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## Grete Hermann as Neo-Kantian Philosopher of Space and Time Representation

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Grete Hermann's essay "Die naturphilosophischen Grundlagen der Quantenmechanik" (1935) has received much deserved scholarly attention in recent years. In this paper, I follow the lead of Elise Crull (2017) who sees in Hermann's work the general outlines of a neo-Kantian interpretation of quantum theory. In full support of this view, I focus on Hermann's central claim that limited spatio-temporal, and even analogically causal, representations of events exist within an overall relational structure of entangled quantum mechanical states that defy any unified spatio-temporal description. In my view, Hermann also advances an important transcendental argument that perspectival spatio-temporal representations of nature have their foundations in general relational networks that are not spatio-temporal. The key point is that the adoption of a perspectival system within the general network induces the representation but only for that context. These ideas are consistent with a perspectival subject–object principle in Kant and also with Weyl's work on Lie groups and their representations.

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## Editors' note

Our dear friend and colleague, Erik Banks (1970–2017) died unexpectedly on 18 August 2017. He is the author of *The Realistic Empiricism of Mach, James and Russell*, of *Ernst Mach's World-Elements*, and of numerous essays. His contributions to the history of analytic philosophy, and especially to the study of neutral monism and figures such as Mach and Hermann Grassmann, enabled a renewal of serious attention to these areas in Anglophone contexts. Characteristically, Erik was laying the bricks on several intellectual projects when he passed away. We are grateful to be able to publish one of his finished structures.

We were forced by circumstances to make the final copyedits to this article. Out of respect for Erik's intentions for the paper, we have remained as close to the original typescript as possible.

–S. E. & L. P.

## 1. Introduction

Attention to Grete Hermann's seminal 1935 essay "Die naturphilosophischen Grundlagen der Quantenmechanik" has increased markedly in recent years after decades of neglect (see Herzenberg 2008). She is currently seen as an important neo-Kantian philosopher of science and this trend is only accelerating. As Elise Crull (2017) has insisted, the topics of Hermann's essay which first drew most attention (her trenchant critique of von Neumann's no-hidden variables proof and her claim that one could claim (partial) "retrocausal" reconstructions of past events in quantum mechanics) were not actually the primary motivations, nor the major conclusion, of her original essay. Rather her relational, or one might even say perspectival, view of quantum mechanics, and its correspondence with neo-Kantian views, formed in the neo-Friesian school of Leonard Nelson, and further elaborated by her in collaboration with the Leipziger Kreis around Heisenberg, is her central accomplishment.[1] What has been shown by Hermann according to Crull is nothing less than "the contours of a viable Kantian interpretation of quantum mechanics" (Crull 2017, 14).

In this paper I will explore Hermann's neo-Kantian ideas about physical space and time representation as expressed in her philosophy of quantum mechanics. I wish to emphasize especially how Hermann's own "perspectival" view helps us to understand the shocking power of quantum mechanics to flout spatio-temporal description on the one hand, while on the other hand exhibiting classical features and partial spatio-temporal representations *analogically*, but *only* relative to a certain perspective or experimental context, as Hermann repeatedly emphasizes in her work. The surprising conclusion, I think, is that the full set of quantum mechanical relations in some ways gives us the *foundation* for the classical spatio-temporal representation of systems and their interactions, in a limited way when a certain context is chosen from within the general network of relations that are not generally representable spatio-temporally.

Thus, the flouting of unified space and time description, and the splitting of different perspectival spatio-temporal views of natural systems (which one would have thought was the most un-Kantian thing possible) is, I think, a natural consequence of Hermann's perspectivalism and her notion of a *Spaltung der*

[1]Max Jammer (1974, 207), who is critical of many of Hermann's views, nevertheless valued her "relational" interpretation of quantum mechanics highly, even giving her a certain priority over Bohr, and encouraged future authors to consider her interpretation of quantum mechanics seriously.

*Wahrheit* for different classical perspectives presented from inside an essentially quantum world of relations. Many interpretations promise to do this, of course, but I hope to show how a flexible neo-Kantian approach like Hermann's can potentially serve us as a powerful tool in the jungle of quantum mechanics interpretation without either adding or subtracting anything from the traditional formalism of the theory.

I will first offer a general overview of Hermann's "relational" philosophy of quantum mechanics, and draw some comparisons with contemporaries such as Schlick and Cassirer. Then I will try to connect her with older traditions in Kant and Helmholtz. I will look in particular at the second antinomy (an analysis of which is strongly suggested in her essay) and consider Kant's use of a perspectival "observer–object" principle as an intellectual foundation for space and time representation (as highlighted by Friedman 2012), very similar to what resulted later from the work of Helmholtz and Lie on transformation groups and geometry. Of course, as Weyl discovered, Lie groups and their representations also play an equally fundamental role in quantum theory and it may be that Hermann's groundbreaking idea that classical spatio-temporal relationships emerge from quantum ones "perspectivally" could eventually be worked out here in a more complete exploration of these ideas.

## 2. Hermann's "Relational Conception" of Quantum Mechanics[2]

Grete Hermann's intellectual pedigree was impressive indeed. She was a doctoral student of Emmy Noether at Göttingen and a student of Leonard Nelson, a neo-Friesian philosopher and close associate of David Hilbert. As part of her ongoing studies, she became part of Heisenberg's Leipzig seminar on quantum mechanics, as a result of which she wrote her most important essay "Die naturphilosophischen Grundlagen der Quantenmechanik" (1935). She is also mentioned in Heisenberg's essay "Quantenmechanik und Kantische Philosophie" (in Heisenberg 1979), although the three-way conversation described there between Hermann, Heisenberg and a young Friedrich von Weizsäcker seems only to describe an early stage in Hermann's development as she forged on toward a Kantian view that would be rigorously consistent with the new science of quantum theory.

Hermann's earliest concern seems to have been to understand how causality (in her own special Nelsonian view of what that means) could be reconciled with the basic indeterminism and uncertainty so characteristic of quantum theory. To take an example heatedly discussed in Heisenberg's essay, if an atom decays and this is essentially an indeterministic process with no prior cause, is it correct to say that some overarching principle such as "every event has a determining prior cause" is in fact false? Is there a way to understand the principle of causation differently, and how can we rule out any investigation of prior causes already and for all time?

Heisenberg answers this question in two ways in this period: one, that since position and momentum of a particle cannot be known simultaneously one cannot completely define the initial state of a system as a function of $(p, q)$ in phase space anyway, so the causal principle is not false but inapplicable, which Max Born also held. Two, the information quantum mechanics gives is maximal and cannot be added to without destroying other accessible information about the system. He uses the decaying atom example to show that if, for example, we do not know in which direction the atom throws off a particle, a momentum uncertainty, then we can allow these possibilities to interfere and obtain a pattern that carries definite physical information about measurements on the system (for example the information that when interference is completely destructive, a particle cannot

[2] This is Jammer's word for her position and Bohr's as well after his response to Einstein's EPR paper.

possibly be detected at a given position). But an investigation of the direction will destroy the interference and this information will be lost (Heisenberg 1979, 66–67).

