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The present paper is concerned with Helmholtz's theory of measurement. It will be argued that an adequate understanding of this theory depends on how Helmholtz's application of the concepts of perception and coincidence is interpreted. In contrast both to conventionalist and (neo-)Kantian readings of Helmholtz's theory, a more realistic interpretation will be suggested.

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1. Introduction

In his 1870 article "Über den Ursprung und die Bedeutung der geometrischen Axiome," Hermann von Helmholtz established a "principle of congruence" according to which perception, especially spatial perception, has its foundation in the coincidence of point pairs in rigid bodies "with the same fixed point pairs in space." This principle of congruence in turn lay at the very bottom of Helmholtz's theory of measurement: when executing measurements, certain sections of the respective measuring devices are brought into congruence with the entities measured, so that, for example, certain point pairs of a measuring rod coincide with the endpoints of a given spatial extension. Thus the experience of perceptual coincidences is, according to Helmholtz, the fundamental fact on which the theory of measurement is grounded.

As is well known, Helmholtz's—supposedly Kantian—conception was, at least to some extent, appropriated by Moritz Schlick who, in his 1918 *Allgemeine Erkenntnislehre*, stressed the epistemological importance of what he called the "method of coincidences." However, in his 1921 comments on Helmholtz's *Epistemological Writings*, Schlick modified the original Helmholtzian approach by a (Poincaréan-inspired) *conventionalist reading* which turned out to be highly decisive for his philosophical reconstruction of Albert Einstein's general theory of relativity. As will be shown in the course of this paper, Schlick's interpretation gave rise to an extended historiographic controversy with Alberto Coffa on the one side and (among others) Michael Friedman and Thomas Ryckman on the other. While Coffa sided with Schlick in seeing in the "real" Helmholtz an important forerunner of a "conventionalized" form of empiricism, Friedman and Ryckman insisted on the fundamental role of Kantian elements in Helmholtz's theory. On the whole, it will be argued that an adequate understanding of Helmholtz's determination of the relation of perception and coincidence depends on the question whether Helmholtz's account can—without any serious loss—be separated from the Kantian (apriorist) tradition in which Helmholtz himself without any doubt located his entire point of view. I answer this question with a tentative "yes," but this does not imply that Schlick's (and Coffa's) conventionalist reading is the only way to go. A more realistic interpretation is worth considering in some detail.

2. Helmholtz's Theory of Measurement in Outline

Let us begin with a preliminary review of Helmholtz's theory of measurement. There can be no doubt that the most encompassing presentation of this theory is to be found in Helmholtz's seminal "Zählen und Messen" from 1887. As the title already indicates, Helmholtz focuses in this paper on the connection between number and measurement. As Olivier Darrigol (2003, 516) has pointed out, Helmholtz was one of the first physicists who reflected on *measurability* rather than on measurement as such; that is, he raised the question under which conditions is measurement possible at all. More precisely, Helmholtz's perspective was an *epistemological* one. Thus his essay starts with the diagnosis that "[a]lthough numbering and measuring are the foundations of the most fruitful, sure and exact scientific method known to us all, relatively little work has been done on their epistemological foundations" (Helmholtz 1887, 72). Helmholtz takes up exactly these epistemological foundations.

Now the pivotal point of Helmholtz's whole approach is that counting and measuring have their epistemological foundation in certain empirical—perceptual—facts. In his 1868 paper on "the facts that lie at the basis of geometry," Helmholtz had already argued that the axioms of geometry are *not a priori* in a Kantian sense (Helmholtz 1868, 149). In his 1887 essay, he attempts to show the same for the axioms of *arithmetic*. Or in his own words:

> [I]f the empiricist theory—which I besides others advocate—regards the axioms of geometry no longer as propositions unprovable and without need of proof, it must also justify itself regarding the origin of the axioms of arithmetic, which are correspondingly related to the form of intuition of time. (Helmholtz 1887, 72)

So whereas the axioms of geometry pertain to the concept of space, the axioms of arithmetic pertain to the concept of time, which, according to Helmholtz, should be conceived as a form of intuition. Relying on Hermann Grassmann's *Ausdehnungslehre* from 1878, Helmholtz specifies seven such axioms, among them the *transitivity of equality* (i.e., the proposition that if two magnitudes are both alike with a third, they are alike amongst themselves), the *associative law of addition*, "$(a + b) + c = a + (b + c)$", and the *commutative law of addition*, "$a + b = b + a$".[1] Furthermore, Helmholtz is of the opinion that cardinal numbers can be reduced to ordinal numbers, for otherwise it would be impossible to account for the *temporal order* of the underlying perceptual sequences (Helmholtz 1887, 75–76). Thus time, as a form of intuition, provides us with a series of ordinal positions, such as *first*, *second*, *third*, etc., whereby these positions are denominated by "arbitrarily chosen symbols" (1887, 76), such as *one*, *two*, *three*, etc.

This procedural conception of numbers (and their significance) provides the basis for the systematic claim that the axioms of arithmetic are *applicable* to concrete objects. More specifically, according to the view defended by Helmholtz, measurement amounts to the assignment of numerical values to physical magnitudes. Proceeding from what was to him a crucial assumption that arithmetic (or the theory of pure numbers) be considered as "a method constructed upon purely psychological facts" (1887, 75), he first elucidates the basic concepts of his theory of measurement (1887, 88–89). One of these concepts results from the consideration of objects that are alike in a particular respect. Helmholtz calls them *units* and, in a next step, defines a *denominate number* (*benannte Zahl*) as the relation of a pure (cardinal) number and the corresponding unit. Furthermore, by *magnitudes* (*Größen*) Helmholtz means properties of objects standing in the relation of being greater or smaller than other properties of the same kind. The *value* of a magnitude is, according to Helmholtz, expressed by a denominate number, and the procedure by which the respective denominate number becomes determined is, Helmholtz holds, the measurement of the magnitude.

These are the basic elements of Helmholtz's theory of measurement.[2] However, a further point needs to be emphasized in this connection: it is instructive to observe how Helmholtz attempts to combine his (psychologically motivated) arithmetic-based metrological conception with his inquiry into the foundations of *geometry*. In the 1887 essay, under the heading "Distances between two points," he discusses the notion, "congruent coincidence" (1887, 92), which already played an essential role in his former writings on geometry. Recall that, for Helmholtz, magnitudes are those properties of objects which are capable of being distinguished into greater or smaller by comparison to other properties of the same kind. The central metrological question, then, is, "what is the *objective sense* of our expressing relationships between real objects as magnitudes, by using de-

[1] For an extended discussion of the contemporary *mathematical context* of Helmholtz's account, see Darrigol (2003, 520–35).

