Journal for the History of Analytical Philosophy

Volume 11, Number 4

ISSN: 2159-0303

jhaponline.org

Carnap's Geometrical Methodology: Explication as a transfer principle

Matteo De Benedetto

In this paper, I will offer a novel perspective on Carnapian explication, understanding it as a philosophical analogue of the transfer principle methodology that originated in nineteenth-century projective geometry. Building upon the historical influence that projective geometry exerted on Carnap's philosophy, I will show how explication can be modeled as a kind of transfer principle that connects, relative to a given task and normatively constrained by the desiderata chosen by the explicators, the functional properties of concepts belonging to different conceptual frameworks. Moreover, I will demonstrate how, in light of this characterization, we can better appreciate the evolution of Carnap's metaphilosophy.

Carnap's Geometrical Methodology: Explication as a transfer principle

Matteo De Benedetto

1. Introduction

The last two decades have witnessed a revival of interest in Carnap's philosophy (e.g., Friedman 1999; Carsten and Awodey 2004; Friedman and Creath 2008; Leitgeb and Carus 2020). As a consequence of this recent historical scholarship, the concept of explication is now considered a pillar of Carnap's mature thought (see Carus 2007; Wagner 2012). The originality of explication as a philosophical method has also been appreciated in recent discussions of conceptual engineering and constructive methodologies in analytic philosophy (e.g., Justus 2012; Brun 2016; Cappelen, Plunkett and Burgess 2020). Despite attracting all this interest, the exact nature and scope of Carnapian explication remain unclear.

In this paper, I will offer a novel interpretation of Carnapian explication by virtue of an analogy with an established mathematical methodology. Specifically, I will argue that explication can be understood as a philosophical analogue of the transfer principle methodology that originated in nineteenth century geometry (Gray 2007; Schiemer 2020a). Transfer principles are analytically defined mappings between different mathematical domains that preserve certain projective properties of mathematical objects. I will argue that Carnapian explication can be seen as a philosophical analogue of such principles that preserves certain functional properties of concepts across different conceptual frameworks. In the light of this interpretation, I will show how we can better understand the evolution of some important features of Carnap's metaphilosophy, such as his structuralism and the exceptionality of logic in his thought.

In Section 2, I will present Carnapian explication. In Section 3, I will focus on the methodology of transfer principles, sketching its history by focusing on three important moments: projective duality, Hesse's principle of transfer, and Klein's transfer by mapping. In Section 4, I will trace the historical connections between projective geometry and Carnapian explication. In Section 5, I will show how Carnapian explication can be modeled as a philosophical transfer principle. In Section 6, I will stress the implications of my interpretation of Carnapian explication for our understanding of Carnap's overall metaphilosophy. Finally, I will draw some general conclusions on what my proposal achieves.

2. Carnapian Explication

In this section, I will present Carnapian explication, i.e., the philosophical methodology with which Carnap explicitly identifies his later work. In his earlier works, Carnap conceptualizes his efforts as examples of rational reconstruction; starting in *Meaning and Necessity* (1947) he first introduces the notion of explication. Biographically, the explication period corresponds to Carnap's life in the USA. Intellectually, the methodology of explication corresponds to Carnap's increasing focus on inductive logic (Sznajder 2018) and it is paradigmatically exemplified by the work contained in the *Logical Foundations of Probability* (1950b). The first chapter of this book is, in fact, explicitly devoted to presenting the procedure of explication, while the work on the concept of probability contained in the rest of the book is meant as an example of philosophical explication. Carnap presents explication with the following words:

> By the procedure of *explication* we mean the transformation of an inexact, prescientific concept, the *explicandum*, onto a new exact concept, the *explicatum*. Although the explicandum cannot be given

> in exact terms, it should be made as clear as possible by informal explanations and examples. (Carnap 1950b, 3, original emphases)

Explication involves then the transformation of an inexact concept into a more exact one. More accurately, as Carnap makes clear in his reply to Strawson's critique of explication (Strawson 1963; Carnap 1963b), the exactness of a concept has to be understood relative to a certain task or goal. Explication then replaces a certain concept, inadequate for a certain task, with another, more adequate concept. This dependency on a given task or goal distinguishes explication from other philosophical methodologies that have a more absolute character, such as (most types of) conceptual analysis (see Carus 2012a).

Explication is traditionally seen as a two-step procedure.[1] First of all, one has to clarify the explicandum, trying to explicitly state the intended meaning of the concept that one wants to explicate. Since the explicandum is usually expressed in a natural language, an exact definition is not required. What Carnap (1950b, 3–5) requires from the explicator, instead, is to state some positive and negative instances of the explicandum, together with some description or (partial) rules of use. This step clarifies and (if necessary) disambiguates the concept that one seeks to explicate. It is, in fact, possible for the explicator, in trying to clarify the explicandum, to realize that there are two or more different concepts that are ambiguously grouped in natural language within a single notion (see Quinon 2019). A famous example of this phenomenon occurs in Carnap's (1950b) explication of probability. In clarifying this concept, Carnap in fact realizes that behind the intuitive understanding of probability lie two different notions: the logical and the frequentist concepts of probability. In clarifying the explicandum, the explicator also freely chooses the context of the explicandum that she wants to explicate. It is, in fact, often the case that a given explication wants to replace only some contexts or uses of the intuitive notion. A paradigmatic example of this decision of context is Tarski's (1933) opening remark before his explication of the concept of truth, where he states that he is interested in explicating the context of truth-assertions like "'snow is white' is true" and not in explicating uses such as "you are a true friend".

The second step of explication involves the formulation of the explicatum in a certain target theory via an explicit definition or by stating its rules of use (Fig. 1).

$$\underset{\text{intuitive concept}}{ED} \xrightarrow{\text{clarification}} \underset{\text{clarified concept}}{ED^*} \xrightarrow{\text{formulation}} \underset{\text{explicatum}}{ET}$$

Figure 1: The two-step structure of Carnapian explication.

