI* is language-game dependent. For Wittgenstein, grammar is always anchored in a particular language game. The distinction is not global, it is local and always internal to a language-game. Thus, a sentence which is empirical in a certain language-game can be grammatical in another without any problem. There is no contradiction in this as long as one does not confuse the two language-games. Talk about norms and facts cannot be off-ground; they should always be relativized to a particular language-game. What Wittgenstein criticizes is not those who use the kinematical model but those who fail to situate this model in its broader environment. This is how I read the end of *PI* 194. Speaking of phrases such as "possibility of movement", Wittgenstein remarks:

> We mind about the kind of expressions we use concerning these things; we do not understand them, however, but misinterpret them. When we do philosophy we are like savages, primitive people, who hear the expressions of civilized men, put a false interpretation on them, and then draw the queerest conclusions from it. (*PI* 194)

As we saw, Reuleaux never confused the abstract machine (the one studied by theoretical kinematics) with the real machine (the one studied in applied kinematics). He repeatedly said that real machines are never perfect and that they are always mixtures of machine and kosmical systems. Reuleaux never wanted to absorb classical mechanics into kinematics; he never confused the rules of his calculus with the real laws causing movements in nature. (On the contrary, to clear this confusion is the goal of his distinction between machine and kosmos.) Thus, Reuleaux is here on the side of civilized men: by framing a new kind of expressions, his only goal was to free kinematics from the tutelage of mechanics.[40]

It is not when we do kinematics but when "we do philosophy" that we are like savages. What does Wittgenstein mean by "doing philosophy" here? The question is certainly too broad to be answered properly. But I would like to suggest that Wittgenstein is not equating doing philosophy to confusing rule and fact, as both Kripke and Baker and Hacker seem to believe. Indeed, there is no substantial theory of rule and fact in *PI* (as there was in *TLP*) which could allow to give a precise meaning to such a confusion. Grammar is always rooted in a particular language-game. And we do philosophy precisely when we loose sight of the differences between language-games. What Wittgenstein criticizes in the passage is not the machine model itself but the temptation of extending it to any case of rule-following, i.e., the temptation to

[40] Baker and Hacker (2009) seem to suggest that Wittgenstein criticizes in *PI* 193–94 a confusion committed by some (unknown) scientists. If this the case, I think they are wrong.

base a general theory of what a rule is on this particular case.[41] Reuleaux had a clear view of the relations between kinematic and the various adjacent scientific fields; he did not project this model to any kind of rule-following, nor did he consider the geometric pre-determination of the machinic movements as the conceptual matrix of what a rule is. He was not doing philosophy.

On my reading, the dispositionalist mistake comes from the temptation to erect the geometric predetermination one finds in kinematics as a general model of rule-following. The confusion between fact and rule is then not the source of the mistake; it is a byproduct of a deeper temptation to project, in an uncontrolled way, a certain language game on others. In *PI* 193–94, Wittgenstein would fight this inclination by exposing its source—the model or "picture" that feeds it. By describing the particular context in which the "geometric" approach to the rule is legitimate and relevant and by identifying precisely the language game which it is rooted into, Wittgenstein would provide the means to defuse the fascination it induces, and thereby block the tendency to export it in all cases of rule-following. In brief, Wittgenstein would seek here, as always, to "teach us differences".[42]

My reading gives a great power of seduction to the machine model, and one might wonder if there really are so many minds that have fallen under the spell of what seems to be a very particular conceptual framework. Who are the philosophers who have been tempted to generalize the kinematic model? In the end, I would like to suggest that one of them is probably the author of the *TLP* himself, who, by virtue of his training (and taste), has been completely immersed in this scientific practice. The kinematic paradigm, set up by Reuleaux, seems to have had a profound impact on the thought of the first Wittgenstein,[43] and this could explain why the author of *PI* gives it such importance. But the detailed defense of this last hypothesis will have to wait for another occasion.

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Sébastien Gandon
Université Clermont Auvergne
sgandon0@gmail.com

[41] I therefore agree with Wilson (2017) that the distinction between the unique and global logical space of the *TLP* and the different spaces of possibilities (the different language-games) of *PI* plays a role in Wittgenstein's developments on machines. But the role I give to Reuleaux in my story is different from his.

[42] Wittgenstein once told Drury that if the book needed a motto, he would use the quotation from *King Lear*: "I'll teach you differences".

[43] For instance, in the first sections of the *TLP*, the relations between objects and states of affairs are molded after the relations between links and kinematical chains: "In a state of affairs objects fit into one another, like the links [*Glieder*] of a chain [*Kette*]" (2.03). Wittgenstein, exactly like Reuleaux, insists on the facts that the objects (resp. links) contain in themselves their possibilities to combine with the other objects in the state of affairs (resp. chain). And, exactly like in Reuleaux, these constraints are incorporated in the grammar of the names that represent the objects. One can even find in the *TLP* an equivalent to Reuleaux's developments about rigidity: "Objects, the unalterable [*Feste*], and the subsistent are one and the same. Objects are what is unalterable and subsistent; their configuration is what is changing and unstable" (2.027–2.0271).

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