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Review: *Bolzano's Theoretical Philosophy, An Introduction*, by Sandra Lapointe

Jan Šebestík

Until recently, Bolzano's philosophy aroused only marginal interest in the English speaking world.[1] Today, the *Mathematical Works of Bernard Bolzano* by Steve Russ, two partial translations of the *Wissenschaftslehre* (*WL*) and two translations by Paul Rusnock and Rolf George are available (*On the Mathematical Method and Correspondence with Exner* and *Selected Writings on Ethics and Politics*); Oxford University Press will soon publish a complete translation of the four volumes of the *WL* by the same translators. Two important monographs appeared: *Bernard Bolzano's Life and Work* by Edgar Morscher and *Bolzano's Philosophy and the Emergence of Modern Mathematics* by Rusnock.

Even though the situation has improved considerably during the recent years, Bolzano's work remains relatively little known. Sandra Lapointe's book fills this important gap in the history of analytic philosophy.

The Introduction begins with Bolzano's carrier as a teacher of the Science of [Catholic] Religion and his dismissal because of the incompatibility of his ideas on the system of government of the Austrian Monarchy and the organization of the society with the mandatory university curriculum. Lapointe depicts the mechanics of Austrian higher education, particularly of philosophy, with a characteristic quote from Karl Rosenkranz: "In Austria, philosophy does not exist at all […]" (p. 5). One should not forget, however, that Rosenkranz was a Hegelian and that both Bolzano and Franz Exner, the Prague professor of philosophy, were declared opponents of Hegel's philosophy.[2] Lapointe rightly explains Bolzano's lack of success by his style and by his theoretical preoccupations, which were closer to pre-Kantian philosophy and "were therefore judged obsolete by his German contemporaries" (p. 5). She recalls his posthumous influence on Husserl, on other Brentano's students Benno Kerry and Kazimierz Twardowski, on Alwin Korselt and on several Polish philosophers.

The discussion makes for a firm bridge to what's perhaps Bolzano's most celebrated innovation: the concept of proposition in itself. As Bolzano tells it, propositions are the primary bearers of truth. At the same time, the propositions are abstract entities, to be distinguished from sentences and mental states. Propositions are composed of ideas that are not themselves propositions.

According to Lapointe, "The motivation behind Bolzano's antipsychologism as well as the semantic realism […] were the actual needs of scientific practice—in mathematics in particular—which is essentially based on demonstration" (p. 9).

Lapointe devotes chapter 1 to Bolzano's relationship to Kant and to German philosophy. In 1798, a ban was imposed on Kant's writings on religion and politics in the Austrian monarchy (see note 1, p. 158). Nonetheless, his popularity in Germany and the radical novelty of Kant's work, particularly the *Critique of Pure Reason*, attracted scores of gifted Austrian students who were dissatisfied with the poverty of the current teaching of philosophy. Lapointe's presentation of latter 18th century philosophical currents includes the cameo roles played by Wolffians, Lockeans and contemporary logicians discussed by Bolzano.

Lapointe thinks that Bolzano was "a fierce opponent of critical philosophy as a whole" (p. 11), but this should be taken *cum grano salis*: as he himself admits, it is from Kant that he got the decisive impetus for the fundamental distinctions of *a priori* - *a posteriori*, analytic - synthetic, intuitions - concepts, although he was not satisfied with Kant's definitions of these concepts (Bolzano 1977, 67–68). His strongest objection was addressed to Kant's concept of pure intuition, which is self-contradictory and cannot therefore play a role in the foundation of mathematical knowledge. For Bolzano, mathematical truths are purely conceptual and are grounded in axioms while Kant appeals to synthetic judgments *a priori* founded in pure intuition.

Chapter 2, On Decomposition, takes over the theme of Lapointe's French book *Qu'est-ce que l'analyse* (2008). Against Kant, for whom "every given concept can be defined through analysis" (p. 21), she advances the idea of a new, Bolzanian concept of decomposition that takes into account the cases where the components of an idea are not identical to the ideas of the properties of its objects (p. 25). In other terms, Bolzano repudiates the picture theory of ideas: the structure of ideas is not an image of the structure of their objects. Nevertheless, I would point out that Bolzano also has an important concept that corresponds to the result of Kantian analysis: the concept of content (*Inhalt*) of an idea which enumerates all its components.