The purpose of explication is the substitution, relative to a specific function-context, of a less satisfactory concept with a (more) satisfactory one. However, this substitution is always partial, since the explicandum plays a crucial role in the assessment of the overall success of an explication and, as such, it is never replaced entirely by its successor. This is the so-called dialectical or open-ended character of explication that has been highlighted by several scholars (see Stein 1992; Carus 2007, 2012b; Uebel 2012). Explication is, moreover, an inherently pragmatic procedure, i.e., its adequacy is not a matter of rightness or wrongness, but of what is more or less satisfactory for the task that the explicator has in mind. Judging this adequacy is, then, never an all-or-nothing matter. The explicator has always a certain degree of freedom in choosing the explicatum for substituting a given concept. In Carnap's (1950a) late terminology, as Stein stresses, questions about explication adequacy are thus external questions:

> The explicatum, as an exactly characterized concept, belongs to some formalized discourse—some 'framework'. The explicandum... belongs ipso facto to a mode of discourse outside that

[1] Note, however, that in certain complex cases, explication arguably involves an additional mid-step. For details, see De Benedetto (2021).

framework. Therefore any question about the relation of the explicatum to the explicandum is an 'external' question; this holds, in particular, of the question whether an explication is adequate. (Stein 1992, 280).

The adequacy of an explication is, thus, an external question, bound to be pragmatically discussed, relative to a given goal and context, outside scientific frameworks with the normative tools of instrumental rationality (Carus 2007, 2017; Steinberger 2016).

Even though explication is not a matter of being right or wrong, one can still judge whether an explication is a good one or a bad one. Relative to a specific purpose or function, one can state certain pragmatic meta-principles that a concept has to respect in order to qualify as a good explicatum for a certain explicandum. Carnap (1950b, 5–8) stated four desiderata that a good explicatum has to respect:

- Similarity: to the extent to which the other desiderata allow it, the explicatum ought to be similar to the explicandum (exact similarity, i.e., identity, is explicitly not required).
- Fruitfulness: the explicatum ought to be connected with other scientific concepts, in order to make as many generalizations as possible expressible within the theory in which it is framed.
- Exactness: rules of use of the explicatum ought to be stated in an exact form (e.g., definitions, axioms).
- Simplicity: the explicatum ought to be as simple as the other desiderata allow it to be.

These principles give a hint of the virtues that a good explicatum has to possess, but they are intrinsically pluralist in their intent.[2] Carnap, in fact, stresses that it is always possible to have different explicata that are equally adequate with respect to a given explicandum. The adequacy of a given explication is always relative to the goal of the explicators and always dependent on the desiderata that explicators want to impose. In assessing this adequacy, the original explicandum works as a central measure of the satisfactoriness of the explicatum. Just like in engineering sciences, the satisfactoriness of a certain tool can be judged only with respect to its goals, its predecessors, and its alternatives (see Richardson 2013). This centrality of the explicandum in the assessment of the overall success of an explication allows what Carus (2007; 2012b) calls the "feedback-relation" between evolved and constructed languages in Carnap's metaphilosophy. Formally constructed languages, in fact, offer replacements (i.e., explicata) for particular parts (i.e., explicanda) of evolved ones, which are judged externally to the constructed frameworks by the pragmatic mode of discourse typical of evolved languages. The procedure of explication can then be seen as a bridge between different (types of) conceptual frameworks. Explication bridges different frameworks in an inherently pluralist and goal-dependent way, connecting parts of different languages that can perform a similar function with respect to a specific problem at hand.

3. The Methodology of Transfer Principles

We saw in the last section Carnap's method of explication and its characteristics. In Section 5, I will argue that explication can be seen as a philosophical analogue of a certain kind of geometrical methodology, i.e., the methodology of transfer principles. In this section, I will present this geometrical methodology, from both an historical and a theoretical point of view.

[2] The exact nature of these principles is not so clear, as it is shown by the many different ways in which philosophers have tried to further specify these desiderata (e.g., Brun 2016, 2020; De Benedetto 2022; Dutilh Novaes and Reck 2017; Hanna 1967; Justus 2012; Quinon 2019; Reck 2012).

Roughly speaking, transfer principles are principles that state a systematic correlation between the projective properties (and the related truths and theorems) of one domain of mathematical objects and the projective properties of some other domain of mathematical objects. Historically, the use of transfer principles in geometry constituted an important strand of nineteenth-century mathematical methodology closely connected with the rise of projective geometry and the related abstract-turn of geometry (Gray 2007, 2008). In order to present the methodology of transfer principles, I will describe here three important moments in the history of such principles: the duality principle (Eder 2021), Hesse's principle of transfer (Hesse 1866a,b), and Klein's transfer by mapping (Klein 1872).

The first step in our brief history of transfer principles in projective geometry is the principle of projective duality (Gray 2007, 53–62). This principle, in its planar version, states that by interchanging the words "line" and "point" in any theorem of planar geometry we get a second theorem, somehow dual to the original one. As an example, we can look at the relation between Pascal's theorem and Brianchon's theorem:

Pascal's Theorem. If a simple hexagon is inscribed in a nondegenerate conic, then the points of intersection of its opposite sides are collinear.

Brianchon's Theorem. If a simple hexagon has elements of a nondegenerate line conic for its sides, then the lines joining opposite vertices are concurrent.

Brianchon's theorem can be obtained from Pascal's theorem by substituting every (implicit and explicit) occurrence of the word "point" by the word "line", and *vice versa*. Thus, we say that the two theorems are duals of each other. The duality principle constituted a major milestone in the rise of projective geometry (see Hawkins 1988). One of the reasons of its importance is that this principle constitutes one of the first meta-theoretical principles of modern geometry. The duality principle does not, in fact, state a relation between mathematical objects, but between theorems. This meta-theoretical status of the duality principle was recognized also by the projective geometers of the time, who discussed at length the possible applications and justifications of this principle (see Gray 2007; Eder 2019).

In the history of transfer principles, the principle of duality occupies an important place for at least three reasons. First, the principle of duality can be seen as a proto-example of a transfer principle, establishing a systematic correlation between the set of theorems of planar geometry and the set of their duals by virtue of the interchangeability of basic geometrical elements such as points and lines. Despite the fact that projective duality connects objects within one single mathematical domain (and not, as mature examples of transfer principles, across different domains), this principle champions the idea of systematically correlating the projective properties of mathematical objects through adequate mappings, an idea that, as we shall see, constitutes the conceptual core of the transfer principle methodology. Secondly, the duality principle is important methodologically for the history of transfer principles due to its meta-theoretical status, i.e., because it explicitly focuses the attention of geometers on second-order connections between the properties of objects (e.g., theorems, truths) and not, as it was customary, on first-order connections between geometrical elements (see Tappenden 2005; Eder 2021). Third, historically, the geometrical work on transfer principles arises from the need to justify duality phenomena and the tentative extension of the related method of dualization to different mathematical domains (see Gray 2007, 161–72, Eder 2021; Schiemer 2020a).