Hermann was clearly not happy with this explanation and probed further the question of whether there could be hidden variables in her well known critique of a circle in von Neumann's proof. (For the relation of Heisenberg and Hermann on the hidden variables issue, see Crull and Bacciagalupi 2009.) However Hermann's eventual conclusion was not that hidden variables were needed to complete quantum mechanics, but rather that they are superfluous, because, she says, the quantum theory already provides for a knowledge of the causes one desires, but only *relative* to a given experimental context or question one asks of the system.

Hermann uses von Weizsäcker's analysis of the well-known gamma ray quantum microscope experiment to illustrate her view. In this example, we send an electron with indeterminate momentum and position into a plane and illuminate it with a light source. A photographic plate is positioned *either* in the focal plane of the lens of the microscope (in which case we will know the momentum imparted to the photon by the recoil with the electron but not the position where the collision occurred)[3] *or* in the objective image plane further on through the aperture of the microscope (in which case we will know the position of the collision with the electron but not the momentum change imparted to the photon).

Until one or the other measurement is made, or, as Hermann would say, the experimental context is determined, the correct total quantum mechanical description of the state is the tensor product of the state space of the electron *and* the state space of the photon constrained by the requirement that momentum be conserved in the interaction. The description of the entangled state of the object and the measuring device is the correct, and complete, physical description of the system. By considering the overall entangled state we have a simple example of what Hermann called the general "network of relations" between the two systems. That is, in the most complete description of a system in quantum mechanics we have a network of states of various systems entangled with each other and not factorizable into the separate systems of the electron and the photon.

When one separates out an aspect of the "photon system" to measure (either its position or momentum depending on where one puts the plate) one can then obtain information about the other system for that same observational context. The positional information about the photon will in turn tell me *where* the electron was but nothing but a range about the momentum of the collision; the other observation in the focal plane of the microscope will tell me what the momentum of the collision was, but because the incident wave is a plane wave with a set of parallel rays representing the particle paths, nothing definite about the position. In short the whole wave function for the entangled system in phase space, or general network of relations is the fundamental thing for Hermann and the choice of context is like the choosing of an embedded *perspective* from which to view the multidimensional and abstract wave function. If no context is chosen and we do not measure the photon at all then this combined state *remains* the correct physical description, and will remain so long after the systems have separated.

As many authors have noted, the microscope experiment is an anticipation of the EPR experiment, as was also apparently realized at the time (see Jammer 1974, 179–80). The Leipzig group (presumably including Hermann) regarded the experiment as a good illustration that the space-time separation of the systems cannot affect entanglement if the correct description of the state

[3]This is because the lens will focus a plane wave with parallel rays into a spherical wave front that converges on a point at the focal length. If the measurement is not made here, a spherical wave front will be passed on into the aperture of the microscope and on into the objective image plane converging there where a positional measurement will occur on the plate.

remains the combined wave function of both systems (Jammer 1974, 179–80). If this flouts our intuitive notions of how physically separated systems behave in space and time, then so much the worse for these intuitions; the Leipzig group had already insisted that the abstract wave function of the two entangled systems in phase space is not to be visualized as a system evolving in space and time anyway.

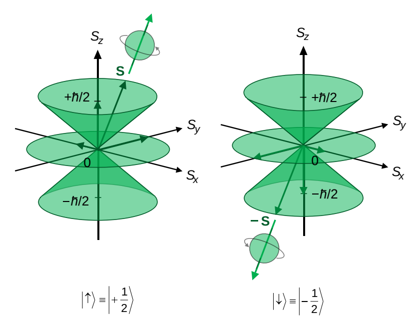
But if quantum mechanics is essentially an abstract set of relations or tensor products, then how *does* something like classical spatio-temporal behavior and a sequence of events emerge? Hermann's answer: once an experimental context is chosen, we project the entangled state onto the *eigenvectors* of a Hermitian operator and obtain probabilities of measurements. Then of course there is a measurement and one of these *eigenstates* is realized in a probabilistic manner. *For these properties chosen to measure the first system*, and in that basis, Hermann says we can then obtain a partial, as it were "retrocausal," description of what happened to the *other* system, which can now be treated as if it were separated from the first, and treated as if the interaction between the two systems were a classical one, like a collision in space and time.

This appearance of limited, classicalized view of separable systems within quantum theory, insofar as they are described with a complete set of commuting observables (CSCO), all of which have the same basis *eigenvectors*, was noted also by Heisenberg in *The Physical Principles of the Quantum Theory* in 1930 and would surely have been known to Hermann, who explicitly used this essay for her own work. He says of the limited classical behavior of quantum systems:

> As examples of cases in which causal relations do exist, the following may be mentioned: the conservation theorems for energy and momentum are contained in the quantum theory, for the energies and momenta of different parts of the same system are commutative quantities. Furthermore, the principal axes of $q$ at time $t$ are only infinitesimally different from the principal axes of $q$ at time $d + dt$. Hence if two position measurements are carried out in rapid succession it is practically certain that the electron will be in almost the same place both times. (Heisenberg 1930, 58–59)

It might also be pointed out that when two observations of the *same* observable are carried out sufficiently close to each other, the behavior of the system is always consistent with classical physics. For two measurements of spin carried out close together, it cannot for example be found that a spin state in a given direction could suddenly change faster in *that* direction than an applied magnetic field could be expected to torque it classically. If I carry out another close measurement of this direction it cannot switch instantaneously to the opposite direction on this axis in less than the given time it could do so classically. It *can* change instantaneously to a new value if a different direction of spin is measured and in this case there is, once again, simply *no* space-time description of such an instantaneous change.[4]

[4]The quantum mechanical interpretation is that the total spin and its component in one direction commute and can be measured simultaneously but that the spin actually precesses (metaphorically) in the two other unknown directions. It is however interesting that the direction of the spin in the known direction reverses the direction of the metaphorical precession in the other incompatible variables. The instantaneously measured change in a new component direction is *not* due to any applied physical torque that acts to change the direction of spin of the particle (see figure below):




(Public domain diagram by Maschen, `https://commons.wikimedia.org/wiki/File:Spin_half_angular_momentum.svg`, accessed 2 January 2018.)

The limited, classicalized behavior of a quantum system or a set of them is contextual only. If we confine our view to only the maximal set of mutually commuting operators (MMCO) determined by the basis of common *eigenvectors*, to this degree only do we develop a quasi-classical story about "what happened" in an intuitive spatio-temporal and causal format we understand and partially visualize in space and time. So for Hermann the appearance of a quasi-classical story about a system is a perspective-relative matter but the classical seeming behavior on the chosen description is for all that a legitimate partial spatio-temporal representation of what would otherwise be an abstract evolution of the wave function of the two entangled systems in higher dimensional phase space. So roughly speaking Hermann's answer is this: classical behavior and visualization is a matter of occupying, or being embedded in, a *perspective* or point of view from within a much broader system of relations, and not from outside of it looking in. When the interior perspective is adopted, it will seem in a limited way that the relations observed between the observer system and object system are semi-classical ones but only if the properties of the system are confined to the experimental context of the CSCO.