In the chapter 3, Meaning and Analysis, we are in the heart of Bolzano's doctrine. We learn that, for Bolzano, propositions are the "'*Sinn*' of sentences" (p. 29) and that indexicals and other context sensitive expressions must be made explicit and eliminated. This amounts to completing underdetermined utterances such as "It is snowing" by the determinations of time and space. As a consequence, *all* sentences express eternal propositions. Bolzano is quite explicit about this (see Bolzano 2004, p. 141) and his amendment is today generally accepted. We only should keep in mind that one and the same inscription can express many different propositions.

Lapointe goes astray with intuitions: Bolzanian intuitions cannot be "indexical components of our beliefs" (p. 31 and elsewhere): indexicals are linguistic devices whose reference depends on the context, while intuitions are fixed ideas, each different from others (see Bolzano 2004: *On the Mathematical method*, § 6, and *Correspondence with Exner*, p. 50-53, 91-92 and 196). Only the word 'this' which designates intuitions is indexical.

Lapointe then recalls Bolzano's important distinction between "'what words are designed to convey' from what a speaker 'intends to convey with them'" (p. 34) and treats the structure of propositions and ideas, interpretation (*Auslegung*) and redundancy. On page 39, she explains the Bolzanian concept of existence as the objectuality (non-emptiness), a second-order predicate, of the corresponding idea.

Bolzano's definition of inclusion is much simpler than that proposed by Lapointe p. 38: A is included in B if A and B are compatible and if "all objects subsumed by A are also subsumed by B" (Bolzano 1837, I, § 95, p. 444). An incomplete quotation on p. 41 induces Lapointe to attribute only a metaphoric value to Kant's notion of inclusion.

In chapter 4, Substitutional theory, the author explains the Bolzanian concept of logical form and his method of variation, in particular the concept of universal *Gültigkeit* of a proposition (or of a propositional form, *Satzform*), and its degree of *Gültigkeit*, i. e. its probability. At the same time she recalls that for Bolzano, logic is not a purely formal science, because some important distinctions, e. g. between *a priori* and *a posteriori* propositions, are material. She stresses the difference between our concept of deducibility and Bolzano's *Ableitbarkeit*: for him, among other things, the premises must be compatible with the conclusions.

Sometimes Lapointe hesitates. On page 163, note 3, she qualifies Bolzano's talk about exchangeable [= variable] components in propositions as metaphorical, although he explains precisely what he means by the words *variation* and *variable*, and she has it right on p. 46. On the same page, and similarly also p. 60 and 63, the author speaks about "the associated set of propositions containing members all sharing some fixed vocabulary". Propositions may share ideas, but not a vocabulary, which depends on language.

Which ideas are admitted for substitutions? Lapointe says that Bolzano never treated the problem of category mistakes. It is true that in “Caius is a man”, for ‘Caius’ we may substitute ‘Sempronius’ and obtain a true proposition, but also ‘rose’ or ‘triangle’, and obtain a false proposition. Nevertheless, as Paul Rusnock reminds me, the signs A, B, C in the syllogistic form Barbara for instance can

> mean very different things, but not quite anything we may choose. They must signify ideas such that B is an idea which can be predicated of all A and C one which can be predicated of all B. Thus it can be seen that the objects A, B, and C are not left indeterminate as to all their characteristics, but only as to some of them. (Bolzano 1837, § 7, p. 28).

Contrary to the author's opinion that the universal *Gültigkeit* “is usually taken to have little comparative import” (p. 51), I think that it is Bolzano's merit to point out this important concept, which is very often used in sciences where we keep logical concepts as well as fundamental concepts of each science fixed, e.g. number, function, mass, weight, field, etc., varying only their instances or values.

Pages 57-58 contain one of the most original contributions of this book: an explicit treatment of quantification according to Bolzano's indications, which he himself never undertook and never used in his mathematical writings. Just a small remark here: while universal quantification is presupposed when using an idea in the position of subject—never in the position of predicate—(the word ‘all’ is redundant and can be omitted before a subject-idea), existence is, as we have seen, a second order predicate. This is a clever idea, but the link between universal and existential quantification is, if not lost, disrupted.