The next stop in our brief history of transfer principles is Hesse's (1866a) work in analytic geometry and, in particular, the specific principle of transfer that he introduces in Hesse (1866b). Hesse's principle states that every point in the complex projective plane can be mapped to a pair of points in the complex projective line (and *vice versa*). Thanks to this mapping, theorems about pairs of points on a line can be transformed into theorems about points in a plane (and *vice versa*), and, as Hesse (1866b, 15) puts it, thereby the geometry of the plane and the geometry of the line can be reduced to each other. Hesse understands transfer principles such as his own as generalizations of projective duality that map projective invariants across different mathematical domains. This search for projective invariants was justified by Hesse's understanding of duality phenomena, which, in line with the analytic kind of geometry championed by Plücker (see Gray 2007, 166–77), understands projective duality as a specific example of the general possibility of analytically reinterpreting geometrical configurations, understood as possible interpretations of abstract equations. Such an analytic understanding of duality phenomena and transfer principles is evident in Hesse's (1866b, 15–16) own presentation of his principle of transfer, where the principle is characterized via a quadratic equation that specifies the mapping function between points in the plane and pairs of points in the line.

Hesse's principle of transfer represents an important example of a a transfer principle, establishing a systematic correlation between the projective properties of one domain of mathematical objects (i.e., pairs of points on a line) and the properties of another domain (i.e., points in a plane). Hesse's principle constitutes moreover a step towards abstraction and generality in our brief history of transfer principles, since Hesse explicitly stresses the virtues of his methodology as rooted in its meta-theoretical and abstract nature. Transfer principles like Hesse's own principle of transfer allow, according to Hesse, geometers to reduce geometries to one another, isolating in this way the projective properties common to different geometrical configurations.

Finally, the third moment in the history of transfer principles that I am going to describe in this section is Felix Klein's method of transfer by mapping (Klein 1872). This method represents Klein's notion of geometrical equivalence and it can be spelled out, in modern terms, as follows. According to Klein, a geometry G can be characterized as a tuple $\langle A, B\rangle$, made of a manifold A and a transformation group B acting on the manifold. Then, for any two geometries G, G', Klein's transfer by mapping method states that the two geometries are equivalent if and only if there exists a mapping between the manifolds of the two geometries $f : A \rightarrow A'$ that induces an isomorphism between the corresponding transformation groups B, B'. Thanks to such a mapping, theorems of one geometry can be translated into theorems of the other geometry (and *vice versa*). Klein saw this method as a generalization of both Hesse's principle of transfer and projective duality, since it prescribes a general abstract rule for transferring geometrical information from one domain to the other, that is, a general method for justifying and obtaining specific transfer principles between two geometrical domain. Klein (1872, 224–25) actively exemplifies his method of transfer by mapping by presenting several specific examples of transfer principles, such as, for instance, the one establishing the geometrical equivalence between pairs of points on a conic and the plane whose straight lines intersect them. From Klein's perspective, the method of transfer by mapping showcases the methodological virtues of the Erlangen program and its group-theoretic conception of geometry (Wussing 1984). The transfer by mapping method encapsulates, in fact, the central idea of the Erlangen program that all that there is of geometrical importance in a geometry can be encoded via group of transformations and some abstract algebraic relations between these groups (see Biagioli 2020). This geometrical knowledge can then be transformed into other geometrical knowledge by virtue of abstract mappings, establishing mathematical and epistemological relations between geometries. It is in this sense that the method of transfer by mapping can be seen as the central methodology of the Erlangen program's efforts in geometrical classification.

Klein's method of transfer by mapping generalizes both Hesse's principle of transfer and the principle of projective duality. Klein's method establishes, in fact, a formal correlation between the geometrical properties of any two geometries (understood as the combination of a formal manifold and a transformation group acting on this manifold), making, from Klein's perspective, both the duality principle and Hesse's principle of transfer two specific cases of Klein's method. Seen from the abstract perspective of the Erlangen program, in fact, all the specific transfer principles and related duality phenomena studied by projective geometers are just specific instances of the general idea of connecting the geometrically relevant properties of different mathematical domains via suitable algebraic mappings.

We have then briefly seen three important moments in the history of transfer principles. The evolution of the methodology of transfer principles sketched above shows us a trend towards an increasing abstraction and generality in the systematic correlations between the projective properties of mathematical domains prescribed by these principles. We saw, in fact, how, at first, in the duality principle, the correlation inscribed in this proto-example of a transfer principle amounts to the mere interchangeability of two basic elements of a single geometrical domain. Moreover, the metatheoretical properties and the justification of such correlation remain, mostly, implicit. By looking at Hesse's principle of transfer, we saw instead an example of a full-fledged transfer principles, correlating different mathematical domains. Moreover, Hesse makes explicit methodological characteristics and advantages of transfer principles like his own, such as the economy and the epistemological reduction provided by it. Finally, in Klein's general method of transfer by mapping we find a whole conception of geometry and geometrical equivalence that is built upon abstract transfer principles. According to the Erlangen Program, specific transfer principles can be developed to transfer mathematical knowledge from a given geometry to another geometrically-equivalent domain, by virtue of completely abstract relations between two formal manifolds and the related transformation groups acting on them.

Despite the differences in the degree of generality and abstraction exhibited by the examples of transfer principles that we briefly analyzed in this section, we can see a common methodology embodied by transfer principles, in their many different applications and instantiations in nineteenth-century projective geometry. The methodology embodied by transfer principles centers around the search for invariant projective properties of different geometries. This search is carried out by abstract mappings that preserve the underlying projective structure of different geometrical configurations. By virtue of these mappings, geometrically relevant properties such as theorems, truths, and projective invariants can be transferred from one geometry to another one. In this way, the methodology of transfer principles pushes geometers to focus their attention on the preservation of invariant structures across different geometries, downgrading at the same time the geometrical significance of the properties and the nature of specific geometrical elements. This emphasis on structure-preserving mappings and the related indifference for the nature of geometrical elements fostered by the methodology of transfer principle make this nineteenth-century methodology an important precursor of modern mathematical structuralism (see Reck and Price 2000; Gray 2008; Schiemer 2020a).

4. The Influence of Projective Geometry on Explication

We have seen now both methodologies at the center of this paper: Carnapian explication and transfer principles. It is now time to see how these two methodologies are historically related to each other. Specifically, in this section I will show how the kind of geometrical thinking exemplified by the methodology of transfer principles influenced Carnapian explication.

