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Review: *Dummett on Abstract Objects*, by George Duke

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The problem of mathematical objects, and of abstract objects in general, is one which occupied Michael Dummett throughout his long and distinguished career, from his early critique of the nominalism of Nelson Goodman and W.V. Quine, running through much of his work on Frege, and on the origins of analytical philosophy, to papers published in his last decade. To a very considerable extent, his thought about the problem is entangled with his interpretation and criticism of Frege's philosophy, and especially of his philosophy of mathematics—so much so that it is not merely very difficult, but probably ill-advised, to separate the two. That his view on the problem underwent significant changes during the half century in which he wrote about it is hardly to be wondered at—it would have been remarkable, had it not done so.

One major—perhaps the single most important—shift involves Dummett's changing attitude towards Frege's famous Context Principle, particularly when it is understood as concerning reference. In one of its formulations in *Die Grundlagen der Arithmetik* (1884), the principle runs: 'Nur in Zusammenhange eines Satzes bedeutet die Wörter etwas'[1]. In their ordinary usage, the verb 'bedeuten' and the corresponding noun 'Bedeutung' exhibit much the same range of meaning as 'mean' and 'meaning' do in English. In particular, 'bedeuten' may mean 'mean', 'signify', or 'stand for'. Thus although these formulations predate Frege's explicit introduction of the sense-reference distinction in 'Über Sinn und Bedeutung' (1892), and his technical use of 'Bedeutung' for reference or semantic value, an interpretation of the principle as concerning

reference is certainly not ruled out. When it is so interpreted, the principle may be taken as denying that if a word is to have reference, it must always be possible to point to, or otherwise separately identify, what it stands for, and as insisting that it is sufficient that the word figures in a complete sentence which may be used to say something true[2]. The principle may then be used—or so Dummett argued in his earlier work—both to pinpoint the error of nominalism, and to justify taking abstract singular terms, such as numerals and other complex terms for numbers, as having reference to abstract objects. By the time of *Frege philosophy of language*, however, Dummett had developed serious reservations about the sense in which reference could legitimately be ascribed to terms for abstract objects, and especially terms for what, by analogy with pure sets, he calls 'pure' abstract objects, such as numbers, and the pure sets themselves. The principal source of these reservations lay in the fact that there appears to be nothing, in the case of abstract terms, analogous to the identification, in the case of terms for concrete objects, of an object as the referent of the term, as part of the process of determining the truth-value of sentences incorporating the term. Proposing a distinction between two notions of reference—a 'realistic' conception of reference as a relation to something 'external', in accordance with the name-bearer prototype, and reference as semantic role—Dummett contends that whereas terms for concrete objects may be regarded as having reference in both senses, in the case of abstract terms, the name-bearer prototype breaks down and we have only reference in the sense of semantic role—reference in a 'thin' sense, whose objects are 'internal to language'.

That there is a tension between Dummett's earlier fulsome endorsement of the Context Principle and his later emphasis on the name-bearer prototype and insistence upon the possibility of extra-sentential identification of objects as integral to 'realistic' reference is, I think, rightly emphasized by George Duke throughout

the central chapters of his recent study of Dummett's evolving view of abstract objects. Duke's overall aim is to provide what he himself terms a 'qualified defence' of Dummett's 'tolerant reductionist' account of abstract objects. Dummett is best understood, he holds, as claiming that abstract singular terms have reference only in a 'thin' and 'language-internal' sense—'in the sense that they possess semantic role, but … can only be regarded as possessing bearers in an attenuated sense that acknowledges the role played by language in their constitution' (p.177). A central contention of the book—and one of the main ways in which Duke's defence of Dummett's position is qualified—is that Dummett's hostility to psychologism prevents him from 'acknowledging the extent to which abstract objects—at least those employed in mathematical theories—are constituted through our linguistic practices'. More specifically, Duke contends that while some of Dummett's remarks suggest that mathematical objects 'have an essentially linguistic character in a way that concrete objects do not', he 'fails to develop this point systematically… [and] … to articulate a clear position on the referential status of mathematical objects' (p.143). To remedy this shortcoming, Duke believes, we must draw on Husserl's work on 'meaning-constitution'. This, he claims, 'illuminates the links between our everyday understanding of number and the formal arithmetic that is constructed on its basis, whilst also helping to explain the sense in which it is legitimate to regard numbers as objects' (p.165). When Dummett's 'intermediate position' is developed in this way, it is—Duke argues—'more plausible than its rivals'—platonism and nominalism. Platonism 'renders our epistemic access to abstract entities inscrutable in so far as it relies on a mysterious and superfluous notion of mind and language independence', whereas 'extreme nominalism… in denying that it is legitimate to ascribe a reference to abstract singular terms at all—contradicts our capacity to form true sentences containing reference to abstract entities, while more sophisticated variants