[2] For a recent (and very insightful) *detailed reconstruction* of this theory, see Biagioli (2016, chap. 4).

nominate numbers; and under what conditions can we do this?" (1887, 75; emphasis added). As he makes plain in his discussion of the notion "congruent coincidence," the simplest geometrical structure for which a magnitude is specifiable is the "distance between a pair of points" (1887, 92). The question of the objectivity of measurement thus becomes the problem of giving specific values to distances.

According to Helmholtz, what is needed in the first place to cope with this problem is a *method of comparison*, or what he alternatively calls the "method of likening distance for two pairs of points" (1887, 92). The application of this method allows the determination of the likeness of distances. It thereby becomes possible to account for the fact that two point pairs that are congruent with a third point pair are also congruent among themselves. However, in order to determine that distances are not only like or unlike but stand in the relation of greater or smaller to each other, the concept of *length* is needed. Helmholtz writes:

> We make the *length* of two bounded straight lines alike if the distance between the end points is alike, thus when the latter can be placed in congruence, whereby the lines too coincide congruently. To this extent the concept of length gives something more than does the concept of distance. (Helmholtz 1887, 93)

Thus, for example, if we have two point pairs a, b and a, c, which—when considered as parallel straight lines—coincide at a and which at the same time *differ* in distance, then either b falls upon the line ac or c upon the line ab. Accordingly, we are in a position to determine which of the two lines is smaller (shorter) or greater (longer) than the other one. In order to do so, it is not enough to merely compare the distances. Rather, it must be possible to calculate the greater distance as the sum of the smaller and their respective difference. In short, a *method of addition* must be at hand as well.[3]

As Moritz Schlick pointed out in his comments on Helmholtz's *Epistemological Writings*, the conception of "congruent coincidence" had tremendous impact on the development of modern physics. Thus Schlick declares:

> 'Congruence' is established by observing the coincidence of material points. All physical measurements can be reduced to this same principle, since any reading of any of our instruments is brought about with the help of coincidences of observable parts with points on a scale, etc. Helmholtz' proposition can therefore be extended to the truth that no occurrences whatsoever can be ascertained physically other than meetings of points, and from this Einstein has logically drawn the conclusion that all physical laws should contain basically only statements about such coincidences. (Schlick 1921, 33–34)

Given this assessment, it appears warranted to have a closer look at the *epistemological presuppositions* on which the coincidence concept is based.

3. From Perception to Coincidence

To begin with, Helmholtz's account of coincidence forms an integral part of his thinking about geometry and space in general. As said before, he employed the coincidence concept already in earlier writings. Thus in his 1870 "Über den Ursprung und die Bedeutung der geometrischen Axiome," Helmholtz offered the following definition of a *rigid body*:

> [T]here must exist, between the coordinates of any two points belonging to a fixed body, an equation which expresses an unchanged spatial relationship between the two points . . . for any motion of the body, and one which is the same for congruent point pairs, while point pairs are congruent, if they can successively coincide with the same point pair fixed in space. (Helmholtz 1870, 15)

[3] In the introduction to his lectures on theoretical physics from 1893, Helmholtz clarifies this point as follows: "The method of comparison does not provide us with an answer to the question: which of the unequal magnitudes is the greatest? . . . Only the method of addition determines the concepts of smaller and greater" (Helmholtz 1903, 36). See, in this connection, also Biagioli (2016, 96–98).

Given this definition, it can be stated that measuring amounts to repeatedly applying an identical measuring *rod*. For a measuring rod can be characterized as a body on which at least two points are marked, the separation of which remains constant. These points in turn are brought into coincidence with corresponding points of the object to be measured.

Going one step further, Helmholtz claims that all geometrical measurements rest ultimately on a "principle of congruence" (1870, 18). This principle of congruence implies the *free mobility* of rigid bodies and is as such tightly connected with the assumption that "all measuring instruments which we take to be fixed, actually are bodies of unchanging form" (1870, 19). Thus by transporting a measuring rod from one place to another its non-deformability is presupposed. As concerns measuring instruments in general, this means that they "undergo no kinds of distortion other than those which we know, such as those of temperature change, or the small extensions which ensue from the different effect of gravity in a changed location" (1870, 19). What in principle remains unaltered are the coincidences of pairs of points.

Now the presupposition of the free mobility of rigid bodies has, according to Helmholtz, its epistemological foundation in certain facts of perception. In his own words:

> When we measure, we are only doing, with the best and most reliable auxiliary means known to us, the same thing as what we otherwise ordinarily ascertain through observation by visual estimation and touch, or through pacing something off. In the latter case, our own body with its organs is the measuring instrument which we carry around in space. (Helmholtz 1870, 19)

So the crux of the matter is that the assumption of the free mobility of rigid bodies in contexts of measurement has its origin in our own bodily movements and the respective perceptual experiences. That is, the assumption of free mobility is conceived of as being derivable from our experience of moving in arbitrary directions: up to, away from and around the objects "in" space. On the whole, any apriorist (Kantian) understanding of space and spatial perception seems to be ruled out by such a maneuver. Consequently, the question to be raised next is whether Helmholtz's discussion of the connection between perception and coincidence entails a thoroughly *empiricist* understanding of the epistemology of measurement.

4. Was Helmholtz an Empiricist?

It is interesting to see that the first reactions to Helmholtz's theory of measurement, as he had presented it in his 1887 essay, came from the side of contemporary mathematicians. Most of these reactions were negative. Thus, for example, Gottlob Frege explicitly rejected Helmholtz's attempt at a psychological grounding of arithmetic, arguing for a purely logical foundation instead. Richard Dedekind went in the very same direction, claiming that "arithmetic is part of logic" and that the concept of number is "entirely independent of the notions or intuitions of space and time" (Dedekind 1888/1901, 31). Perhaps the most vehement reaction came from Georg Cantor, according to whom . . .

> Helmholtz and Kronecker plead the extreme empirico-psychological standpoint with a vigor which one would scarcely deem credible, if it did not here appear twice embodied in flesh and blood . . . With these investigators, numbers shall be *signs* first and foremost, but not indeed signs for concepts that relate to sets, but *symbols for things counted by the subjective number process*. (Cantor 1887, 382; quoted from Darrigol 2003, 558)

The mathematician Leopold Kronecker had indeed defended a view of arithmetic that was rather close to the conception proposed by Helmholtz. Kronecker's essay "Über den Zahlbegriff" (1887) appeared in the same volume as Helmholtz's "Zählen und Messen," and it was Cantor's conviction that both Helmholtz and Kronecker represented a brute "academic-positivistic skepticism" (Cantor 1887, 383; quoted from Darrigol 2003, 558) that prevented an adequate understanding of the status of numbers and of arithmetic in general.