The influence that the kind of geometrical thinking exemplified by transfer principles exerted on Carnap's explication can be divided into two kinds: a direct and an indirect kind of influence. The methods and ideas of projective geometry influenced directly Carnap's development of explication, as some textual evidence in Carnap's explication writings and a more general assessment of Carnap's intellectual development arguably show. Furthermore, the specific kind of geometrical thinking at work in the transfer principle methodology shaped the main philosophical and logical traditions inside which Carnap's thought developed. Through this cluster of intellectual connections, projective geometry arguably influenced also indirectly Carnap's development of explication.

The direct influence of projective geometry can be traced throughout the whole development of Carnap's thought. The obvious place to start is Carnap's own geometrical work, i.e., his dissertation *Der Raum* (1922). In his dissertation, Carnap seeks to clarify the different concepts of space (i.e., formal, intuitive, and physical) and their possible mathematical versions. The importance of Carnap's geometrical work for understanding Carnap's philosophical methods and ideals has been stressed by many scholars (e.g., Richardson 1997, 2003; Mormann 2008). Carnap's favorite strategy of rationally reconstructing a given phenomenon and distinguishing several specific conceptual alternatives for (dis)solving a given philosophical problem is, in fact, already at work in Carnap's distinction and reconstruction of different concepts of space. Moreover, the mathematical tools and ideas of projective geometry, such as the fundamental concept of implicit definitions or the many different structural characterizations of a space, continue to be central elements of Carnap's technical toolbox for all his career (see, Richardson 2003). On a more general level, geometrical conventionalism has a major role in shaping Carnap's own conventionalism about meaning and his related views on languages, frameworks, and analyticity (see Coffa 1986; Creath 1992; Mormann 2008).

On top of the general influence that projective geometry exerted on Carnap's thought, projective geometry arguably also influences Carnap's specific development of the explication methodology. In his most detailed presentation of explication, i.e., in the first chapter of the *Logical Foundations of Probability*, Carnap (1950b) acknowledges in fact a certain debt in coming up with the idea of explication to Karl Menger's (1943) work on geometrical definitions. Menger (1979a) himself, in the introduction to the second volume of his collected papers, stresses how his remarks about geometrical definition influenced Carnap, who considered them "paradigms of what he called the explication of concepts as well as of the treatment of explications as such" (Menger 1979a, 7).

Indeed, looking at Menger's (1943; 1979b) papers on geometrical definitions, one can find many methodological similarities between his reflections on geometrical definitions and Carnap's description of explication. Menger stresses, in fact, how, in technically defining a given geometrical concept, one ought to stay faithful to some core uses of the everyday counterpart of the concept under focus. This is what Menger (1979b, 208) calls the formal postulate of geometrical definitions and it clearly resembles Carnap's similarity desideratum for explication. At the same time, Menger (1979b, 208–209) stresses how geometrical definitions need to satisfy another postulate, i.e., the material postulate, that prescribes that these definitions ought to maximize fruitfulness and to allow us to create new interesting mathematics. This goal is exactly the one encapsulated by Carnap's desideratum of fruitfulness. On top of the similar methodology and constraints between Menger's discussion of geometrical definitions and Carnap's explication, there is also a more general agreement in the pluralist and voluntarist spirit of the two projects. Menger (1979b, 209) highlights, in fact, how any geometrical definition is always somewhat arbitrary and how there is no absolute right or wrong in offering a new definition of a concept, but just a matter of fruitfulness and satisfactoriness of the new tools. As we saw in Section 2, a similar pragmatist and pluralist spirit pervades Carnap's presentation of explication.

Menger's influence on the development of explication arguably demonstrates how the original domain of the ideas that influenced Carnap's development of explication is geometry. Moreover, Menger's ideas on geometrical definitions exemplify the kind of methodological thinking, typical of modern geometry, that sees specific elements of a geometrical domain as mere possible configurations of geometrical entities that are often recombined through abstract mappings. This emphasis on the abstract recombination of geometrical elements can be seen at work in Menger's (1979b, 207) example of what a good geometrical definition ought to be, i.e., the notion of dimension. Menger (1979b, 210–11) first shows how traditional characterizations of geometrical dimension, such as, for instance, quantity-based ones are inadequate in modern geometry, since elements of allegedly different quantity such as lines, planes, and cubes can be mapped into a 1-1 correspondence to each other by virtue of suitable abstract mappings that preserve the geometrically relevant properties. Then, Menger (1979b, 213) argues that his own set-thoretic definition of dimension is instead useful to modern geometers, since its classification of the dimension of geometrical objects respects (most uses of) our intuitive concept of dimension, while being based upon structurally invariant features. In this way, Menger's case study on the notion of dimension exemplifies how, in modern geometry, geometrically fruitful notions are the ones the characterization of which is based upon structurally invariant properties of geometrical objects. Thus, Menger's work on geometrical definition emphasizes the possibility of recombining geometrical elements through abstract mappings while preserving geometrically relevant properties, an idea that, as we saw in the last section, was paradigmatically championed in projective geometry by the methodology of transfer principles.

On top of the direct connections between the kind of geometrical thinking exemplified by the methodology of transfer principles and Carnapian explication, there are also several strands of indirect influence that the former arguably exerted on the latter. Projective geometry and its methodology had in fact a huge influence in shaping both major philosophical traditions that constituted Carnap's philosophical background, i.e., neo-Kantianism and logicism. For what concerns neo-Kantianism, the centrality of nineteenth-century geometry in neo-Kantian philosophy has been stressed by several scholars, especially in relation to the cluster of problems posed by non-Euclidean geometries and the related structural/abstract turn in geometrical knowledge for broadly Kantian views of mathematical knowledge (see Coffa 1991; Richardson 1997; Friedman 2000). In relation to logicism, instead, recent scholarship has stressed the importance of projective geometry and specifically of projective duality for understanding Frege's philosophical background and his logicist ideal (see Wilson 1992; Tappenden 2005, 2006; Eder 2019). Moreover, modern logic itself, especially in the model-theoretic tradition, is built upon the conceptual toolbox of projective geometry. Fundamental aspects of model-theoretic thinking, such as the idea of re-interpreting non-logical language, can be seen as direct generalizations of the emphasis that projective geometers put in recombining algebraic relations between geometrical elements through transfer principles (see Eder 2021). This connection between model-theoretic thinking and projective geometry is particularly important for Carnap's explication for two reasons. First, as reconstructed in detail by Awodey and Carus (2003, 2007, 2009) and Carus (2007), the ideal of explication arose in Carnap together with the re-discovery of semantics and the related break-out from the syntax cage. Thus, Carnap's explication should be seen as closely connected with the tools and the ways of thinking of formal semantics. Secondly, Carnap always held the model-theoretic construction of a logical language as a paradigmatic example of a conceptual or linguistic framework. Since, as we saw in Section 2, the method of explication is intertwined with the centrality of frameworks in Carnap's philosophy, it seems natural to assume that, if projective geometry strongly shaped model-theoretic notion of truth in a structure, this background also indirectly affected Carnap's methodology.