overlook the role played by our linguistic practice in the constitution of 'thin' objects of reference'. (p.176)

There are two separable questions here—*Is Duke's interpretation and development of Dummett's position correct, or at least plausible?* and—*Is the proposed position really, as he claims, more plausible than its rivals?* I am inclined to think that the answers are *No* and *No*. Justifying them would call for a larger discussion than I can undertake here, but I can say a little in their defence.

Duke holds that Dummett's position in *Frege philosophy of mathematics*, where he first advocates 'tolerant reductionism', remains essentially the same as the 'intermediate position' of *Frege philosophy of language* (cf. p.136). For reasons given elsewhere[3], this seems to me to overlook, or drastically underplay the importance of, a shift between the two works over the relations between 'reference realistically construed as a relation to an external object' (as Dummett puts it) and reference as semantic role.

Dummett's position in the earlier book is that abstract singular terms have reference only in the sense of having a semantic role, where that is a matter of making a contribution to determining the reference of more complex expressions in which they occur, but lack reference realistically construed. It is precisely this which is supposed to differentiate the intermediate from the austere position, according to which abstract terms do not refer at all, and are not even semantically significant parts of sentences containing them. Crispin Wright and I argued, independently[4], that this position is unstable—the main burden of our arguments being that reference as semantic role and reference realistically construed cannot be held apart, in the way that Dummett seemed to require.

Dummett's later discussion continues to deny that abstract terms have realistic reference—but now this denial is to be understood as claiming that in the context of a semantic theory which explains how the reference of complex expressions depends upon that of their parts, terms introduced by contextual definition are

not to be treated as having reference. This is not to say that such terms cannot be said to have reference at all—for they may still be said to refer in a 'thin' sense, but they are 'semantically idle', in the sense that they play no part in determining the truth-values of sentences containing them—i.e. they have no semantic role. The later position is therefore quite different from the earlier. But it is, if anything, even more difficult to see how it can avoid collapsing into the austere position and *in*tolerant reductionism. Once it is denied that abstract terms introduced by contextual definition have even a semantic role, it is quite unclear what differentiates tolerant reductionism from the view that we can indeed intelligibly use sentences incorporating such terms (e.g. 'The shape of this vase = the shape of that one'), but only on condition that they be understood as a mere *façon de parler*—as a potentially misleading variant, devoid of semantically significant structure, on sentences free of even any surface appearance of abstract reference (e.g. 'These vases are geometrically similar') . And it appears equally unstable—in particular, it arguably faces a lethal dilemma over whether to insist on a distinction, parallel to that between thin and thick reference, between thin and thick *existence*. It is, on the one horn, difficult to characterize such a distinction without either begging the question against the platonist or trivializing the issue; but if existence is univocal, it is unclear why it should matter to the platonist whether reference to abstract objects is thick or thin[5].

I imagine that Duke would concede that it is at best unlikely that Dummett would have welcomed the interpretation and development of his view along the Husserlian lines proposed. His claim is, I take it, that—like it or no—some such account of the 'constitution' of thin objects is needed if the desired intermediate position is to be viable. In view of the difficulties rehearsed just now, it is a good question whether Dummett's position could be improved by maintaining, along Husserlian lines, that thin objects of reference are somehow constituted by our linguistic behaviour.

I suppose that Duke would embrace the first horn of my dilemma, and claim that there *are* two kinds of existence—mind- or language-dependent and mind- and language-independent. Numbers and other abstracta, on Duke's version of Dummett's tolerant view, enjoy only the former, whereas concrete objects but not, save on a traditional platonist view, by numbers, etc., enjoy the latter.