Max Jammer, in his otherwise sympathetic review of Hermann's ideas, questioned whether her classicalized partial reconstruction of the event should be considered a *causal* account of a collision between an electron and a photon (Jammer 1974, 209–10). Jammer believed that Hermann had shown only how an electron at a certain position *could have* led to the detection of a photon at another position, but not that it necessarily had to be so, not even in retrospect (the electron did *not* after all have a definite momentum at the position where the collision occurred).

Hermann was clear that her case was not a past attribution of a position *and* a momentum to the electron as in Heisenberg's "time of flight" example where we can conceivably violate the uncertainty principle but only in the unverifiable past (Heisenberg 1930, 24–25). An electron at a position colliding with a photon did not *have* to lead to the photon being detected where it was, it simply did in this case, and for this context, which could not have been predicted *a priori* (see Heisenberg 1930, 24). In this respect Hermann's view of causality as an incomplete analogy for ordering appearances in a certain experimental context, differs completely from Schlick's view that the *Kausalprinzip* is a linguistic schema for making predictive statements (Schlick 1931, 1936). As she says repeatedly, forward prediction of final states by initial states is not the main thing, nor is it necessary for a causal *interpretation* of events that have already happened, in which the two states are already given and then of course the sense of prediction has changed completely:

> [Q]uantum mechanics requires a *criterion* for causality and takes it, like classical physics, from the possibility of predicting future events. In contrast to classical physics, however, it has broken with the hitherto almost self-evident appearing presupposition that each causal claim may be verified directly by the prediction of the effect. In all cases of in-principle unpredictable events, the causal explanation quantum mechanics gives for them can only be verified indirectly by inferring backwards from these events to their cause, and by further deriving, from the assumption that this cause was present, predictions of forthcoming events whose occurrence can be verified empirically. (Hermann 1935/2017, 262)

Jammer's seems to me an overly harsh interpretation of what Hermann meant. She also denies that the resulting measured position of the photon was predictable even in principle, simply by determining *where* the electron was (Hermann 1935/2017, 257). She does however say that given the reverse derivation of the position of the electron from the position measurement on the photon, *in a context where the two positions are both given*, then they can be retrospectively connected as ordered by cause and effect, in the sense of representing them in a sequence, where the one event *was* followed spatio-temporally by the other. We must imagine the *two* determined positions of the systems being given and drawing in thought a connection between these positions in the reconstruction of a collision event. It is quite a different thing to go backwards to a time when *only the position of the electron is known* and say that the position of the photon is not yet known but predicted. How could that be the case? I can't believe that is the correct interpretation of what she meant.

I think, in consideration of what Hermann will say later about causality, she only means that in a given context, an interpretive causal *analogy* becomes possible within which the *two given positional measurements* can be ordered together, and a limited analogical principle of cause and effect can be extended that allows us to order the states *as a sequence of space-time perspectives of the positions of particles undergoing a collision*. But that context will require both measurements of position to be assumed and the separability of the two systems. The important thing for Hermann is that even indeterministic events admit, *post-hoc*, of being placed in this kind of sequential space-time format as a viable *analogy* or format for a very limited, contextual representation of what has already occurred, not to *predict* what will happen given only one datum, the position of the electron.

Incidentally Hermann's very subtle Nelsonian view of causation as an *analogy* only for connecting appearances (of which more later) appears to me to fit well with Kant's own special interpretation of causality, which is also, in a certain way, analogical and post-hoc. Kant uses the famous example in which the sides of a house *could* have been experienced in any order and there is no "succession in the object" as he says. The *a priori* prediction of the sequence is *not* the issue and is in fact impossible given that one is free to see a house in many different sequences of perspectival views of it. Nevertheless, the *category* of causation applies even to this example. Once a given sequence of views has occurred for the observer in a certain order he has to retrospectively connect, synthesize, these views in *that* order for that sequence to add up to the experience of viewing a house. Once the sequence of views is apprehended, it cannot be altered in memory or imagination, upsetting the space or time order of the perceptions. Removing the space and time "places" occupied by the views in the sequence in effect destroys the *idea of a sequence* and replaces it with a random shuffle of views in which one cannot know where or when each view is to be placed. That removal of an objective sequence of views would undermine and destroy the experience of *having viewed* a house and not a random jumble of views that do not constitute an object. Even Kant's famous discussion is mostly about ordering or synthesizing the views of an event that have already been apprehended.

Hermann's view of causality is also, of course, perspective dependent and thus her view of what creates a foundation for a perspective is the primary framework, within which we can return to the question of what she really meant by relative causal reconstructions in a given perspective.

## 3. A New Transcendental Argument

The question of how quantum mechanics seems to flout space, time, and causality was very prominent in Heisenberg's circle. This discussion was, of course, heavily influenced by Bohr, who also saw the space and time representation of quantum states as one of its most problematic new aspects. As Crull points out (2017, 8–11), of the three versions of complementarity that Hermann located in Bohr's work, the most central was the complementary relationship between the intuitive descriptions of the natural processes in space and time and the unintuitive and abstract evolution of the state of the system in phase space which utterly flouts spatio-temporal representation (Hermann 1935/2017, 265–67).

In Heisenberg's 1930 lectures he too presents a Kierkegaardian "either-or" where, on the one hand, we can obtain a description of "[p]henomena described in terms of space and time, *but* uncertainty principle", or, on the other hand, we can stick with the "causal relationship expressed by mathematical laws *but* physical description of phenomena in space and time impossible" (Heisenberg 1930, 65). This statement by Heisenberg is a little different from Hermann's view, since she regarded the analogical causal description as belonging to the (albeit limited) description of processes in space and time, that is, on the classical side of the divide, whereas Heisenberg regards the evolution of the Schrödinger equation of the combined system as the causal or deterministic element although the evolution is on abstract phase space and the spatio-temporal description does not apply to this evolution except in special cases where the degrees of freedom of the system coincide with the spatial coordinates. Hermann also *sometimes* spoke of causal relations in connection with the idea that the complete network of relationships is causally determined, that is on the quantum side by the Schrödinger equation (Hermann 1936, 343).

Von Weizsäcker is quoted in Heisenberg's essay about discussions in Leipzig as saying that Kant's spatio-temporal account of experience of objects "is essentially correct" and a viable format for the organizing of experience, but only if "relativized" to each experimental context (Heisenberg 1979, 70). Von Weizsäcker also points out that quantum mechanics does not need to respect the classical categories of space, time or causality "except in this relativized way." In that sense, he says, the Kantian categories are like the law of the lever, correct but only in a relativized and incomplete domain of application (Heisenberg 1979, 73–75). Clearly some of this must have influenced Hermann's mature view. In fact, Heisenberg seems to be attributing aspects of Hermann's later view to von Weizsäcker! Heisenberg decisively credited Hermann with the breakthrough to a relational conception of quantum mechanics and limited or contextual space, time, and causal representation: "that's it, what we've been trying to say for so long!" he exclaimed to her (see Jammer 1974, 209).