To sum up, we have seen how the methodology of transfer principles has several interesting historical connections with the development of Carnap's explication. Specifically, we saw the rich geometrical context of Carnap's philosophical methodology and we noticed that Carnap explicitly acknowledged an intellectual debt in developing explication to Menger's work on geometrical definitions, a work that emphasizes the fruitfulness of characterizing geometrical notions in a structurally invariant way. Moreover, we recalled how the results and methods of projective geometry shaped the philosophical background and practice of both Neo-Kantianism and Logicism, i.e., the two major philosophical traditions inside which Carnap's thought developed. Finally, we described how projective duality and the methodology of transfer principles shaped many central concepts of model-theory, a part of modern logic that was pivotal in Carnap's development of the ideal of explication.

5. Explication as a Transfer Principle between Conceptual Frameworks

We saw, then, the rich historical connections between the methodology of transfer principles and Carnapian explication. I will now demonstrate how explication can be adequately understood and modeled as a philosophical transfer principle.

Before modeling Carnapian explication as a kind of transfer principle, I must briefly recall the description of explication that I gave in Section 2. The main characters of explication are concepts. Explication involves the transformation of a concept, the explicandum, onto one or many concepts, the explicatum (or explicata, if more than one). This transformation is meant to have, as an output, the partial replacement of the explicandum with an explicatum, with the idea that the latter is a more adequate tool for a given task than the former. A fundamental caveat of explication is that the explicandum and the explicatum belong to different frameworks. Thus, explication connects concepts from different frameworks with the idea that they can perform a similar role in helping us to solve a given philosophical or scientific task. This connecting role between different frameworks is key for understanding how explication can be seen as a transfer principle. Remember, in fact, the informal characterization of transfer principles that we saw in Section 3: principles that state a systematic correlation between the projective properties of one domain of mathematical objects and the projective properties of some other domain of mathematical objects.

This characterization of transfer principles can be applied to describe explication, as well. In order to do that, we need to change the specific terms involved in the definition of transfer principles with their conceptual analogues in Carnap's metaphilosophy. First, Carnapian explication does not involve objects, but concepts. Moreover, explication does not connect (only) mathematical domains, but more generally conceptual frameworks of various kinds. As stressed by Carnap (1963b) in his reply to Strawson in the Schilpp volume, the intended scope of explication is the whole domain of philosophical and scientific frameworks, ranging from very informal frameworks loosely describing parts of natural language to completely regimented formal languages reconstructing parts of exact sciences. Finally, and most importantly, explication does not correlate the projective properties of the concepts involved, but what we can call the functional properties of the explicanda and the explicata, i.e., the functions that these concepts are able to perform in relation to a given philosophical or scientific task. As we saw in Section 2, Carnap stresses several times that the overall goal of explication is to partially replace a given concept with another one that is a more adequate tool in the context of a given task. Thus, the explicatum ought to perform roughly the same function that the explicandum used to perform, but in a better way, that is, in a way that more adequately satisfies the goals and the values of the explicators (see Carnap 1963b, 966).

This emphasis on the functional properties of concepts can be seen more clearly by looking at specific cases of explication. Take, for instance, Tarski's (1933) explication of truth. Tarski's goal, in explicating certain uses of our intuitive notion of truth, was to find a formal predicate that was able to perform the same meta-semantic function of expressing the correspondence between a given proposition and a given fact of the world that our informal notion of truth performs in our intuitive truth-talk (a function that Tarski encapsulated in the T-schema, see Tarski 1933; Horsten 2011). At the same time, Tarski wanted a truth predicate that was fully formalized, in order for using it for talking about truth within fully formalized languages, and that was completely devoid of any metaphysical connotation. The same emphasis on the functional properties of concepts can be seen guiding Turing's (1936) explication of our intuitive notion of computation. Turing's search for a formal theory of computability can be in fact understood as the explication task of finding a formal notion that performs the function of singling out the class of problems that can be solved by a given (idealized) agent with the non-ingenuous recursive use of simple operations, a function that our intuitive talk of effective procedure performs in our language (and a function that Turing encapsulated in his informal axioms for computability, see Turing 1936; Sieg 2002; De Benedetto 2021). At the same time, Turing wanted a notion of effective calculability that was fully formalized, so that it would be possible to quantify over the whole range of effectively calculable functions in order to decide what could not be done by effective means (solving in this way several connected problem such as the *Entscheidungsproblem*). A third specific example of the functional properties of concepts related by explication can be seen by looking at the invention of the scientific concept of temperature, understood as an explication of our intuitive concepts of warmer and colder (see Carnap 1950b). Seen in this way, the scientific concept of temperature performs roughly the same functions of our intuitive concepts (e.g., sorting phenomena based on their temperature), but in a way far more adequate for the goals and values of scientists (e.g., by being based upon an objective scale and by allowing richer measurement scales, see Chang 2004). Explication seems then a matter of correlating the functional properties of concepts, finding an explicatum that is able to perform, in a way more adequate to the goals and values of the explicators, roughly the same function that an explicandum performed relative to a given philosophical or scientific task.

In order to apply our general definition of transfer principles to explication, it seems that we need to make three major changes: replacing mathematical objects with concepts, mathematical domains with conceptual frameworks, and projective properties with functional properties relative to a given philosophical or scientific task. Thanks to this triple change, we get the following tentative definition of explication as a transfer principle:

> **Explication *qua* Transfer Principle** (tentative):
> Explication states a systematic correlation, relative to a given philosophical or scientific task, between the functional properties of the explicandum, belonging to a given conceptual framework, and the functional properties of the explicata, belonging to one or more other conceptual frameworks.