This unattractive view might be thought objectionable on the ground that it compromises the objectivity of arithmetic—if the natural numbers are mind-dependent, must not truths about them be so as well? Perhaps this objection could be resisted—we might agree that works of literature and musical compositions are mind-dependent abstracta, but still hold that there are objective truths about them. But even if such a view can evade this and other objections, Duke's contention that it is more plausible than its rivals may certainly be contested. In particular, I think we should contest his claim that the platonist must rely on a 'mysterious and superfluous notion of mind and language independence' which 'renders our epistemic access to abstract entities inscrutable'. That a modest form of platonism based on Hume's principle as an implicit definition of the number operator can supply a route to *a priori* knowledge about the natural numbers—and so answer Paul Benacerraf's celebrated challenge—is, of course, the central thesis of the neo-Fregean version of logicism which Wright and I have defended in several places. I can only suppose that Duke thinks he has disposed of its claims in his discussion of our work in chapter 6—that it either fails to qualify as a genuine version of platonism or succumbs to some other objection(s) presented there. The former is suggested by a passage where he quotes our characterization of arithmetical platonism as the view 'that number words have reference, and that their reference is to objects—objects which, on any reasonable account of the abstract-concrete distinction, must be reckoned to lie on the abstract side of it'[6], and complains that this

seems at first glance a rather weak form of platonism. … For the logicist platonist 'there is no good distinction between an expression's functioning as a singular term according to syntactic criteria and its being appropriate to construe its semantics referentially'. This formulation asserts only that there are logical and mathematical objects, however, not that such objects have genuine mind and language independence. This raises the question of in what genuine mind and language independent existence of such entities could be taken to consist.[7]

It is certainly true that the embedded quotation says nothing about mind or language independence—but that is hardly surprising, since it is lifted from a context in which Wright's sole concerned to explain how the platonist will resist a reductive construal of equivalences such as Hume's principle and the Direction Equivalence which he sees as introducing terms for abstract objects (numbers and directions) on the basis of an equivalence relation on entities of some other kind (concepts and lines). Mind and language independence were not the issue. But the suggestion that we have nothing to say on the matter is very wide of the mark. A large part of point of the Fregean approach—which appears to have completely eluded Duke—is to reduce questions about the mind and language independent existence of objects to the question whether true statements featuring terms for those objects are objectively true, i.e. true in a mind and language independent sense. One perfectly good, unmysterious, and relevant explication of that idea is that a statement is mind and language independently true iff it would have been true, even if there had been no creature capable of making it. There is, of course, a much stronger notion of objectivity, viz. objectivity in the sense of Dummett's realist, who thinks that some statements are apt for radically evidence-transcendent truth—that they may be true, or false, in ways which outstrip our capacities, even in principle, to determine their truth-value. Whether mathematical, or any other, statements are objectively truth-valued in this sense is highly problematic, and need not concern us here.

If the charge that our view somehow fails to make space for the mind- and language-independent existence of abstract objects is not made out, does Duke offer any other reason for discounting it? Here I can only report that while I have found a distressingly large number of misunderstandings or misrepresentations of our view, I have been able to detect no objection to which we have not already replied. The misunderstandings range from relatively minor to quite major. For example, at p.119, Hume's principle is described as introducing 'a term forming-operator of second-level that transforms a statement about the number of objects falling under a concept into one that makes reference to objects'! Even this relatively minor gaffe is potentially quite serious. The term-forming operator does not transform any statement at all, much less a statement about the number of objects falling under a concept—it operates on a predicate which stands for a concept, or property, to form a term which, if the relevant instance of the left hand side is true, stands for an object. Perhaps this is just a careless slip, but I fear it betokens a more serious failure—to grasp the sense in which numbers and other abstracta are, as Wright and I put it in recent work[8], metaphysically lightweight. If all goes well, a second-order abstraction such as Hume's principle defines a function from properties to objects. Given the abundant theory of properties we advocate, the conditions for property existence are extremely undemanding—roughly, all that is required is that there could be a predicate with a suitable satisfaction condition. Since, if the abstraction is in good standing, a suitable equivalence relation on properties suffices for the existence of the corresponding objects, that too is equally undemanding.