In view of these objections it appears plausible that Helmholtz, in fact, was an empiricist (or even positivist). Further evidence for this diagnosis is provided, not only by Helmholtz's own characterization of his theory as "empiricist" (see the quotation at the beginning of Section 2), but also by his celebrated thought experiment with the mirrored sphere, which pertains directly to the realm of geometry (see Helmholtz 1870, 20). As is well known, Helmholtz, in this thought experiment, lets us think of the image of the world in a convex mirror. The inhabitants of this world would, from our perspective, perceive straight lines as curved and rigid motions as distorted. The reflected bodies and measuring rods in the mirror image would appear as expanding and contracting along the curvature of the mirror. However, the respective measuring *outcomes* would, from the perspective of the inhabitants of the mirror world, agree with ours. They would think of their *own* space as Euclidean, and our space, on the other hand, would appear to them as that of a convex mirror. That is, there seems, according to Helmholtz's thought experiment, to be no way to determine which of the two perspectives—our own or the one of the inhabitants of the mirror world—is the "correct" one. Since all bodies (including the supposedly rigid measuring rods) in the mirror world are equally distorted, the respective *congruent coincidences* would remain unchanged. Yet, as soon as we reflect on the underlying *laws of mechanics*, a choice between the two perspectives can obviously be made. It is for this reason that Helmholtz comments on the situation in the thought experiment as follows:

> [I]f the men of the two worlds could converse together, then neither would be able to convince the other that he had the true and the other the distorted situation. I cannot even recognise that such a question has at all any sense, as long as we introduce no mechanical considerations. (Helmholtz 1870, 20)

In short, the laws of mechanics play, for Helmholtz, the decisive role for empirically determining the "true" geometry. Accordingly, geometry and mechanics are to be seen as inseparable from each other, and exactly this seems to entail an empiricist conception.[4]

However, this sort of interpretation is somewhat controversial (if not to say perplexing). As Robert DiSalle very aptly remarks, "precisely what distinguished Helmholtz's empiricism from its more naïve predecessors—his appreciation of the physical meaning of our concepts of congruence and straightness—invited its reinterpretation as conventionalism" (DiSalle 2006a, 125). What exactly is at stake? Is there any *crucial difference* between an empiricist and a conventionalist conception of geometry and measurement? And if so, what would be the *systematic consequences* of such a contraposition of views?

First of all, it is important to realize that the putative empirical character of the principles of geometry appeared to rest on a problematic premise. More precisely, there seemed to be an element in Helmholtz's conception that lay beyond empirical control. Recall that, for Helmholtz, it was a matter of fact that rigid bodies are capable of being moved around in space without changing their shape or dimension. If a body failed to satisfy this condition, then, Helmholtz apparently presupposed, this could be empirically determined by comparing that body with *another* body that indeed fulfilled the condition of being rigid. How-

[4] Robert DiSalle attributes to Helmholtz a "relatively sophisticated empiricism" and specifies it as follows: "Before Helmholtz, the empiricist alternative to Kant was a naïve one, based on the idea that geometrical postulates are arrived at by straightforward induction . . . . Helmholtz saw a subtler connection between geometry and experience, not directly of the postulates of geometry but of underlying physical facts" (DiSalle 2006a, 124). As we will see in a moment, this assessment has to be qualified in light of the subsequent reception of Helmholtz's alleged "relativization of geometry."

ever, an infinite regress seemed to be inevitable under that very presupposition: Comparing one body with another obviously necessitated to avail a *further* body with the property of being rigid, this further body standing in need of being compared with still another "actually" rigid body, and so on. So, how can this regress be stopped? As is well known, it was Henri Poincaré who gave the answer that Helmholtz's allegedly empirical geometrical principles were, in fact, definitions or, better, *stipulations*. Accordingly, conventionalism—rather than empiricism—seemed to follow (see, in this connection, the respective discussion in DiSalle 2006b, 79–97).

Yet an interpretation of geometry and measurement along purely conventionalist lines runs the risk of losing touch with what can be empirically determined. Why, then, not *combine* empiricism with conventionalism? Exactly this was the strategy adopted by Schlick. In one of his comments to Helmholtz's 1870 geometrical paper he pointed out:

> What kind of sense is there in saying that a body is actually rigid? According to Helmholtz's definition of a fixed body . . . , this would presuppose that one could speak of the distance between two points 'of space' without regard to bodies; but it is beyond doubt that without such bodies one cannot ascertain and measure the distance in any way. . . . If the content of the concept 'actually' is to be such that it can be empirically tested and ascertained, then there remains only the expedient . . . to declare those bodies to be rigid which, when used as measuring rods, lead to the simplest physics. . . . Thus, what has to count as 'actually' rigid is then not determined by a logical necessity or intuition, but by a convention, a definition. (Schlick 1921, 34)

As can be easily inferred from this comment, Schlick embraced the Poincaréan conventionalist "expedient."

However, at the same time, Schlick promoted an empiricist—and likewise "realistic"—reading of the relationship between perception and coincidence in Helmholtz's sense. Thus in his 1918 *Allgemeine Erkenntnislehre* he established what he called the "method of coincidences" (Schlick 1918/1974, 272). It was this method which already figured prominently in his 1917 *Raum und Zeit in der gegenwärtigen Physik*. There, he described it as follows:

> In order to fix a point in space, we must in some way or other, directly or indirectly, point to it: we must make the point of a pair of compasses, or a finger, or the intersection of cross-wires, coincide with it (i.e. bring about a time-space coincidence of two elements which are usually apart). Now these coincidences always occur consistently for all the intuitional spaces of the various senses and for various individuals. It is just on account of this that a 'point' is defined which is objective, i.e. independent of individual experiences and valid for all. . . . Upon close investigation, we find that we arrive at the construction of physical space and time by just this method of coincidences and by no other process. (Schlick 1917/1979, 262–63)

The idea is that by applying the method of coincidences the psychological description of subjective spatial perception and the physical description of objective spatial structures are brought together. According to Schlick, we construct the objective physical ordering on the basis of *singularities* in our various perceptual spaces. These singularities are nothing else but concrete sections of the whole perceptual situation. To use one of Schlick's examples: when I see the tip of my pencil touch my finger, I have two perceptual singularities at the same time, one in my visual field and one in my tactile field. Both fields have a completely different intuitive (qualitative) spatiality: my visual perception has no intuitive relation to my tactile perception. By the method of coincidences, however, I bring both fields into relation. I construct a single *nonintuitive* ordering which contains both the pencil and my finger. Completely abstracting from the qualitative peculiarities of my various perceptual spaces I thereby generate a single point in objective—physical—space.