Chapter 5 treats the important concept of analyticity. Lapointe compares Bolzano and Kant, shows the inadequacy of the latter's conception. Pages 60, 66 and elsewhere: the reader would like to know what the semantic regularities mentioned by the author are. Page 63 is puzzling: Lapointe quotes Bolzano's definition, but her paraphrase suggests a typo (it is not, unfortunately; see p. 163, note 7): she simply reproduces the definition of universal *Gültigkeit* from p. 47. The good definition of broader analyticity is, of course, Bolzano's: a proposition is analytically true (false) if it contains at least one idea such that all its objectual substitution instances are true (false), and Lapointe has it right on p. 69. Thus, “Caius, who is a bachelor, is unmarried” (p. 64) is analytic, because it contains an idea (namely “Caius”) whose variation yields only truths. The first lines of p. 67 are misleading, because an analytic proposition is universally valid with respect to the idea(s) that are free for variation in it.

After universal *Gültigkeit* and analyticity comes the most important concept: that of logical analyticity, which is a close relative to Quine’s concept of logical truth. As for synonymy (p. 69–71), I propose the following: two expressions designating ideas or propositions are synonymous if they are composed from the same parts in the same order or having the same structure. Logical equivalence is surely not enough: 'equilateral triangle' and 'equiangular triangle' are equivalent, but not synonymous.

In chapter 6, *Ableitbarkeit* and *Abfolge*, Lapointe presents these two fundamental relations of Bolzano’s logic. In the note 2, p. 165, she claims that “it is difficult to say what this [the possibility of transferring extensional relations among ideas to propositions] tells us about his logic”. Well, it tells us first how to construct the logic of relations among propositions from the logic of classes, second, that they are isomorphic, which is an important truth, and that Bolzano's logic is not a logic of truth functions (although it is possible to reconstruct it in his system) but of extensional relations, among them, the relation of *Ableitbarkeit.*

The formula p. 74, lines 6–7, expresses symmetry; for asymmetry, one must take the negation of the consequent. In fact, *Ableitbarkeit* is not symmetric. Asymmetry holds only for *Ableitbarkeit* without reciprocity; mutual *Ableitbarkeit* is equivalence. The author thinks that Bolzano "does not systematically uphold the distinction" between general and logical *Ableitbarkeit* (p. 75). But at the beginning of the *Schlusslehre*, one of the most beautiful passages of the whole work (Bolzano 1837, § 223, p. 392 and 395, and again Bolzano 2004, *On the mathematical method*, §8, p. 55), he is quite explicit:

> Moreover, according to the very wide sense in which I have taken the word deducibility (*Ableitbarkeit,* §155) the validity or invalidity of some deductions can be assessed only if we have knowledge of matters outside logic. Thus from the proposition 'this is a triangle' we may deduce the proposition 'this is a figure the sum of whose angles equals two right angles' (with respect to the idea 'this'), and from the proposition 'Caius is a man', we can deduce the proposition 'Caius has an immortal soul' with respect to the idea 'Caius'). […] But to realize this, we must know two truths, namely that the sum of the angles in any triangle equals two right angles, and that the souls of all men are immortal. Since these are truths which are not at all concerned with logical objects, i.e. with the nature of concepts and propositions, or rules according to which we must proceed in scientific exposition, nobody will demand that logic should teach deductions of that sort. Hence, what can be expected in this place is only a description of those modes of deduction whose correctness can be shown from logical concepts alone, or, what comes to the same thing, which can be expressed in the forms of truths, in which nothing is mentioned except concepts, propositions, and other logical objects. (Transl. of Rusnock and George).

In Bolzano's logic, one cannot derive anything from contradictory premises (p. 76), but one can nevertheless draw conclusions from false premises under the condition of their compatibility. Next comes Bolzano's probabilistic logic, the concept of grounding, the relation between *Ableitbarkeit* and *Abfolge* and the notion of exact or adequate *Ableitbarkeit*. The two formulas p. 80 are incorrect; it should be 1) "then the probability of non-T = 1–μ", 3) "the probability of T > ½" (similarly in the next sentence: T instead of M).