What Hermann contributed, with her perspectival interpretation, was in a way very different from what Bohr, Heisenberg and others were saying, at least originally, about the classical-quantum divide, and about the need to retain the classical language or a classical interpretation of the measuring device on one side of a "cut" in order to interpret experimental observations. Their argument was rather positivistic, not Kantian, along the lines of: "we must interpret the measurement device classically, or else what do the measurements mean? If the device and the object are both treated as an entangled system the device would be capable of giving measurements that were superpositions of the classical variables and this would be nonsensical."

But this tenet of the orthodox Copenhagen interpretation was also strongly challenged at the time. For von Laue, Einstein, and Schrödinger, for example, there was no such requirement to retain the classical language for all time. As Einstein wrote to Schrödinger, if position and momentum and other non-commuting variables were subject to "a shaky game" (Fine 1986, 18), then why not find other *combined* observables (not hidden variables) and try to construct a causal story of atomic processes or systems evolving in this new, ever-changing basis? Einstein had pulled off the trick before of combining energy and momentum into a spatio-temporally invariant energy-momentum vector so why not something along these lines?

Because of its completeness and basis independence, Hilbert space (a space of orthogonal *functions*) has the built-in property that any state in the most complex superposition for some classical operator like energy will also be an *eigenstate* for *some* other Hermitian operator on some very abstract basis indeed. But this mystery operator is just a mathematical possibility; it may be forbiddingly complicated, or even impossible to realize three-dimensionally. Even if one did, what would this strange property represent physically, what would the basis functions be, and would they even be tractable to analysis?

Hermann was certainly aware of this rather desperate "way out" of the shaky game and rejected it (1935/2017, 240). Some neo-Kantians like Cassirer however welcomed the break with the past and encouraged a complete rejection of intuition, for example the intuition that particles have any distinct identity, and an acceptance of the abstract ideas of functions and invariants in its place (see Cassirer 1956; Mormann 2015). Russell (1927, chap. 4) also praised the elimination of the material atom and intuitive space and time in favor of Heisenberg's abstract matrix atom, warning that more extreme revisions to our intuitive concepts of space and time could be expected to result from quantum theory than even from relativity.[5] Earlier, Ernst Mach had pushed for an "abstract physics," freed of the intuitive, or psychological, spatio-temporal and material format, and free to explore abstract invariant functional relationships wherever these might lead (see Banks 2003). Given these open questions, is the choice of classical language and a semi-classical description, comporting with a limited space-time and causal understanding, more than just an anthropomorphism or subjectively valid analogy? This is the question for Hermann's view and it is the key to her whole interpretation but not for the positivistic reasons given by orthodox Copenhageners.

The question is this: why are we duty-bound to find some intuitive spatio-temporal format within a world that, like Hermann, we recognize as essentially a non-intuitive quantum phenomenon? Why do we bother with these partially intuitive spatio-temporal and causal reconstructions within quantum mechanics except for our subjective convenience, or a misplaced allegiance to intuition? Hermann was absolutely clear that quantum mechanics in its complete form always describes entangled systems and an entangled universe in "a network of relations whose foundations ultimately remain indefinite" (Hermann 1935/2017, 275). Crull describes Hermann's view as "fundamentally quantum" and only relatively contextual (Crull 2017, 15). But within this relational network, by a suitable choice of an observable basis we "can make a cut through them," choosing a basis of commuting observables, and classicalize some of these relations among those observables that are not only convenient for our powers of visualization but which also, as she says, are "indispensable" for "the development of physical knowledge" about the systems we investigate (Hermann 1935/2017, 273). Such knowledge is of course analogical and is a partial representation at best of the full set of relations:

> [T]he quantum mechanical description by which, on the basis of some observation, a physicist determines his system, does not characterize this system completely and absolutely, but (so to speak) reveals only one aspect of it—precisely the aspect that presents itself to the researcher on the basis of the observation made here. (Hermann 1935/2017, 256)

Nevertheless, the limited aspect or classicalized perspective gives real knowledge and is not a *mere* representation; nature really does behave quasi-spatio-temporally and even (in a limited way) causally under those given limiting conditions and relative to context. The representation is perspective-dependent but not thereby unreal or illusory. Perhaps Leibniz's term fits these limited aspects best: they are *well-founded phenomena*.

And so here is the real point: we can understand what space, time and a special analogical causal representation really are by virtue of quantum mechanics, we can understand the conditions under which such a representation is possible and what actually determines its possibility physically. The puzzle isn't to find spatio-temporal and causal foundations for quantum mechanics in general, via classical hidden variables. No such foundations are possible according to Hermann. Rather, the

[5][*From the editors:* Erik Banks indicated that "Russell (1913)" also supports this view. However, that text was not listed in the bibliography, and we are not sure which text he intended to cite. Given that, we have deleted the reference from the main text.]

puzzle is, in reverse, how to let quantum mechanics reveal for us the emergence of classical spatio-temporal physics as a "perspectival fact" contained within quantum mechanics. Quantum mechanics is therefore not to be taken just to *flout* all spatio-temporal and causal representations of physical systems, as it seems at first, by considering only the grandest set of relations of systems in an entangled, evolving wavefunction. No, quantum mechanics also gives, under special conditions, the *foundations for classicalized spatio-temporal representations of nature* we never were able to penetrate within classical physics—or even relativity.[6] I believe this is a true transcendental argument if there ever was one.

The key to the new transcendental argument à la Hermann is her perspectivalism. For Hermann the whole quantum mechanical wave function of entangled systems is foundationless because it is a "perspectiveless" shifting jumble of all interrelated possible perspectives an observer inhabits and takes upon a system, involving entangled relations of virtually any variables subject to the ingenuity of the physicist. This basis is entangled with that, and this basis projects its basis vectors onto that one, as the system evolves deterministically in time according to the Schrödinger equation. The wave function is a complete state description and needs no hidden variables because we can already obtain all the information about causal connections that is potentially accessible simply by investigating one or another aspect of the network, i.e., adopting limited perspectives within it which carry limited causal stories and representations of its states and how they succeed each other in events. A quantum state *as such* has no preferred perspective or basis description in just one set of observables, for example the position or momentum basis; it is always multiply expressible (even when it is in an *eigenstate* for a given observable). The adoption of a perspective within this overall network of relations determines a basis and all interior relations are then to be judged from this standpoint (Hermann 1935/2017, 256). From this perspective, classical behavior and the possibility of spatio-temporal, and analogical causal description, emerges but only in a limited way relative to the observables accessible in the perspective. Most of the time the initial adoption of a perspective is a probabilistic matter, the system will not be in an *eigenstate* chosen by the observer, although it will always be in a state suitable for some "ideal observer" and his perspectival system of basis *eigenvectors*. But this isn't the important thing; the important thing is our ability to adopt a perspective in which we can separate out, provisionally, the two entangled systems of the measured object, the photon system, from the object, electron system and thus seemingly arrive at a limited classicized understanding of their relationship.