There has indeed already been extensive work on Schlick's method of coincidences and its relation to Helmholtz's theory of perception and measurement (see, for example, Friedman 1997, 25–28; Pulte 2006, 198–99; Neuber 2012a, 174–77; Oberdan 2015, 41–42). For the present concerns, it may suffice to note that Schlick saw a close connection between the method of coincidences and Einstein's general theory of relativity. Hence, he concluded that by taking seriously his own analysis of space and time one will encounter "just that significance of space and time which Einstein has recognized to be essential and unique for physics, where he has established it in its full right" (Schlick 1917/1979, 262–63).[5] And, it should be added, it is exactly here that Schlick locates the heritage of Helmholtz's alleged empiricism.

However, as we have seen before, Schlick does *not* agree with Helmholtz's definition of a rigid body. According to Schlick, that definition—which fundamentally contains the assumption of congruence—is blatantly circular. Or, in his own words:

> This definition reduces congruence (the equality of two tracts) to the coincidence of point pairs in rigid bodies 'with the same fixed point pairs in space' and thus presupposes that 'points in space' can be distinguished and held fixed. This presupposition was explicitly made by Helmholtz . . . , but for this he had to presuppose in turn the existence of 'certain spatial structures which are regarded as unchangeable and rigid'. Unalterability and rigidity . . . cannot for its own part again be specified with the help of that definition of congruence, for one would otherwise clearly go round in a circle. For this reason the definition seems not to be logically satisfactory. (Schlick 1921, 31)

[5]Einstein's own account of the importance of coincidence can be found in his 1916 seminal paper "Die Grundlage der allgemeinen Relativitätstheorie." There he writes: "All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meeting of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time" (Einstein 1916/1923, 117). For an elaboration of this assertion, see Howard (1999).

One escapes the circle, Schlick continues, "only by stipulating by convention that certain bodies are to be regarded as rigid, and one chooses these bodies such that the choice leads to the simplest system of describing nature" (1921, 31). Here again, it is conventions that provide the alternative to Helmholtz's original conception.

To be sure, empiricism plays an essential role for Schlick's account of space and spatial measurement. But it must be seen that the empiricist component is *significantly qualified* by the integration of conventions in the Poincaréan sense. In order to determine the rigidity of bodies, we must, according to Schlick, rely on conventions. But then rigidity—as well as congruence—is dependent on what we are *doing* (and not on what there *is*). In short, if one follows the interpretation provided by Schlick, then Helmholtzian and Poincaréan elements must be combined in such a way, that a sophisticated—"conventionalized"—form of empiricism results.

5. The (neo-)Kantian Predicament

It was Alberto Coffa who initiated the more recent discussion about Schlick's appropriation of Helmholtz's theory. According to Coffa, "Schlick was the first to attempt a systematic formulation of the picture of knowledge implicit in Helmholtz's writings" (Coffa 1991, 171–72). In Coffa's view, Schlick's conventionalized empiricism in fact contributed to an adequate *reinterpretation* of Helmholtz's original position. However, that diagnosis did not remain uncontested. Michael Friedman, for example, thinks that Coffa and other commentators "have been far too quick . . . simply to take Schlick's attempt at appropriation [of Helmholtz's ideas] at face value" (Friedman 1997, 44 n 14). And, to quote another example, Thomas Ryckman is of the opinion that one must sharply distinguish between "Helmholtz and Schlick's Helmholtz" (Ryckman 2005, 67). So, one may ask, what is the alluded un-Schlickean alternative?

Following Friedman's view it is tempting to say that the alternative is through and through Kantian. More carefully speaking, it is the apparent *transcendental element* in Helmholtz's theory which in Friedman's reading plays the essential role. Consequently, the free mobility of rigid bodies required for congruence should, on this reading, be understood as the basic fact or, more aptly, *the condition of the possibility* of spatial measurement. And indeed: according to Helmholtz, one must distinguish between space, on the one hand, and spatial measurement, on the other. Whereas space forms part of theoretical knowledge, spatial measurement forms part of the concrete practice of empirical science. The axioms of geometry, Helmholtz maintains, are embodied in the system of spatial measurement. Therefore, he thinks that they are *not* synthetic *a priori*. On the other hand, space itself—and with it the free mobility of rigid bodies—is for Helmholtz the *precondition* of all measurement and, consequently, of the axioms of geometry as well. Or as Helmholtz himself famously put it: "Space can be transcendental without the axioms being so" (1977, 149).

So the assumption of the free mobility of rigid bodies forms part of our concept of space as a precondition of all measurement. Space—via its overarching feature of free mobility—is for Helmholtz "a *given form* of intuition, possessed *prior to all experience*" (Helmholtz 1870, 124), and therefore transcendental. Concerning the axioms of geometry, Helmholtz recognized the possibility of different (both Euclidean and non-Euclidean) systems of such axioms, thereby being an empiricist in so far as he thought that the decision about which system physically "fits" is a matter of empirical discovery.[6] Thus, on the whole, it can be said that "Helmholtz argued against the Kantian philosophy of geometry while retaining an inherently Kantian theory of space" (Ryckman 2005, 73–74).[7]

Now the problem with this line of interpretation is that it immediately prompts the question whether Helmholtz did contradict himself. To be sure, on Ryckman's reading, Helmholtz's definition of rigid bodies definitely does not amount to a stipulation, but must be considered as a constraint imposed by the *a priori* form of spatiality itself. However, the problem arises as soon as this a priori form is equated with the concept of intuition. As Francesca Biagioli has recently made plain, "Helmholtz argued against the Kantian philosophy of geometry because he did not admit a meaningful use of 'pure intuition' as distinguished from psychological intuition" (Biagioli 2016, 16). Likewise Joel Michell, in his essay on the origins of the "representational" theory of measurement, already pointed out that "Helmholtz's reinterpretation of Kant . . . attempts to locate the Kantian forms of intuition and categories within the nervous system. It is the nervous system which constructs experience" (Michell 1993, 190). So we are obviously faced with some sort of *(neo-)Kantian predicament*: space and with it the free mobility of rigid bodies is, according to Helmholtz, to be seen as an *a priori* form of intuition; on the other hand, this form of intuition is analyzed in terms of empirio-perceptual psychology alone. But then it cannot be *a priori* in the Kantian sense anymore. In short, the (neo-Kantian) predicament entails the overtly un-Kantian strategy of *naturalizing* both the transcendental and the *a priori*.[8]

[6]See, for example (and especially), Helmholtz (1868, 25): "[S]uch a system of propositions is given an actual content, which can be confirmed or refuted by experience, but which for just that reason can also be obtained by experience."