This tentative definition makes transparent the influence of projective geometry on Carnap's explication. The focus of transfer principles on structure-preserving mappings between different mathematical domains has a philosophical analogue in Carnap's emphasis on functional-properties-preserving mappings between concepts belonging to different philosophical and scientific frameworks. Moreover, also the other core methodological insight that transfer principles arguably exhibit, i.e., indifference to the nature of geometrical elements and the related emphasis on recombining and transforming mathematical objects, has a direct analogue in Carnap's emphasis on the necessity of replacing philosophical and scientific concepts and his related distrust of absolute metaphysical questions on the nature of philosophical entities. For Carnap, what matters are the functional properties of concepts that help us solve problems in science and philosophy. Thus, we must explicate our conceptual tools whenever we can, without any reverential fear of changing the tools and the language of philosophy (see Jeffrey 1994).

Yet, this tentative characterization of explication as a transfer principle leaves out an important component of Carnapian explication, i.e., its normative dimension. The systematic connection between an explicandum and an explicatum has an inherent normative dimension; the explicatum is, at least relative to the context/task chosen by the explicators, a better tool than the explicandum. We saw, in fact, this normative dimension of explication clearly present in all the three specific examples of explication we mentioned before: truth, computability, and temperature. Relative to the goals and values of the explicators, all three explicata (i.e., Tarski's T predicate, Turing computability, and the scientific concept of temperature) are more adequate concepts for the task at hand than their respective explicanda (i.e., our intuitive concepts of truth, effective calculability, and warmer-colder), because they enjoy certain properties that their predecessors do not possess (respectively, being framed within a fully formalized language and being devoid of any metaphysical connotations; allowing for universal quantification over the whole range of computable function; and being based upon objective properties of bodies and allowing for richer measurement scales). This normative dimension of explication does not have a clear analogue in the transfer principle methodology, which, as we saw in Section 3, involved symmetric principles between different mathematical domains. Moreover, the normative dimension of explication creates often another asymmetry between the explicandum and the explicatum that is also left out by the above tentative definition: the explicandum is often an informal concept. As such, the framework of the explicandum is often only what we can call a quasi-framework, i.e., a vaguely, open-ended, informally characterized part of an evolved language. From a quasi-framework, no precise mapping can be drawn, but only perhaps a quasi-mapping, i.e., an informally characterized function with a vague domain. This asymmetry between the precision of the frameworks to which the explicandum and the explicatum belong seems to constitute another asymmetry between the procedure of explication and the transfer principle methodology, since the latter methodology usually connects well-defined mathematical domains to well-defined mathematical domains.

Explication, then, differently from a transfer principle, is often a normative bridge between different kinds of frameworks and, because of that, it seems to be more than a mere matter of mappings. Although the whole procedure of explication cannot be reduced to a pure matter of mappings, systematically correlating the functional properties does indeed constitute a central part of it. The question becomes now how to improve our tentative definition of explication as a transfer principle in order to adequately include also the two missing components we identified: the normative dimension of explication and the vague characterization of the quasi-framework of the explicandum.

The solution to our second issue, i.e., how the quasi-framework of the explicandum gets characterized, is hinted by Carnap himself, in his discussion of the two different steps that constitute the procedure of explication. As we saw in Section 2, in fact, explication is traditionally divided into two steps: the clarification of the explicandum and the formulation of the explicatum. The first step points us to a possible solution to our issue. In the clarification step, according to Carnap, the explicators ought to clarify and disambiguate the explicandum, choosing at the same time the specific context on which the explication will focus. By clarifying, disambiguating, and choosing the context of the explicandum, the explicators also implicitly characterize the quasi-framework that will constitute the domain of the mapping between the functional properties of the explicandum and the ones of the explicatum. The systematic correlation drawn by explication does not start with the explicandum, but with the clarified explicandum. This change in the domain of the mapping solves the issue of how the domain of the quasi-mapping behind explication is characterized.

A solution to the other issue we identified above, i.e., how to include the normative dimension of explication into our mapping-based characterization, can be also provided by recalling another important element of explication that was left out in our tentative definition, i.e., the desiderata that an explicatum ought to satisfy. We saw in Section 2, in fact, that the adequacy of an explication is not a yes-or-no matter, but it is instead a matter of relative satisfaction of the explicator in relation to the task for which a given explicatum is sought. This relative satisfaction is spelled out by Carnap in terms of an open-ended list of virtues that an explicatum ought to possess, i.e., the desiderata of an explication. Example of common desiderata are the aforementioned theoretical virtues listed by Carnap, such as similarity, fruitfulness, exactness, and simplicity.[3] These desiderata are Carnap's way of making precise the normative dimension of explication. An explicatum is a more adequate concept than the explicandum for solving a certain task because it satisfies the desiderata freely chosen by the explicators. The desiderata of an explication normatively constrain the mapping between the functional properties of the explicandum and the explicatum, that is, they determine which concepts, among those that can perform roughly the same function of the explicandum, are better than the explicandum for the task and values of the explicators. This pivotal role of explication desiderata can be seen at work in all three examples of explication we mentioned before. For instance, I mentioned before how, in explicating our intuitive notion of truth, the normative goal of Tarski was to find a concept of truth that could be applied to fully formalized languages and that was devoid of any metaphysical connotation. These goals can be represented as certain specifications of Carnap's exactness and similarity desiderata, by assuming that Tarski wanted an explicatum so exact that it could be formulated within a fully formal meta-theory and not similar to the intuitive notion of truth in its metaphysical uses (but only in disquotational uses). In this way, these desiderata can be seen as constraining normatively the possible explicata of our intuitive notion of truth, mapping the functional properties of the explicandum only to explicata that satisfy these (specifications of the) desiderata to a reasonable extent. This is why Tarski's chosen T-predicate is a good explicatum, relative to Tarski's goals and values, of our intuitive truth, while, say, Aristotle's correspondence-based notion of truth and Hegel's notion of historical truth are not. Despite all three of these notions of truth sharing with our intuitive notion of truth the functional properties that Tarski wanted to preserve (i.e., expressing the correspondence between propositions and facts of the world), only the T-predicate satisfies the desiderata that Tarski imposed on his explication (i.e., being so exact as to be implementable within a fully formalized language and being similar to the intuitive notion in the disquotational uses, but dissimilar in the metaphysical ones) and thus constitutes an improved concept for Tarski's goals. Analogously, Turing's normative goals in explicating our intuitive notion of effective calculability and the goals of the scientific community in explicating our intuitive temperature talk can be suitably represented as explication desiderata that normatively constrain the possible explicata of a given explication. The desiderata of an explication represent then the constraints that the normative dimension of explication imposes on the mappings between the functional properties of the explicandum and the explicata.