So, to me at any rate, this is the really wondrous thing about Hermann's view. She explains spatio-temporal representation as something that emerges perspectivally from within quantum mechanics itself. This position dovetails quite nicely with her neo-Kantian views, as I will show below, and, I think, even with historical aspects of Kant's own views on space and time representation.

## 4. Hermann and Wave Function Collapse

A very interesting document is Grete Hermann's terse comment "Zum Vortrag Schlicks" on a Schlick lecture, published posthumously in *Erkenntnis* in 1936 ("Quantentheorie und Erkennbarkeit der Natur"). Here she responds to a passage where Schlick seemingly directly criticizes her "nachträglich" (*post-hoc*)

[6] I mean that in classical and relativity physics, space and time or space-time is always in some sense a given background or drafting board rather than a phenomenon to be explained and understood. We can analyze and understand space-time from within space-time but we cannot discuss any further foundations in those theories. Quantum theory may well be different in this respect.

interpretation of causal relations in the microscope experiment (Schlick 1936, 321–22). For Schlick, as always, causal knowledge is the *forward-looking* verificationist prediction of functional relationships, not postdiction (see Schlick 1931). The previous position of the electron as revealed by the present experiment does not mean that this causal relationship really existed in the past, since *at* the time of the collision the electron really had no position and "was really indeterminate." The correct description *then* was the entangled wavefunction of both systems. And of course even knowing the position retrospectively doesn't permit the prediction of the photon's position, not even in the retrospective story. But this only matters if one is a *forward* verificationist about causation and Hermann certainly was not that, having fully embraced the indeterminism of quantum mechanics. Schlick also criticizes Kant's principle of causality as purely an *a priori* principle for the *forward* prediction of causal relations. But that was not Hermann's understanding of Kant via Nelson and Fries, as upholding instead the claim of "the causal connectedness of nature" through relations, which she says is indeed upheld in the relational interpretation of quantum mechanics (Hermann 1936, 343). The continued insistence that causation be understood as forward prediction *a priori* seems to have drawn her ire.

The more interesting part of her response, however, concerns the fact that she apparently did not regard wave function collapse as a physical process in space and time. The collapse of the wave packet was not a physical effect of a "wave previously spread over space suddenly becoming confined to a small region" in what would have to be an instantaneous process faster than light (Hermann 1936, 342).

What was Hermann's view of wave function collapse, or instantaneous state transitions? One possible answer is that she may have already been a Bohrian although of a very special kind, a "no-collapse Bohrian," (perhaps even before Bohr himself). Scholars have already determined important relationships between Hermann and Bohr that have been characterized as Kantian, in particular the use of analogies when our knowledge is not intuitively presented but symbolic and analogical in nature (Chevalley 1994; Crull 2017, 7–14, 17–19). Jammer credits Hermann with a "relational interpretation" of the microscope experiment that Bohr later arrived at after EPR (Jammer 1974, 208). Clearly she thought of a measurement as a kind of "classicalizing" of the total relational system relative to that basis wherein the measurement is made. Yet she also insists on keeping the total network of relations and interpreting the classicalized sections as a perspective or relative contexts contained within the overall set. And it is *always* the overall set of relations that provides the foundation for this form of representation. But does the adoption of a perspective then necessitate wave function collapse as part of her view, or is adopting a perspective more like occupying a branch in a relative state, or no collapse, theory?

Hermann's view will not involve collapse if we read her like Bohr, as interpreted by Don Howard. According to Howard, when all is said and done, Bohr held the following (very nuanced) view:

> What Bohr meant by a "classical" description was not a description in terms of classical mechanics or electrodynamics. It was, instead, a description wherein one assumes that object and instrument are separable, that they do not form an entangled pair. But since they do form an entangled pair, how is such a "classical" description possible? What I think Bohr meant is this (see Howard 1979, 1994): Given a pure state correctly describing any system, including a joint system consisting of an entangled instrument–object pair, and given an experimental context, in the form of a maximal set of comeasureable observables, one can write down a mixture that gives for all observables in that context exactly the same statistical predictions as are given by the pure state. But then, with respect to the observables measurable in that context, one proceeds as if the instrument and object were not entangled. One can

> speak as if the measurement reveals a property of the object alone, and one can regard the statistics as ordinary ignorance statistics, the experiment being taken to reveal a definite, though previously unknown value of the parameter in question. One short step now to complementarity, for the mutual incompatibility of the experimental contexts for measuring conjugate observables implies that different contextualized "classical" descriptions in terms of mixtures of the aforementioned kind are required for incompatible observables. Relativizing to experimental context makes possible an unambiguous, "objective" account of the object as not entangled with the instrument and, in so doing, implies complementarity. No wave packet collapse. No antirealism. No subjectivism. (Howard 2004, 675)

If we have a pure state for an entangled system, we can write down a density operator for this state in the basis of a chosen observable and obtain *another* density operator such that, for the observer in that basis, the state will seem to behave empirically with regard to measurement *as if* it were a classical mixture, obtaining the probabilities of a measurement as a kind of perspectival effect of our point of view on the entangled system rather than as a physical process of wave function collapse. In effect, we have again classicalized the quantum state for the observer who can believe that he or she is dealing with two separable classical systems, albeit probabilistically, but the classical format is simply a perspectival effect of being embedded in an overall relational network which *does not collapse* and which continues to provide from within it the foundations for the classical context in space and time. The overall relational network remains in a quantum mechanical pure state albeit differently expressed in another basis. The trace of the square of the density operator will still reveal the difference, of course, even to the embedded observer. So the observer will know he is still within a network of relations that is behaving in a limited classical way from his perspective.

As Howard shows, we can perspectivally and in a limited way represent the probabilistic nature of quantum mechanics as if it were classical or statistical uncertainty, as occurs when we provisionally separate the measured and the measuring system and classical behavior seems to emerge from quantum behavior. We can thus classicalize sections of quantum mechanics by taking up a certain internal perspectival view of the relational network which is really still *always* a network of relations (and our classicalized mixture *really* a pure state in Howard's example).[7]

Hermann's remarkably similar view about the probabilistic collision of electron and photon is that when the measurement occurs it is unpredictable in principle, of course, but potentially comprehensible as *a classical statistical uncertainty* in the causal story created by adopting that perspective. A pure quantum state indeed behaves like a classical mixture to an observer occupying that relative context. In that perspective, the quantum event can be comprehended *as if classically statistical* and is thus amenable to a retrospective reconstruction of the collision *as if it were* a sequence of positional events connected together into paths through space and time.