A more serious misunderstanding concerns our account of implicit definition. Duke writes as follows:

…Wright and Hale's appeal to implicit definitions as determining a meaning-constituting pattern of use for certain expressions seems a long way from traditional mathematical Platonism. Most problematic from this perspective is their assertion that 'one clear desideratum' of a satisfying account of explanation via implicit definition is that it 'must leave room for the capacity of such explanations to create a meaning' … The assertion that implicit definitions allow us to construct meanings seems to conflict directly with the claim that we can construe the referential commitment embodied in the [Left Hand Side] of Dir^9 and N= in a manner analogous to that for concrete singular terms, which presumably denote objects that are not the product of an act of 'invention'. (p.123)

The clear implication here is that by holding that implicit definitions can create meanings, we somehow commit ourselves to holding that the objects to which the terms introduced by an abstraction principle, taken as an implicit definition, have reference (if corresponding instances of the Right Hand Side are true) are themselves the product of some creative act. This is a travesty of our view. As we have been at pains to emphasize all along, there is a clear and indisputable difference between inventing meanings or concepts, on the one hand, and creating objects on the other. If, for example, Hume's principle succeeds as an implicit definition of the number operator, it ensures that a complex singular term such as 'the number of aardvarks' has a meaning, at least provided that 'aardvark' does; but whether that singular term has reference to an object—the number of aardvarks—is not a matter settled by stipulation or creative definition.

The foregoing remarks are almost entirely concerned with the last two chapters of Duke's book. The preceding five chapters contain extended discussions of the Fregean notion of an object and the syntactic priority thesis, Frege's and Dummett's criticisms of Husserl's psychologism, the context principle, Frege's notion of reference and Dummett's account of it, and the abstract-concrete distinction. As far as I have been able to get clear about them, Duke's aims, in broadest outline, are as follows:

(Chapter 1) to show, first, that Dummett tends to read back into Frege's talk of objects and concepts an ontological significance beyond what Frege intended, and that Frege's real concern was with the objectivity of mathematics, rather than mathematical objects, and, second, that the syntactic priority thesis should be rejected, both as an interpretation of Frege and in its own right;

(Chapter 2) to separate out what is correct in Frege's and Dummett's opposition to psychologism, partly by way of a distinction between weak and strong psychologism (p.43), and to make a case that while much of their criticism of Husserl is justified, his late work on 'the genesis of ideal objects' and 'the role of played by language in the constitution of ideal meaning' can make a valuable contribution to debates about abstract entities (p.55);

(Chapter 3) to chart Dummett's interpretation of the philosophical significance of the context principle, from his early appeals to it in his critique of nominalism through to his later, more qualified assessment of its import;

(Chapter 4) to advance an interpretation of Frege's notion of the reference (Bedeutung) of an expression according to which 'having a Bedeutung is a matter … internal to the language formalized by a semantic theory, based on a previous ontological decision regarding the objects in the domain of quantification'(p.86) which requires revising, if not abandoning, the context principle (ibid); and to trace the shift in Dummett's conception of reference—which involves separating the name-bearer prototype and semantic role as two components, and 'privileging the model of meaning for concrete singular terms, taking it as prescriptive for a model of meaning for abstract singular terms'—and the problems to which Duke believes it gives rise (p.87);

(Chapter 5) to air the difficulties involve in giving an account of the distinction between abstract and concrete objects, and in

particular, the difficulties afflicting Dummett's account of it, including what Duke takes to be his 'contradictory … theory of pure abstract objects', and to propose an improved account drawing upon Husserl's account of the 'constitution' of mathematical objects.

While I read these chapters with interest, I have to report that I found them largely unconvincing. There is much in them with which to disagree, both on substantial philosophical matters and on questions of interpretation, especially of Frege and Dummett. Husserl's doctrines remain as unclear to me as they were before I studied this book, and I remain somewhat sceptical that they can be used to any philosophically illuminative purpose.