[7]In the same vein, Helmut Pulte speaks of Helmholtz's idea of an "a priori-rigidity" (Pulte 2006, 199). And Liesbet De Kock points out that "Helmholtz's criticism of Kant's analysis of (pure) intuition did not amount to a categorical rejection of the *a priori* altogether" (De Kock 2016, 31).

[8]As Liesbet De Kock (2016, 27 n 40) makes clear, this sort of "physiological neo-Kantianism" provoked the critical reaction by Hermann Cohen (and other members of the Marburg School of neo-Kantianism). The essential tension lurking in the background is described by de Kock as follows: "Helmholtz's nativist interpretation of the *a priori* in terms of physiological organization transforms it from a formal condition to an empirically verifiable fact. Hence, the necessity of the *a priori* as a constitutive condition of experience is dissolved into the contingency of the empirical subject, which is why Kant . . . explicitly set this kind of 'physiology of understanding' apart from his critical project" (2016, 27 n 40). However, it should be seen that while Helmholtz was writing,

Admittedly, Ryckman is entirely aware of the problems associated with this strategy. In fact, his own view of the matter is *not* committed to the naturalistic paradigm. But it is highly questionable whether his (and Friedman's) Kantian reading of Helmholtz's theory indeed reveals the "real" Helmholtz. In my view, the *spirit* of Helmholtz's theory was significantly less Kantian than suggested by Ryckman (or by Friedman). Yet this does not necessarily mean that the only way to go is following Schlick (and Coffa) by taking the Poincaréan conventionalist "expedient." A more *realistic* reinterpretation is available, and I shall in what follows indicate what this reinterpretation amounts to.

6. Kaila, Helmholtz, and Metrological Structural Realism

Historically, my central point of reference is a monograph by Eino Kaila who, early in his career, defended a certain form of "critical" realism and, later on, attempted to combine this point of view with the fundamental tenets of logical empiricism.[9] The monograph in question is titled *On the Concept of Reality in Physical Science* which appeared originally in German in 1941. Chapter V of this small book is devoted to the theory of measurement which, according to Kaila, is "the nucleus of the general theory of physical science" (Kaila 1941, 157). As he points out, there are two opposing views within the theory of measurement: a "conventionalist" ("positivist", "formalistic"), on the one hand, and an "empiricist" ("realist") on the other" (1941, 157). The first view is, according to Kaila, represented by the writings of Poincaré and especially Ernst Mach, the second by the writings of Helmholtz and Norman Robert Campbell.

Now the crucial point in Kaila's own conception is that it is based on the notion of *invariance*. "In knowledge," he contends, "we are always concerned with 'invariances' alone" (1941, 131). Accordingly:

> The aim of exact science is to discover the higher invariances of the domain of experience in question. We shall show that "physico-scientific reality" (as to its content) consists in nothing other than the system of the higher invariances of the everyday physical world and thus (in the last analysis) "immediate experience". (Kaila 1941, 152)

The connection with the theory of measurement becomes evident, when Kaila discusses the two opposing views. The conventionalist view, he points out, implies that measurement consists in the mere imposition of numerals onto phenomena, whereas the empiricist (realist) view initially contends that the correlation between phenomena and numerals must be *discovered* by executing measurements. Kaila writes:

> According to the conventionalist view, . . . one starts in every measurement from arbitrary stipulations of some kind and finds, if one is lucky, by assigning the measured numerical values to the phenomena according to the conventions adopted, some kind of lawful dependencies of the measured values; according to the empiricist view, one starts (in all theoretically important cases) conversely from the existence of some kind of lawful relationship which first is only qualitatively known and searches then for a principle of correlation of the measured numerical values by means of which this lawful relationship can be quantitatively expressed. (Kaila 1941, 158)

it was an entirely open question how best to interpret Kant on the question of naturalism. E.g., neo-Kantians of the "first generation" such as Friedrich Albert Lange (1866) thought a psychological/physiological description of certain structures of the nervous system was precisely how one could provide *scientific confirmation* for Kant's theory of the *a priori*. Exactly this, though, was contested by Cohen and other neo-Kantians of the "second generation" such as Ernst Cassirer who, explicitly referring to Lange's "genetic" account, spoke of "the deficiencies of the empirico-physiological interpretation of Kantian Apriorism" (Cassirer 1929, 215). More comprehensive discussions of the attempt to naturalize the Kantian *a priori* are to be found in Allison (1995) and in Patton (2009). For an extended discussion of Cohen's reception of Helmholtz's interpretation of Kant, see Biagioli (2016, 71–77).

[9] For an instructive overview of Kaila's philosophical development, see von Wright (1979). See further Niiniluoto (1992).

The conventionalist consequently aims at a convention-based most economical description of measurable data, while the empiricist is looking for invariant and, at the same time, quantitatively expressible relationships among such data. The decisive contrast, then, is to be found between economy, on the one hand, and invariance, on the other.[10]

On the whole, Kaila rejects the conventionalist standpoint since, as he maintains, measurement is more than a mere arithmetical method for the economical designation of properties. Hence he repeatedly declares the principle of economy as being merely an aspect (or consequence) of the more universal and pervasive principle of invariance (Kaila 1941, 154, 191–92).[11]

This is not the place to go into the details of Kaila's metrological conception.[12] However, it is nonetheless important to elaborate on what exactly Kaila means by "empiricism" (as opposed to "conventionalism" and equated with "realism"), and how Helmholtz fits into the resulting programmatic picture. The first thing to realize in this connection is that Kaila heavily relies on Helmholtz's 1887 paper, when he analyzes the notion of *magnitude*. A magnitude, according to Kaila, is a denominate number, usually a fraction, associated with a certain physical property (Kaila 1941, 185–86). Furthermore, Kaila sharply distinguishes between "topological" and "metrical" determinations. Whereas topological determinations merely determine at what point two properties are attributed equal or different numerical values, metrical determinations determine the point at which the *difference* between two properties is equal to or different from two other (tokens of the same type of) properties. The fact that the temperature of boiling water is higher than that of freezing water, for example, is a topological determination, whereas the fact that the rise of temperature by, say, 1° at different places on a scale means the same temperature increase is a metrical determination. Thus the Helmholtzian *principle of congruence* plays an essential role in the latter case: by way of metrical determinations it becomes possible to specify that "this here is to be equal to that there." Consequently, the subdivision of the respective scale is all-decisive. It allows for the further specification of "how the different positive and negative fractions are to be assigned to the different points on the scale" (1941, 186).