[7] There are obvious connections between the relational approach and the relative state interpretations of quantum mechanics, the "one world–many observers" versions in particular (see for example Bitbol 1991). Making the classical world emerge from quantum foundations is a main goal of the interpretation of quantum mechanics. The difference here is the use of the "Kantian way of thinking" to accomplish this, which is the real novelty of Hermann's approach I believe and of course her historical priority is also important to recognize. Epistemologically, there may be important connections between Hermann's neo-Kantianism and *Spaltung der Wahrheit* and the program of "perspectival realism"—see Massimi (2016)—although I do not have space here to explore this. As Crull mentions, although Hermann's ideas were generated by thinking about quantum mechanics, she also thought about branching out and applying her philosophical ideas more generally and it is quite possible that she saw them as a kind of general epistemology of science similar to what perspectival realism is proposing. I do not have enough knowledge of the movement to judge whether this is so.

## 5. Causal Relations as Analogies, Hermann's Neo-Kantian Heritage and a "Spaltung der Wahrheit"

I now want to tie in Grete Hermann's relational view of quantum mechanics to the neo-Kantian lineage of ideas, in hopes of backing Crull's claim that Hermann has indeed revealed the outlines of an *echt* Kantian approach to quantum mechanics and to connect what I have called above her "transcendental argument" supporting space-time and causal representation, with its ultimate Kantian roots, and in my own way backing Hermann's claim that limited contexts and perspectival reconstructions within quantum mechanics represent in their own way *real knowledge* and not *mere* representations suitable to human intuition. (This objection might be urged against Howard's no-collapse Bohr as well as Hermann: what does it matter that quantum relations "seem like" classicalized relations, such as the measurement of mixtures, from the limited perspective of an observer? Is this not the philosophy of the *Als-Ob*?)

As was said before, Hermann was part of Leonard Nelson's neo-Friesian school. Not everyone is familiar with Fries and his version of Kantianism. This is unfortunate because, according to Beiser (2014), Jakob Fries was the real founder of neo-Kantianism, in so far as that word denotes a movement of scientifically minded philosophers who rejected German Idealism and instead embraced Kant's work as a philosophy suitable for scientists as well as philosophers. Fries was considered a psychological Kantian by some (see Hatfield 1991), but Beiser says that he is better understood as introducing what came to be known as the metalanguage–object language distinction (which was incidentally one subject of Nelson's collaboration with Hilbert and the metamathematics program at Göttingen). For Fries, we must separate the metalinguistic psychological justification of synthetic *a priori* principles *as necessary to our thought*, from the conditions of their validity taken as axioms without further proof or justification (Beiser 2014, 31–32). Hermann herself points out that Fries rejected Kant's distinction between our knowledge of phenomena and the unknown thing in itself (Hermann 1935/2017, 271). She defends this view in Heisenberg's essay where she corrects von Weiszäcker (1935/2017, 257) and says that the atom is expressly *not* a *Ding an sich* but a "transcendental object of experience," that is a function unifying sensory data into an object of experience, not something beyond. The thing in itself is something "we know not what", a simple expression defined by our ignorance of it, and not an independently subsisting thing that is causing phenomena. For example quantum mechanics is essentially "foundationless" for her and as such needs no *Ding an sich* for foundation. There is simply a self-sustaining system of relations and literally nothing beneath that level.

Hermann also explicitly calls attention to two Kantian themes in her work: to the importance of the antinomies in understanding the *Ding an sich*, and to causation as an "analogy." In the second antinomy, Kant discusses the "conflict of ideas" between the Leibnizian view where matter, space, and time must be grounded in simples, or monads, and the continuum view where the smallest parts of nature or space and time are made up of still further relations with further structure (time is made up of extended times not instants, space of spatially extended parts not points) and no simple thing is empirically discoverable. If we take the continuum view, Kant thinks, the world would dissolve into an ungrounded system of mere relations without ultimate relata, which he cannot accept. And if we take the view of the *Monadology*, we seem to be unable to generate a continuum out of simple elements like monads, as Leibniz actually attempted to do in his later years.

As Kant explains at the end of this discussion, what makes appearances always complex, and never the perception of the absolutely uncomplex and simple, is the fact that we do not stand outside reality from a God's eye view (as one does in the

*Monadology*). We always occupy a perspective *within* the system of experience, which means it is automatically divided up between a subject and an object, or points in space or time occupied as opposed to other points represented from the chosen perspective. No experience, no perception, not even of the self, can therefore actually *be* simple if it is already embedded in this kind of subject and object representation.

As Hermann now points out, and this is a clear result of her work with Nelson, causation is *fully* part of the phenomenal continuum side of the antinomy. Appearances are not *caused* by unknown things in themselves, causation is only applicable within appearances. As she says, when we understand changes in natural phenomena like motion through differential equations, or when we analyze the continuous mass-density of a body, we do so not by attributing properties to simples or points, but by working entirely within extended regions, by making them smaller and smaller but always considering neighboring points as well, never the point in itself (Hermann 1935/2017, 273). Causation, she points out, *cannot* be the discrete following of a state by a successive state by cause and effect, because in a continuum there is no unique successor state. Similarly one cannot locate an instantaneous velocity acquired at a point in free fall, the velocity is also changing continuously, nor can one define a continuous mass density as a property at a point, one can only consider smaller and smaller extended regions and think of the point as the limiting case of mass densities at neighboring points (1935/2017, 273). It would seem, then, that the continuum of space and time and the limited causal principle are applicable only when we occupy a perspective within appearances and it is only in that sense that these spatio-temporal *and causal* aspects of reality seem to us to have a real foundation as a *representation* of relationships which may or may not be spatio-temporal.

Hence, Hermann sums up:

> The concepts of substance and causality, and with them those of things in space and of their states evolving in accordance with natural laws, are not thereby eliminated from physical consideration. It has only become apparent that they do not describe adequately the natural events but are rather used as mere analogies. Qua such analogies, however, they have their use—indeed they are indispensable for the development of physical knowledge. It is enough to make a 'cut' somewhere in the investigation of this relational network across which the relational connections cannot be traced, and in this limited perspective one conceives of things in space and of causal links for processes. (Hermann 1935/2017, 273)

The causal connection of appearances, for example, the retrospective connection between the positions of the photon and the electron that we represent as a collision, is simply an *analogy* for synthesizing the appearances together within a given perspective (position measurement on the photon). Causation as applied in this format does not get into the representation of *how states cause other states* or how the electron at one position actually causes the photon to bounce off of it and fly into the microscope lens and be detected on the plate. It is at best a reconstruction of the events in the perspective as a collision at a certain place, but since we will always lack the precise momenta in this perspective, it is not representable completely as a classical collision, only *analogous* to one in these respects on which the reconstruction is based. Her Nelsonian view of causation as a perspectival analogy only best informs what she really meant by "causality."

Hermann thus made the explicit connection between the appearance of classicalized systems within quantum mechanics in her relational view, and the appearance of space and time representation and causal analogies in her neo-Kantian philosophical view. In both, classical behavior, and space and time and limited causal representation, appear only when occupying a position or perspectival point of view within an overall system of relations that is fundamentally quantum and not spatio-temporal. *Outside* of occupying a perspective on those relations, or a subject-object condition, there is no such thing as a classical or spatio-temporal representation of nature.