I think it is widely held that some of Dummett's interpretations of Frege owe more to what he thought ought to have been Frege's view than to what Frege actually says. Much the same, I fear, is true of some of Duke's. In particular, his attempt to make out that in his earlier work, Frege is careless of 'the difference between expressions and their denotations', which he supports by the claim that in *Funktion und Begriff* , he characterizes an object as 'anything that is not a function and hence lacks an argument place' commenting that this 'does not clearly differentiate between a property of an expression (to lack an argument place) and the property of whatever the sign is taken to denote, suggesting that we are working from a perspective internal to language' (p.22–3) appears to rest on a straight mistranslation. What Frege actually says, after admitting that he can give no 'regular definition' of an object, is: Hier kann nur gesagt werden: Gegenstand ist alles, was nicht Funktion ist, dessen Ausdruck also keine leere Stelle mit sich führt—'Here it can only be said: An object is anything that is not a function, of which the expression carries with it no empty places'. It is quite clear that it is the *expression*, *not* the *object*, which Frege is describing as having no argument places.

To give just one more example, in a later passage, Duke tries to make out that Dummett seeks to support the syntactic priority thesis by over-interpreting a key passage in *Grundlagen* where Frege introduces the need for a criterion of identity. Frege writes:

…wir haben schon festgestellt, dass unter den Zahlwörtern selbständige Gegenstände zu verstehen sind. Damit is uns eine Gattung von Sätzen gegeben, die einen Sinn haben müssen, der Sätze, welche ein Wiedererkennen ausdrücken. Wenn uns das Zeichen a einen Gegenstand bezeichnen soll, so müssen wir ein Kennzeichen haben, welches überall entscheidet, ob b dasselbe sei wie a, …

According to Duke (p.184, note 18), the most obvious interpretation of these sentences would be:

…now that we have established that numbers are independent and complete objects, we can explain how they are given to us by fixing the senses of the kind of sentence in which they occur. This presupposes that we are able to recognize two different proper names as standing for the same object and can form identity-statements with these two names.

He then objects:

Dummett, however, glosses the principle as that 'in order to grasp what object a name is being used to stand for, it is necessary to know … how to recognize the object as the same again'

Literally translated, Frege's passage runs:

we have already established that self-standing objects are to be understood by number words. Thereby we are given a kind of proposition which must have a sense, the propositions which express a recognition. If the sign a is to signify for us an object, we must have a criterion (literally 'sign' or 'mark') which everywhere decides whether b is the same as a …

I think it will be obvious to the reader whose 'interpretation' is closest to Frege's words. Once again, Duke's criticism appears to be based on a perverse misreading of Frege's German. His wording 'einen Sinn haben müssen' is no accident—he repeats it in Grundlagen §106.

I wish I could recommend this book with more enthusiasm. I learned quite a lot by reading it, but mainly through the effort of thinking out where I thought it had gone wrong.

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Notes

[1] Frege 1884, §62—other formulations occur on page x and in §106 of this work.

[2] Given Frege's later insistence that an expression's lacking reference means that any more complex expression of which it forms part must likewise lack reference, together with his identification of the reference of a declarative sentence with its truth-value, figuring in a false sentence would be equally sufficient for an expression to have reference.

[3] In Hale 1994, §4. Duke is certainly aware of this discussion, since he refers to it; but he offers no counter to its arguments. Indeed, his discussion at pp.130-33 seems to confuse what Wright and I say (in Wright 1983 and Hale 1987) in criticism of Dummett's position in 1973 with my later arguments against his 1991 position.

[4] Wright 1983, §x; Hale 1987 ch.7, §I

[5] This drastically abbreviates an argument given in Hale 1994, at the close of §4. The second horn of the dilemma assumes that existential generalization is allowed with respect to 'thinly referential' terms. If the tolerant reductionist severs that connection, he is in trouble on other grounds.

[6] Hale & Wright 2001, p.7

[7] Duke, p.120. The embedded quotation is from Hale & Wright 2001, p.156

[8] e.g. Hale & Wright 2009

[9] Dir is the Directions equivalence: The direction of line a = the direction of line b if and only a and b are parallel. N=, also known as Hume's principle, is: The number of Fs = the number of Gs if and only if the Fs correspond one-one with the Gs.

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