Given these Helmholtzian-inspired considerations, it becomes possible to make the point that the sort of empiricism de-

[10]In current philosophy of science, empiricism is often *opposed* to realism. In Kaila's case, however, both currents are reconcilable with each other: while empiricism enters the picture by taking seriously the epistemology of measurement, realism is needed for accounting for the ontological status of concrete measurement *outcomes*. More precisely, it is the prominent role Kaila ascribes to what he programmatically calls the "principle" of invariance (Kaila 1941, chap. II) by which his realism is driven. In the 1936 monograph *Über das System der Wirklichkeitsbegriffe*, he explicitly declares that "[t]he different levels of reality . . . correspond to different degrees of invariance" (Kaila 1936, 102). In the 1941 contribution, he coins the related slogan "*The real is what is in some respect (relatively) invariant*" (1941, 185). This is what he later called the "relativization of reality" (1960, 271). Hence, the greater the invariance of a given structure, the greater its reality; furthermore, whatever possesses greater reality is, according to Kaila, of greater generality, and thus more lawlike. Accordingly, by executing measurements quantitative systems of numbers are correlated with—and at the same used to *represent*—qualitative lawful relationships. The contrast to the conventionalist point of view, then, is that these qualitative lawful relationships are supposed to exist independently of the measurement situation itself. From the perspective of, say, Mach this would be nothing but a superfluous metaphysical assumption. In short, both points of view are opposed as to the question of "ontological commitment."

[11]To be sure, for Mach, certain relative invariances in the phenomena are precisely what afford cognitive economy. However, the contrast is a substantial one: a realist (empiricist) like Kaila thinks science aims to discover any and all invariances that exist among the phenomena; whereas a conventionalist (positivist) like Mach thinks science aims to discover only those invariances that afford us some cognitive economy. It is for this reason that Kaila points out that "science is not a minimum problem but a *maximum problem*. It is not a 'parsimonious economy' but a bold adventure, not so much an unperturbed enjoyment of a 'stable world-picture' as a fight against perceptual shocks to which the theoretical mind exposes itself by its generalizations" (Kaila 1941, 154).

[12]For an extended discussion of Kaila's theory of measurement, see Neuber (2012b).

fended by Kaila is more realistic (and hence less conventionalist) than the one defended by Schlick. Recall that the conventionalist "expedient" was the point where Schlick *divorced* himself from Helmholtz's theory of measurement. Kaila, on the other hand, is eager to *shield* this theory from "conventionalization." The essential step in that direction is marked by his distinguishing "essential" from "unessential" conventions (Kaila 1941, 196). As already noted, Kaila rejects the conventionalist account of measurement. This, however, does not mean that he denies that conventions sometimes play a significant role in science—the distinction, therefore, between "essential" and "unessential": for physics, whether distances are measured in miles or in kilometers is immaterial. Opting for one or the other of these two systems is, according to Kaila, a mere incidental convention. Miles can be replaced by kilometers and nothing changes with respect to the relational invariance of distances. Accordingly, the respective conventions are equivalent "from the standpoint of the principle of invariance" (1941, 187). By contrast, there is a substantial difference as to whether or not spatial relations are represented in Euclidean or in non-Euclidean coordinates. What in one coordinate system is invariant may turn out as variant in the other coordinate system. Hence both systems are based on essential conventions of measurement. They are, in other words, different from the standpoint of the principle of invariance.

That being said, Kaila's realistically motivated metrological conception may be summed up as follows: its central objection to the conventionalist is that the latter fails to distinguish both between metrical and topological determinations and between essential and unessential conventions. The conventionalist, according to Kaila, thus treats metrical determinations as if they were topological determinations, and consequently fails to acknowledge the pivotal status of the congruence concept with reference to the actual practice of measurement.[13] Moreover—and intimately related to this—the conventionalist typically conflates essential and unessential conventions, or else allows for entirely arbitrary stipulations, that is, in Kaila's understanding, unessential conventions. Kaila's response to this, in his estimation, erroneous theory is best summed up by his claiming:

> [O]nly such a property can be measured with respect to which there exist lawful relationships of a definite kind, where then the assignment of numerical values is done in such a way that these lawful relationships find expression in the relations of the numerical values. This principle uniquely prescribes all essential measurement conventions. (Kaila 1941, 192)

Here it must be noted firstly that invariant (lawful) systems of relations, that is, *structures*, are accessed by executing measurements insofar as they are made quantitatively expressible by way of their being assigned numerical values. Conversely, it is such structures that effectively confine the range of measurable properties in a lawful and, to this extent, ontologically binding way. Consequently, measurement is dependent on laws of nature and not, as the conventionalist would have it, on mere unessential conventions. Secondly, Kaila would not go so far as to assert that what he designates essential conventions possess the status of Kantian synthetic judgments *a priori*. Following Kant, then, they would figure as conditions of the possibility of experience. Yet, according to Kaila, the reverse is the case: essential conventions are determined empirically by correlating qualitative lawful relations with relations of numerical values. It is this very correlation which "uniquely prescribes all essential measurement conventions" and which, in the last analysis, necessitates the quantitative interpretation of theoretically stipulated systems of descriptions. Hence, the assumption of synthetic judgments *a priori* finds no place in Kaila's metrological conception.[14]

[13] It is in this context that Kaila writes: "Indeed one can say that the *metric is a differential topology*. A metric is given when not only the initial elements but also the differences of these elements, the differences of the differences, etc., can be ordered. Then, but only then one has also attained the full concept of *congruence*" (1941, 196).

[14] Interestingly enough, for Kaila it was Helmholtz who "was perhaps the

All of this leads to a programmatic outlook that might be labeled *metrological structural realism*. This point of view focuses essentially on the relationship between measurement, on the one hand, and invariant systems of relations, that is, structures, on the other. As for the question of how *perception and coincidence* connect in this account, the following answer can now be given. Perception, as Kaila would have it, is the essential starting point for quantitatively establishing invariant structures. It is these invariant structures that determine all measurement procedures. Yet in order to grasp the full *empirical content* of concrete measurement outcomes, a method for fixing the measured quantities is needed. And this is where the concept of coincidence comes in. For it is congruent coincidences by which we proceed from the initial perceptual situation to a *physically objective ordering*. Since congruent coincidences themselves are, according to Kaila, in the first place metrical relations and these metrical relations range over both the immediate perceptual and the non-perceptual realm, the resulting ontological framework would be a structural realist one.