How many possible perspectival systems can be found within the overall system of relations, and will these different systems be compatible or incompatible? Here there occurs what Hermann calls a "Spaltung der Wahrheit," a splitting or fracturing of truth, a term she draws from Friesian philosopher Ernst Friedrich Apelt. For example, in the microscope experiment, it is clear that one can occupy the position basis perspective or the momentum space perspective and occupy very different and incompatible worlds, each of which has a semi-classical or spatio-temporal nature as either particles or waves. The wave story on momentum space corresponding to "the collision of two particles story" in the position basis, for example, is going to look very different indeed and is fundamentally split off from the other perspective.[8]

But since spatio-temporal representation depends on the adoption of a particular perspective, and because the system of relations contains *many* such perspectives within it, the adoption of the one basis simply rules out sharp or classicalized behavior in the other basis, so the choice of one perspective of classicalized perspectives simply excludes the other, and we have a multitude of perspectives instead of a simple unified view of nature in space and time—in a rather startling contrast to relativity where there is a universal stage, not all of which is accessible at any time or place to every other observer, but which does not face the splitting of spatio-temporal perspectives as in Hermann's view of quantum mechanics.

[8]An interesting question I leave out of consideration here is what happens when two observers who have occupied different perspectives compare their measurements made of incompatible observables? Does one observer $A$ in effect "measure" the other $B$ and do those measurements $AB–BA$ commute or not?

## 6. Kant, Helmholtz and Hermann: A Comparison

The critical result, then, of the above investigation of Hermann's views is as follows: quantum mechanics can flout space-time description because its relational network of entangled objects and perspective-independent evolution is more general than the restricted space, time and limited analogical causal representation of natural processes. Non-locality, delayed choice experiments, the sudden and instantaneous changes from one state vector to another, none of this should surprise us if that is true. What Hermann has shown, in my view, is that we should adopt this "fundamentally quantum" view and use it to explain the emergence of spatio-temporal representation from within a system of relations that is not spatio-temporally representable.

This seems to me to be by far the most exciting prospect for Hermann's neo-Kantian view and it connects well with Kant himself (see Banks 2014). The case could be made that within Kant's *Critique of Pure Reason* a "perspectival principle" of subject–object representation also plays *exactly* this primary role in founding the spatio-temporal representation of experience both intuitively and intellectually (Friedman 2012; Banks 2014). Space and time representation is intuitive for us, and not conceptual, in one sense not because it *has* no further foundation or analysis and must be taken as brute, but rather, as in the analogies section, we are always embedded in a perspective and that fact itself is responsible for generating this phenomenon of spatio-temporal representation.

Some of our spatio-temporal representation is clearly a schematic construction such as the construal of dynamical processes and enduring and extended objects of experience which must be synthesized or put together by rules or categories. As an observer occupying a perspective, I always have structured intuitions, these resemble relatively simple "snapshots", like a jumbled stack of photos. Each photo exhibits something like an individualized perspective on a certain object at a given point in space or snapshot of a process in time. My understanding can put them into a series and generate a perspectival object in space according to a rule and my imagination can schematize thereby an intuited object, if only I have a principle which the snapshots are bound to obey on one side (their "affinity" to be so organized) and which the understanding can read out of them as an organizing principle to synthesize an order.

The perspectival or subject–object principle provides just this: all snapshots are comprehensible as perspectival views of an object to a subject embedded in that system of perspectives. For example, to fully synthesize and intuit a three dimensional solid out of the temporal sequence of views of its faces I have to imagine a path which takes up and connects the snapshots so that the process generates a sequence of views perspectivally related from the vantage point of an observing subject who, in passing over the faces in sequence, can experience the object that way. If an object moves in a parabola, since only the present state and position are ever experienced, I must come up with a sequence for my memories and possibly future anticipated positions such that these snapshots offer a perspective sequence of temporal appearances of a process to a subject.

Following Friedman (2012), and being deliberately anachronistic, the individual snapshots of intuition already have the simple non-metric structure of a set of projective planes in a projective space. (This is why, for example, a perspective drawing can represent parallel lines meeting at a vanishing point at infinity, and why intuited space in general is capable of presenting "infinite given magnitude" in intuition.) Space also has an "intellectual" perspectival structure to be read purely by the understanding. In the simplest sense, space is a system of points in which every point is representable from every other, where every point can be the zero, or center of a perspective on the rest of the system. Similarly, time is also, intellectually, a system of perspectives in which any time can be selected as the point of view from which to consider all other times as past present or future relative to it. Today we would say that the perspectival nature of space and time enable us to characterize these relations of points of view as a group of transformations, or rather that a group of transformations and its invariant can always be induced within a self-consistent system of perspectives (Friedman 2012). As Singer (2001, 85) shows, a system of observers at various velocities provides a kind of recipe for constructing a group of transformations of a system with momentum, say a particle, from one allowed observer's perspective on it to that of another observer in the same system, and momentum is thus a conserved property in this system. (I.e., if you take it away in one perspective, say by having the observer occupy the frame of reference of the particle, the perspectives of the other observers shift and the property simply reappears elsewhere for a different observer.)

The adoption of a point of view within such a system, then, taking up the occupied position of a subject from which to represent the other elements, effectively generates the intuitive representation that one is *in* a space, moving about, changing position and experiencing the property of intuited extension and "directedness" toward other perspectives. We get the intuitive representation of extended space and time by being embedded *in* the structure of perspectives it allows us.[9]

[9] Although I do not wish to introduce my own views in this (however speculative) historical essay, I actually think for the perspectival system to be experienced as spatially and temporally extended from the inside, the indeterminacy in the other unmeasurable observables should play a role in the extended representation as well but only in smeared out ranges of values, as it were filling in the gaps or transitions between the sharp values accessible through the allowed perspectives on the system while leaving others blurred. Thus I do not agree with Hermann completely that only the MMCOs (maximal sets of mutually commuting operators) are of interest here in laying the foundations for spatio-temporality. As I've related in other writings (Banks 2013; 2014) the notion of an extension seems to carry with it logically the idea

There are thus very strong correspondences between Hermann's ideas on perspectivalism stated above and the philosophy of geometry in this Kantian vein, where space is essentially a perspectival system realized in intuition and grasped intellectually by the understanding. As Helmholtz discovered in his investigations of geometry, the analytic or intellectual content of space can be understood through its transformation group and the invariant, a curved line element in a space of constant curvature, can be deduced from these facts. Sophus Lie later formalized the concept of a continuous transformation or Lie group (expressible in matrix representation for example, where the matrices are both the "objects" in the group and are also the entities multiplied by matrix multiplication as the group product to obtain the other group members). Lie also "took the derivative" of a path through the continuous group and arrived at the infinitesimal generators of the group transformations (also matrices) in the underlying Lie algebra, with its own algebraic matrix product, the commutator, or Lie bracket, which characterizes the extension of the smallest regions of the manifold.