It would be interesting, she writes on p. 87, to "systematically pick out inferences in which the necessity of the conclusion can be established formally on basis of true premises. But this is not the case". In fact, Bolzano tried to do it already in the *Beyträge zu einer begründeteren Darstellung der Mathematik* (Bolzano 1810, II, § 12, p. 63-68), where he enumerates four simple independent inference schemes that represent the "objective dependence of truths", in contradistinction to other valid schemes where the premises are not objective grounds of the conclusion. Lapointe also errs also when she writes that Bolzano's concept of "exact [adequate] *Ableitbarkeit* remains incidental" (p. 89, line 14-15). On the contrary: all forms of inference of the *Schlusslehre* are *genaue Schlüssen* (II, § 223, p. 393). Under the condition of the greatest possible simplicity of the premises and the conclusion (II, § 221, 7), *all* inference forms of the *Schlusslehre* are *formale Abfolgen*.[3]

In the chapter 7, Justification and Proof, the author's logical analysis is enlarged: it also contains elements of epistemology with concept such as the grasping of grounding relations, certainty, belief, the degree of confidence (*Zuversicht*), etc. At the very beginning of this chapter (p. 92), Lapointe distinguishes grounding (*Abfolge*), objective justification and objective proofs (*Begründungen*). For her, Bolzanian *Begründungen* are "linguistic objects that are meant […] to reliably cause in agents objectively justified knowledge". Similarly (p. 92), proofs are "linguistic representations of a set of propositions". At first sight this seems reasonable and some mathematicians, for example Pierre Cartier, share partly this view. The same thing, however, could be said of every true proposition, of *Ableitbarkeit*, etc. We grasp propositions with our mind and pronounce them, write them, or see them written or hear spoken in a language. But proofs are not dependent on a particular language. Euclid's proofs in Greek and in English are two linguistic representations of the same proof, of the same propositions. I also doubt if "deductive practices […] exclude non-conceptual knowledge" (p. 96); think of astronomy, physics, chemistry …, that contain empirical knowledge based on intuitions (§ 586, p. 406**)**, neither do I agree with all what is written p. 98. If we present a science in the strictly scientific manner, we always should strive to ground (*begründen*) our proofs objectively (although there always are limit cases where pragmatic considerations play a role) and Bolzano tries to do it in all his scientific works.

The title of the chapter 8, *a priori* Knowledge, is definitely non-Bolzanian. This expression is rare in his writings and has several occurrences only when Bolzano discusses the theories of other thinkers who use it, in particular those of Kant, Leibniz and other philosophers**.** In the whole *WL*, there is just a passage on account of Bolzano in II, §133, p. 36-37, then one page about *a priori* and *a posteriori* knowledge in III, §306, 12 with the definitions (p. 202), and ten lines in §586, 3, p. 406). Bolzano prefers to speak about conceptual and empirical propositions whose difference is based on their inner characteristics and not on the relation between propositions and our cognitive faculty. He says that these distinctions "nearly coincide" and that "the truth of most conceptual propositions can be decided by pure thought" (II, § 133). There is also a more pertinent quote about prime numbers than that p. 107: we do not know "of the formula that yields all the prime numbers, when it is still doubtful whether there is such a formula" (Bolzano 2004, 161, missing in Lapointe's bibliography p.171).

Lapointe identifies knowledge by virtue of meaning with *a priori* knowledge (p. 90). But this happens by means of explicit definitions, which may be empirical in the case of empirical ideas.

Our author discusses the question of the connection of the subject and the predicate in a proposition and explains that for Bolzano (and against Kant), *a priori* knowledge, and in particular the axioms of a theory, is not grounded in pure intuitions, but in the concepts themselves. Bolzano is also the father of implicit definition: the primitive concepts that occur in the axioms are defined "on the basis of the use or context" (Bolzano 1837, § 668, 9).

Chapter 9, Things, Collections and Numbers. My English speaking friends Paul Rusnock, Steve Russ and Peter Simons have problems with the translation of some of Bolzano's terms, in particular of *Menge*, which is at the same time a common word for 'a number of' and a technical term for 'set' in German. In some contexts it is possible to say mass, when speaking about collections, maybe multitude, but when Bolzano speaks about the *Menge der Sprünge einer Function*, the only possible translation is 'the set of the leaps of a function'.