Up to this point, Schlick—who, as it were, invented the "method" of coincidences—would agree with Kaila. However, the difference becomes pronounced as the concept of rigid body is taken into account. According to Schlick, rigidity is based on what Kaila would call an unessential convention. Kaila himself, on the other hand, sees—like Helmholtz—a more intimate relation between rigidity and (other) *physical facts*. In his view, "physico-scientific space . . . is obtained . . . through exact metrical determinations and . . . , therefore, by definition has a definite metric" (Kaila 1941, 212). Consequently, Hans Reichenbach's (1928/1958, 16) speculative talk of "universal forces" is explicitly rejected by Kaila. "From our point of view," he writes . . .

> . . . the results of measurements by means of rigid bodies determine the geometry of physico-scientific space just as uniquely and precisely as physical measurements characterize any other structure. Should a physical circle with a physical diameter be moved about in space and should it turn out in the process that the diameter is sometimes too short, sometimes too long, then this is an unequivocal indication that the Gaussian curvature of physico-scientific space is variable—provided of course that no physical forces are responsible for the distortion, i.e., that this distortion is exhibited uniformly in all materials. Since the congruence relations used in spatial measurement would not be disturbed by a distortion of this kind, we cannot introduce any unmeasurable 'universal forces'. (Kaila 1941, 212)

Helmholtz would surely have agreed: the scope of the conventional is significantly restricted by Kaila's principle of invariance, and the problem of rigid bodies finds its solution within the framework of a *definite geometry*.[15]

Coming eventually back to the essential point of Kaila's whole account, the question should be readdressed of how exactly, on Kaila's view, measurement depends on lawful systems of relations. What Joel Michell, in an influential article from 1993, called the "representational theory" of measurement might be quite helpful here. According to this theory:

> The objects measured, their properties and the relationships between them are described as *qualitative*, to distinguish them from numbers and numerical relationships, which are described as *quantitative* . . . . Such qualitative structures, however, may be similar (i.e. isomorphic or homomorphic) to quantitative (numerical) structures. It is in virtue of this structural similarity that numerical systems may be used to *represent* qualitative empirical systems. (Michell 1993, 189)

Exactly the same is implied by Kaila's point of view: when arguing for what he calls "empiricism," the decisive aspect in

first to realize that the Kantian *a priori* philosophy . . . is devoid of content" (1941, 231).

[15]For reasons of space the relation between Helmholtz and the program of a definite geometry cannot be discussed here. But see Carrier (1994).

his argumentation is that a principle of *correlation* must be at work. Accordingly, numerical systems are employed in order to quantitatively express ("pre-metrologically" existing) qualitative structures. Numerical systems, in other words, fulfill a representational function. However, the *effectiveness* of this representational function is by no means a matter of chance. It rather presupposes qualitative structures of a lawful kind. For otherwise it would remain entirely unclear why measurements are repeatable and, when successful, entailed by constant outcomes.

That being said, the contrast to the conventionalist point of view can be made plain again. For example, Mach in his *Principien der Wärmelehre* (1896) claimed that temperature is "nothing but a *characterization*, *designation*, of the thermal state by a *number*. This temperature number has solely the property of an *inventory number* by virtue of which one can recognize the same thermal state" (Mach 1896/1986, 196). Kaila opposes this claim by arguing that by ascribing certain numerals to certain properties these properties are by no means *measured*. Drawing on an analogy introduced by Campbell (1928, 1), Kaila points out that "[w]hen telephones are assigned certain numbers they are not measured, any more than a street is measured by the fact that the houses are assigned numbers. The difference between such assignments and measurements is that the former are completely arbitrary" (Kaila 1941, 187). In order to restrict this kind of arbitrariness, the properties in question must, according to Kaila, "satisfy lawful conditions of a specific kind" (1941, 187). Only if this criterion is fulfilled, Kaila goes on, is it then appropriate to speak of the "true measure of temperature." Accordingly, he is convinced that "the development of the theory of temperature was in fact based on the presupposition preceding all measurement that a *certain thermal quality is constant*" and that "the development of the concepts of thermometric measures becomes intelligible only in view of this principle of invariance" (1941, 159). In short, invariance and lawfulness go hand in hand in Kaila's metrological conception.[16]

7. Conclusion

In this paper, I have argued that the prevailing conventionalist and neo-Kantian reinterpretations of Helmholtz's theory of measurement can be challenged by a more realistic reading. On this more realistic reading, the relation of perception and coincidence is reconstructed in a setting of invariance and structure. It is only within this setting that the empirical content of concrete measurement outcomes can be fully accounted for. Moreover, it has been shown that Kaila's appropriation and elaboration of Helmholtz's theory of measurement allows one to see in Helmholtz an important forerunner of what I discussed under the label "metrological structural realism." The *spirit* of Helmholtz's theory is significantly closer to this particular point of view than to both neo-Kantianism and conventionalism. To be sure, there are indeed some neo-Kantian elements in Helmholtz's theory. But these neo-Kantian elements are, in the last analysis, negligible. A structural realist interpretation appears as the more natural one.

Admittedly, the further development of the suggested metrological structural realism is a challenge of its own. But I dare say it is worth the effort.[17]

[16]In a very similar vein, Eugene Wigner, in his oft-quoted *Symmetries and Reflections*, declares: "To be touchstones for the laws of nature is probably the most important function of invariance principles" (Wigner 1967, 47).

[17]For the outlines of a more diachronic (systematic) exposition of what is implied by "metrological structural realism," see Neuber (2017).

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References

Allison, Henry E., 1995. "On Naturalizing Kant's Transcendental Psychology." *Dialectica* 49: 335–51.

Biagioli, Francesca, 2016. *Space, Number, and Geometry from Helmholtz to Cassirer*. Cham: Springer.

Campbell, Norman R., 1928. *An Account of the Principles of Measurement and Calculation*. London: Longmans, Green and Co.

Cantor, Georg, 1887. *Gesammelte Abhandlungen*. Berlin: Springer.

Carrier, Martin, 1994. "Geometric Facts and Geometric Theory: Helmholtz and 20th-Century Philosophy of Geometry." In *Universalgenie Helmholtz. Rückblick nach 100 Jahren*, edited by L. Krüger, pp. 276–91. Berlin: Akademie-Verlag.