[3] Note here that, as I already stressed in Section 2, this list is extremely dependent on the values of the explicator and the nature of the given task. This can be easily seen by looking at the tentative precisifications of such theoretical virtues available in the philosophical literature. The majority of philosophers who tried to further specify the virtues that an explicatum ought to possess stressed in fact the intrinsic open-endedness and context-dependency of such virtues (e.g., Justus 2012; Brun 2016; Dutilh Novaes and Reck 2017; De Benedetto 2022).

By constraining the possible explicata in this way, the desiderata are formal constraints on the connection established by a given instance of explication. The desiderata constrain the possible values of the transfer-principle action of explication, making the quasi-mapping of explication an intensional mapping that includes, as formal constraints, the values and the goals of the explicators. This is how the desiderata of an explication solve our issue of including the normative dimension of explication in our transfer-principle-like above characterization. The quasi-mapping between the clarified explicandum and the explicatum is normatively constrained by the desiderata chosen by the explicators. The desiderata of an explication, freely chosen and tinkered by the explicator(s) relative to the philosophical or scientific task at hand, determine the acceptable explicata, the functional properties of which can be correlated with the ones of the clarified explicandum. The desiderata of an explication are, thus, analogous to topological or algebraic constraints that mathematicians can impose on correlations between different mathematical domains. Indeed, in formalizing the explication procedure within the theory of conceptual spaces (Gärdenfors 2000), explication desiderata become actual topological constraints on the mapping established by explication between an explicandum and its possible explicata (see De Benedetto 2022, 868–79).

Thanks to this analysis, we can now state an improved characterization of explication as a transfer principle that adequately includes the normative dimension of this procedure, by making explicit the pivotal role of the clarification step and of the desiderata for defining and constraining the domain and the mapping between the functional properties of the clarified explicandum and the ones of the adequate explicata:

Explication *qua* Transfer Principle:
Explication states a quasi-systematic correlation, relative to a given philosophical or scientific task and normatively constrained by the desiderata chosen by the explicators, between the functional properties of the clarified explicandum, belonging to a given conceptual (quasi-)framework, and the functional properties of the explicata, belonging to one or more other conceptual frameworks.

6. The Evolution of Carnap's Metaphilosophy

We saw how explication can be modeled as a philosophical transfer principle that correlates, relative to a given task and to the desiderata freely chosen by the explicators, the functional properties of concepts belonging to different conceptual frameworks. This characterization of explication *qua* transfer principle does not only highlight the historical influence that projective geometry exerted on Carnap's methodology, but it can also help us to understand better the evolution of Carnap's metaphilosophy. Specifically, in this section, we will see how, in the light of this interpretation, we can improve our understanding of the evolution of two central ideals of Carnap's philosophy, i.e., his structuralism and the exceptionality of logic in his thought.

Let us look first at Carnap's structuralism. Several scholars have stressed that much of Carnap's work is pervaded by an emphasis on the structural content of phenomena (e.g., Richardson 1997; Friedman 2011; Schiemer 2020b). Examples of this structuralism can be found in all the major areas in which Carnap worked, from epistemology, where he reconstructed cognitive phenomena via logically abstracting their structural relations (see Carnap 1928a,b), to philosophy of mathematics and philosophy of science, where he reconstructed the logical content of scientific theories as structural in character (Carnap 1934, 1966; Friedman 2011; Schiemer 2020b). The exact character of Carnap's structuralism eschews a unique characterization, changing from work to work consistently with his meta-theoretical views about language and logic (see Schiemer 2020b; Lavers 2016). Thus, in philosophy of mathematics, Carnap can be considered to be defending different structuralist theses in different works (see Schiemer 2020b), while in philosophy of science, Carnap hints at a kind of structuralist reconstruction of scientific theories akin to post-Kuhnian structuralists (see Friedman 2011; Psillos 1999). Such structuralist theses can be seen as stemming from Carnap's methodology of rational reconstruction, according to which rationally reconstructed content is always structural in character due to a certain kind of objectivity and rationality that the structural content of phenomena somehow embodies (the exact reason and framing of the thesis changes with the specific structuralist thesis at issue, see Schiemer 2020b). Despite the differences between them, then, all pre-Tolerance Carnap's structuralist theses involved what, in Reck's and Price's (2000) fine-grained classification of structuralist positions, we can classify as substantive or formalist kinds of structuralism, i.e., structuralist theses that give a non-neutral answer to the semantic and metaphysical implications of the structuralist methodology. More specifically, formalist kinds of structuralism give a negative/deflationary answer to ontological questions about mathematical structures, while substantive kinds of structuralism give a positive metaphysical story of the nature of such structures.

Yet, after the tolerance turn, Carnap cannot be considered a formalist nor a substantive structuralist, in any of the many different versions of such positions available in the philosophical literature. Any such form of structuralism would in fact constitute an unjust prohibition in direct contrast with the conventional freedom of the principle of tolerance. If, according to the principle of tolerance, we cannot set any prohibitions to how we explicate a certain phenomenon, then it seems that we cannot constrain the output of an explication to be structural in content. Moreover, the kind of deflationary meta-ontology championed by Carnap in the post-tolerance phase treats external questions as pragmatic matters, thus banning any form of non-neutral structuralism such as formalist and substantive structuralism. Nonetheless, even the after-tolerance Carnap often has a structuralist approach to its philosophical subject-matter (see Carnap 1966; Friedman 2011). The reconstruction of scientific theories and theoretical terms, for instance, is still conducted via a structuralist methodology that has a strong connection with the post-Kuhn structuralist philosophies of science. Theoretical terms and related scientific laws, according to Carnap, are still best reconstructed as certain kinds of abstract structures. Such a structuralist position has been criticized as constituting an evident violation of the freedom that the principle of tolerance proclaimed (e.g., Lavers 2016).