Lapointe explains the different species of collections: masses (*Mengen*), sums, quantities, pluralities, series, etc. She quotes Bolzano's construction of the sequence of natural numbers from *Reine Zahlenlehre* and commits a fatal error translating the word *gleich* by 'identical' (p.120 and also 121), although already in the second *Lieferung* of the *Beyträge*, *Allgemeine Mathesis* §13, Bolzano explains the difference between equality and identity.[4] Let us look more closely at numbers. To obtain the number 2, one has to take "an object which is equal to the previous member with a new unit of type A" (p. 120 corrected). If A_1 is 1 of the sort A (in Lapointian: of the type A), 2 will be the sum

$$S_2 = A'_1 + A_2 = \{A'_1, A_2\}$$

with A'_1 equal (equivalent) to, but different from A_1. Each number thus contains different units of the sort A and Lapointe's imaginary problem of non-redundancy (p. 121-122) vanishes. To my knowledge, the only correct treatment of this question can be found in Sebestik 1992, p. 346, and in Simons 1999, 223-224.

What follows is an excursion into Bolzano's ontology: the question of universals, the concepts of adherence and the important distinction between objective ideas and the subjective mental states.

The last two chapters deal with Frege and Husserl whose theories are closely connected with those of Bolzano.

The resemblance between Bolzano and Frege is striking, especially with regard to the *Sinn* of sentences and to their grasping, although “there is no evidence whatever that Frege ever read Bolzano” (Dummett 1996, quoted p. 130). I agree with the author's statement, which takes into account the situation of communication, that Bolzano's views “are vastly richer than Frege’s” (p. 130).

With Husserl, the situation is different, because Bolzano played the role of a powerful ally in the criticism of psychologism in the *Logical investigations*. On the other hand, it was Husserl who first drew the attention of philosophers to an author whose logic “far surpasses everything that world literature has to offer in the way of systematic sketch of logic” (Husserl 1970, § 61, p. 222). Contrary to John Stuart Mill, Husserl considers pure logic as a normative discipline “which rests on one or more theoretical disciplines” (quote p. 145). Lapointe widens the debate to include the questions relative to inference and *Begründung*, and reminds Husserl's objections to Bolzano's empiricism in the theory of knowledge.

One cannot but subscribe to her judgment: “Bolzano was not merely a great anticipator; he was also a formidable analyst” (p. 7).

In spite of some controversial passages and errors, and an amount of carelessness, her book is well organized and is full of significant insights—all major Bolzano's logical theories are discussed—and deserves to be carefully studied and confronted with Bolzano’s texts. I hope that it will spark interest in Bolzano and promote further study of his philosophy. It also shows how difficult is the undertaking to read him carefully and to understand his definitions, not to speak of their interpretation in the light of 21st century logic. It contains important notes. Last but not least: the quotations from Bolzano make a precious anthology of his most important passages.

Finally I want to add few words on the problems of terminology and translation.[5] Lapointe practices sometimes misplaced purism: she uses the German words *Gültigkeit*, *Ableitbarkeit* and *Abfolge* without English equivalents although Mill's *System of Logic*, published six years after the *Wissenschaftslehre*, has an occurrence of *validity*, and *ableiten* is a German translation of Latin *deducere.* Why are current translations “misleading”? (p. 163, note 4). On the other hand, she negligently translates *gleich* by *identical*, *ähnlich* by *isomorphic.*

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Notes

[1] I wish to thank Paul Rusnock for his acute criticism and for his helpful suggestions.

[2] Rosenkranz, announced his verdict after the publication of Bolzano 1837 whose last §718 contains a severe critique of the dialectical method and also of Rosenkranz' book on Hegel.

[3] When the premises are equivalent to the conclusion, there might be doubts about *formale Abfolge.* In these cases (e.g. II, §225, 6, §227, 3, and elsewhere), Bolzano describes the inference and comments that the conclusion is also the consequence (*Folge*) of the premises.

[4] French translation in Bolzano 2010, p.148.

[5] On June 20–21, 2013, a round table on the problems of translation was held at the Bolzano conference in Clermont-Ferrand.

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