Cassirer, Ernst, 1929. "Neo-Kantianism." In *Encyclopedia Britannica*, 14th ed., vol. XVI, pp. 215–16.

Coffa, J. Alberto, 1991. *The Semantic Tradition from Kant to Carnap: To the Vienna Station*, edited by L. Wessels. Cambridge: Cambridge University Press.

Darrigol, Olivier, 2003. "Number and Measure: Hermann von Helmholtz at the Crossroads of Mathematics, Physics, and Psychology." *Studies in History and Philosophy of Science* 34: 515–73.

Dedekind, Richard, 1888/1901. *Essays on the Theory of Numbers*. Chicago: Open Court.

De Kock, Liesbet, 2016. "Helmholtz's Kant Revisited (Once More). The All-Pervasive Nature of Helmholtz's Struggle with Kant's *Anschauung*." *Studies in History and Philosophy of Science* 56: 20–32.

DiSalle, Robert, 2006a. "Kant, Helmholtz, and the Meaning of Empiricism." In *The Kantian Legacy in Nineteenth-Century Science*, edited by M. Friedman and A. Nordmann, pp. 123–39. Cambridge, MA: MIT Press.

———, 2006b. *Understanding Space-Time: The Philosophical Development of Physics from Newton to Einstein*. Cambridge: Cambridge University Press.

Einstein, Albert, 1916/1923. "The Foundation of the General Theory of Relativity." In *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity*, translated by W. Perrett and G. B. Jeffrey, pp. 111–64. New York: Dover.

Friedman, Michael, 1997. "Helmholtz's *Zeichentheorie* and Schlick's *Allgemeine Erkenntnislehre*: Early Logical Empiricism and its Nineteenth-Century Background." *Philosophical Topics* 25: 19–50.

Grassmann, Hermann, 1878. *Ausdehungslehre*, 2nd ed. Leipzig: Otto Wigand.

Helmholtz, Hermann von, 1868. "On the Origin and the Significance of the Axioms of Geometry." In Helmholtz (1977), pp. 1–26.

———, 1870. "The Facts in Perception." In Helmholtz (1977), pp. 115–63.

———, 1887. "Numbering and Measuring from an Epistemological Viewpoint." In Helmholtz (1977), pp. 72–103.

———, 1903. *Vorlesungen über theoretische Physik*. Leipzig: Barth.

———, 1977. *Epistemological Writings*, translated by M. Lowe, edited by R. Cohen and Y. Elkana. Dordrecht: Reidel.

Howard, Don, 1999. "Point-Coincidences and Pointer-Coincidences: Einstein on Invariant Structure in Spacetime Theories." In *The History of General Relativity IV: The Expanding Worlds of General Relativity*, edited by H. Goenner, J. Renn, J. Ritter and T. Sauer, pp. 463–500. Boston: Birkhäuser.

Kaila, Eino, 1936. "On the System of the Concepts of Reality. A Contribution to Logical Empiricism." In Kaila (1979), pp. 59–125.

———, 1941. "On the Concept of Reality in Physical Science. Second Contribution to Logical Empiricism." In Kaila (1979), pp. 126–258.

———, 1960. "The Perceptual and Conceptual Components of Everyday Experience." In Kaila (1979), pp. 259–312.

———, 1979. *Reality and Experience: Four Philosophical Essays*, translated by A. Kirschenmann and P. Kirschenmann, edited by R. S. Cohen. Dordrecht: Reidel.

Kronecker, Leopold, 1887. "Über den Zahlbegriff." In *Philosophische Aufsätze, Eduard Zeller zu seinem fünfzigjährigen Doctorjubiläum gewidmet*, pp. 263–74. Leipzig: Fues.

Lange, F. Albert, 1866. *Geschichte des Materialismus und Kritik seiner Bedeutung in der Gegenwart*. Iserlohn: J. Baedeker.

Mach, Ernst, 1896/1986. *Principles of the Theory of Heat: Historically and Critically Elucidated*, edited by B. McGuiness. Dordrecht: Reidel.

Michell, Joel, 1993. "The Origins of the Representational Theory of Measurement: Helmholtz, Hölder, and Russell." *Studies in History and Philosophy of Science* 24: 185–206.

Neuber, Matthias, 2012a. "Helmholtz's Theory of Space and Its Significance for Schlick." *British Journal for the History of Philosophy* 20: 163–80.

———, 2012b. "Invariance, Structure, Measurement—Eino Kaila and the History of Logical Empiricism." *Theoria* 78: 358–83.

———, 2017. "'Denn was man messen kann, das existiert auch'—Grundzüge eines metrologischen Strukturenrealismus." *Zeitschrift für philosophische Forschung* 71.

Niiniluoto, Ilkka, 1992. "Eino Kaila and Scientific Realism." In *Eino Kaila and Logical Empiricism*, edited by I. Niiniluoto, M. Sintonen and G. H. von Wright, pp. 102–16. Helsinki: Acta Philosophica Fennica.

Oberdan, Thomas, 2015. "From Helmholtz to Schlick: The Evolution of the Sign-Theory of Perception." *Studies in History and Philosophy of Science* 52: 35–43.

Patton, Lydia, 2009. "Signs, Toy Models, and the A Priori." *Studies in History and Philosophy of Science* 40: 281–89.

Pulte, Helmut, 2006. "The Space Between Helmholtz and Einstein: Moritz Schlick on Spatial Intuition and the Foundations of Geometry." In *Interactions: Mathematics, Physics and Philosophy, 1860–1930*, edited by V. F. Hendricks, K. F. Jørgensen, J. Lützen and S. A. Pedersen, pp. 185–206. Berlin: Springer.

Reichenbach, Hans, 1928/1958. *The Philosophy of Space and Time*. New York: Dover.

Ryckman, Thomas, 2005. *The Reign of Relativity: Philosophy in Physics 1915–1925*. New York: Oxford University Press.

Schlick, Moritz, 1917/1979. *Philosophical Papers*, vol. 1, translated by P. Heat, edited by H. Mulder and B. van de Felde-Schlick. Dordrecht: Reidel.

———, 1918/1974. *General Theory of Knowledge*, translated by A. Blumberg. LaSalle, IL: Open Court.

———, 1921. "Notes and Comments." In Helmholtz (1977), pp. 26–38.

Von Wright, Georg H., 1979. "Introduction." In Kaila (1979), pp. ix–xlii.

Wigner, Eugene, 1967. *Symmetries and Reflections*. Bloomington: Indiana University Press.