Following Heisenberg's work on matrix mechanics with Born and Jordan, Hermann Weyl (see Woit 2017) was able to apply Lie groups directly to quantum mechanics.[10] As it turns out, every group of transformations on a quantum mechanical system, moving the system along in time, translating it in space, giving it a velocity boost, or rotation, all of the classical space-time symmetries the system exhibits in state space, have as their infinitesimal generator a self-adjoint or Hermitian operator, for the energy, for the momentum, the position, the angular momentum. The conserved quantities of most direct physical interest have this direct relation to space and time symmetry groups. For example, in the case of time representation, the linear time translation group has the simple representation $e^{-iHt/\hbar}$ and the algebraic generator of the transformation is the Hamiltonian (energy) operator $H$ (Woit 2017, sec. 10.2). We exponentiate the generator in the Lie algebra to get the transformations in the Lie group, so if the generator is the energy operator $H$ the matrix exponential is the group operation of advancing the system in time in state space $e^{-iHt/\hbar}$. In fact, every space or time group symmetry of a quantum mechanical system will induce a corresponding Hermitian operator on the state space of the system

of independence or dissociation of parts as well as associations: in an extension, a determinate set of points or quantities should be extended or separated out from each other by associating-dissociating them with another quantity that can change freely and independently of the first. Quantum indeterminacy seems to provide a mechanism for both the linking and the dissociation between non-commuting quantities and their independent *eigenvectors*, which should be looked into as a basis for spatio-temporal representation along the lines Hermann is suggesting. I'll just give two examples that seem suggestive to me. In his fundamental 1925 paper, Heisenberg treats the $x$ coordinate in the energy basis as smeared out and extended into all of the possible transitions between the stationary energy states. In the energy basis, the transitions $\langle n-\tau/x/n\rangle$ are essentially uncertain and are expressed as probabilistic amplitudes "filling up space" between transitions with uncertainty and contributing to the spatial extension of the system even if the $x$'s are not represented with sharp values, in fact because they are not. In the case of spin mentioned above, the uncertainties in $x$ and $y$ spin, by being completely dissociated from the definite $z$ and total spin observables, in a way precess and "fill in empty space" between the allowed perspectives on the other quantities. They don't really precess spatio-temporally or else they would have definite values of $x$ and $y$ during the precession. It is better to think of the values as smeared together into a solid cone. I hope to show in future work, following on Hermann's ideas, that classical spatio-temporal extension is indeed a quantum phenomenon, with foundations there.

[10]As Weyl saw, Born's canonical commutation relation $pq - qp = i\hbar$, is actually a Lie bracket on a Lie algebra (as well as a kind of "derivative of $q$ with respect to $q$"). Exponentiating the generators in the Lie algebra led Weyl to the "Heisenberg group" of upper triangular matrices and to the Schrödinger representation of this group on function space. This group, as Woit points out, is more important for the unique representation it produces of a particle moving freely or in a potential, rather than being considered a true symmetry group (Woit 2017, sec. 13). Of course this group and its representation apply to the whole range of abstract wave functions on phase space, not the individualized perspectives I refer to in this essay as foundations for spatio-temporal representation.

and this operator is the infinitesimal generator of the group action, as well as characterizing the measurable physical quantities on the system. There is thus a direct connection between the choice of operators and the space and time transformations and symmetries of the system that come along with the choice.[11]

The choice of classical language or some intuitable system of perspectives à la Hermann is thus, I believe, revealed to be far more important than it seemed at first. The choice of a Hermitian operator, and its *eigenvectors*, as an "experimental context" also involves one in a choice of a partial space-time group representation with it, which is exactly what Grete Hermann had insisted was really important: the space-time format and system of perspectives *did* encode important "knowledge" of the system, and not merely a need to communicate the measurements in a classical language for convenience's sake or to assuage human intuition.

Although she does not mention these developments to my knowledge, the fundamental results of Weyl surely must be relevant to Hermann's explanation of the emergence of spatio-temporal representations characterized above, and can be seen as a kind of vindication of her position that spatio-temporal representations carry "real knowledge." Hermann says that, in adopting a basis or choosing one of these Hermitian operators as your observable, you enter what is otherwise an abstract perspectiveless system, the general state space, and you occupy a perspective and with it the set of symmetries that are the foundation of the given representation of the system as a spatial or temporal one. Then, the observed system too seems to you to be "extended out" in space and time and suitable for a certain limited and analogical causal sequencing of events, or causal succession of perspectives in a sequence. Nevertheless the overall state space remains the non spatio-temporal foundation for all perspectival relations we might take up in it.

## 7. Conclusion

I think one can now see how a powerful neo-Kantian or perspectivalist research program on spatio-temporal representation could well find its beginnings in Grete Hermann's groundbreaking 1935 work. To this end, let me now summarize the major conclusions of the discussion above:

1. The concept of causality, as modified by Hermann is analogical and regulative rather than predictive and *a priori*, in direct conflict with the views of Schlick, for example. It is applicable only in a domain that is relative to a given context of compatible operators with the same *eigenvectors*, or a choice of MMCOs (maximal sets of mutually commuting operators). Even so, the "causal connection" is not a succession of those contextualized states, one leading to another necessarily, but a connection of states established analogically from already given final to initial stages to understand an experiment that has already happened. Hermann's claims about partial causal reconstruction must be understood with regard to a certain chosen context only, in which relations can be interpreted semi-classically.

2. The reconstruction of causal connections in the above sense can serve for the understanding of transfers of conserved quantities, such as momentum, between two interacting systems or as Heisenberg already indicated the different parts of a system, since these operators always commute. But, if care is taken, even wavefunction collapses that are unpredictable in principle can be understood *as if they were* classicalized measurements of mixed states, even though it is clear that the real states are always pure ones.

[11]Some of the objects and their symmetries are quite strange of course. For example spin ½ particles are symmetrical on *two* complete rotations, others on a half a rotation. The group of translations moreover applies to the wavefunction of systems in state space and not directly to actual objects in space, which emerges from this in the view under consideration here.

3. Hermann offered in 1935 a modernized transcendental argument, in order to account for the possibility of limited spatio-temporal representations of nature with quantum mechanical foundations that are *not* spatio-temporal. These limited spatio-temporal representations that result from the adoption of a certain context are *not* mere appearances, but carry with them real physical knowledge accessible from that perspective, especially if we accept a further connection between Hermann's work and Weyl's, both of which seem to point to the idea that spatio-temporal representation first becomes possible by adopting a limited system of perspectives on a physical system, which, I suggest, would agree in a very deep way with Kant's own views of subject-object representation. However due to the very un-Kantian sounding *Spaltung der Wahrheit*, even for empirical knowledge of nature, these experimental contexts are incompatible and fundamentally separated from each other and it is therefore folly to seek complete spatio-temporal foundations for the complete network of relations described by quantum mechanics.

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