Is there any way we can reconcile Carnap's structuralist leanings with the almost unbounded pluralism of the principle of tolerance? Our characterization of explication as a philosophical transfer principle gives us an answer. We saw in Section 3, in fact, that an emphasis on the structural content of mathematical theories is strongly connected, both historically and conceptually, with the methodology of transfer principles. For these reasons, it has been argued that transfer principles embodied a kind of methodological structuralism (Schiemer 2020a; Reck and Price 2000), i.e., a neutral form of structuralism that emphasizes structures in its methodology. Methodological structuralism, i.e., the kind of structuralist methodology typical of modern mathematics that originated from pre-structuralist nineteenth-century methodologies such as transfer principles, pragmatically understands the study of mathematics as centered around the study of structures, rather than individual properties of certain objects, but it remains completely neutral to the semantic and metaphysical implications of such structure-centered inquiry. Since explication, as we saw in the last section, can be adequately modeled as a transfer principle, I propose that the key for understanding Carnap's after-tolerance structuralism is to understand it as a methodological structuralism. Carnap's philosophical methodology, i.e., explication, *qua* a philosophical transfer principle, I contend, is a structuralist methodology, in that it focuses on correlating the properties of concepts across different frameworks, disregarding the properties of individual concepts and individual languages. This kind of methodological structuralism that explication, *qua* transfer principle, shares with modern mathematical practice, provides then Carnap's philosophy with a structuralism compatible with the freedom prescribed by the principle of tolerance. The methodological structuralism of explication, in fact, just like the one embodied by transfer principles (see Schiemer 2020a), understands the study of structures as a pragmatic choice, remaining absolutely neutral on the semantic and metaphysical implications of this structure-centered inquiry. Such neutralism on metaphysical issues is also the key for understanding how Carnap's structuralist-leaning treatment of theoretical terms in science is compatible with his meta-ontological views, as Michael Friedman (2011) convincingly argued. Carnap's (Psillos 2000) epsilon-based reconstruction of theoretical terms as abstract structures is, in fact, completely neutral in the dispute between instrumentalism and realism, eschewing metaphysical discussion in favor of explication matters, consistently with what the methodology of explication prescribes.

We can see a second example of how understanding Carnapian explication as a philosophical transfer principle helps us to better understand the evolution of Carnap's philosophical ideals by looking at Carnap's logical exceptionalism. I already recalled, in Section 4, the great influence that logicism and the rise of formal logic has on Carnap's thought. It is not surprising then, that Carnap, throughout all his works, analyzes philosophical and scientific phenomena always through the lens of logic and that logical knowledge is always granted a special status in Carnap's philosophy. Examples of this exceptionality of logic can be found in almost every major work of Carnap, from the special status granted to logical abstraction and logical structures in the reconstruction of epistemological phenomena contained in the *Aufbau* (1928a) to the technical efforts towards developing an adequate logic of science in the *Syntax* (1934), as well as in the development of a satisfying intensional semantics or in the construction of several systems of inductive logic in Carnap's later works (1947; 1950b). Just like his structuralism, Carnap's logical exceptionalism takes many specific forms, consistently with the evolution of his philosophy and of the logical knowledge of his time. In philosophy of mathematics, Carnap's logical exceptionalism can be seen as stemming from his non-standard logicism, where logical knowledge is always held to be fundamental for mathematical knowledge (see Schiemer 2020a,b for a survey of Carnap's logicist attempts and the various ways in which he spells out this fundamental nature of logical knowledge). In epistemology and philosophy of language, the exceptionality of logic is instead rooted in the relatively apriori status that, according to Carnap, logical knowledge enjoys in any conceptual framework. Despite the specific different justifications that Carnap's logical exceptionalism is given in the different periods and domains of Carnap's philosophy, Carnap justifies this ideal, before tolerance, always based on certain special properties that logical statements possess. Logic is exceptional because logical statements are tautologies, or because they are analytic, or because they are knownable apriori, and so on. These special properties of logical statements justify, then, the exceptional status of logic in Carnap's pre-tolerance methodology.

However, similarly to what we noticed for the case of Carnap's pre-tolerance structuralism, this kind of logical exceptionalism is arguably inconsistent with Carnap's principle of tolerance, because it unjustly limits the freedom prescribed by this principle. After all, according to this principle, the methods and the knowledge provided by logic should be only one of the many possible ways through which philosophers ought to explicate philosophical and scientific concepts and their alleged special properties are bound to be internal to certain (classes of) linguistic frameworks. Yet, Carnap, in his post-tolerance works, kept granting logical methods an exceptional status in his methodology, as his repeated tentative proposals of a general framework of inductive logic and his logic-centered reconstructions of scientific theories arguably demonstrate.[4] Thus, we are faced again with

[4]It should be noted that, despite logical methods occupied an exceptional place in Carnap's philosophy of science, Carnap arguably left an important

the problem of reconciling the seemingly unbounded freedom prescribed by Carnap's principle of tolerance with Carnap's own philosophical ideals.

The key to understand how this reconciliation is possible, I contend, is given by understanding explication as a philosophical transfer principle. The kind of logical exceptionalism that the late Carnap subscribed to is, in fact, just like his structuralism, only methodological in character and it is stems from the characteristics of explication as a philosophical transfer principle. Logic is exceptional, according to the post-tolerance Carnap, not in virtue of the properties that logical statements possess, but in virtue of the methodological role that logic performs in explication. Logical methods allow explicators to discern and connect, with the maximum clarity, the functional properties that concepts of different frameworks possess. In this way, logic helps explicators to recognize that concepts belonging to radically different linguistic frameworks share similar functional properties and are thus able to perform similar functions in a given scientific and philosophical task. By connecting the functional properties of different conceptual frameworks, logical tools play an exceptional role in Carnap's methodology of explication, allowing explicators to achieve the only absolute ideal permitted by the principle of tolerance, i.e., clarity of methods. Such a methodological justification of logical exceptionalism, just like we noted for the case of methodological structuralism, allows Carnap to pragmatically grant an exceptional role to logical tools in philosophical methodology without imposing any prohibition to the freedom prescribed by his principle of tolerance.

This analysis of Carnap's ideals of structuralism and logical exceptionalism shows us the close connection between the method of explication and the principle of tolerance. Only with the method of explication, Carnap's ideal of tolerance becomes a reality. Explication allows, in fact, Carnap to transform his philosophical ideals, such as structuralism and logical exceptionalism, onto methodological conventions, thereby achieving the quasi-absolute neutrality on metaphysical and ontological disputes that Carnap long sought.

place for non-logical methodologies such as historical and sociological analyses. For a detailed discussion of the relationships between logical and nonlogical methods in Carnap's philosophy of science, see Uebel (2011).

7. Conclusion

Let me recap the main steps of the present work. I started by describing Carnap's procedure of explication and the conceptual bridge-function between different frameworks that this method performs in Carnap's philosophy. Then, we saw how an analogous bridge-function is performed between different mathematical domains by the methodology of transfer principles. After an historical analysis of the rich historical connections between projective geometry and Carnapian explication, I showed how explication can be modeled as a philosophical kind of transfer principle, connecting, relative to a given task and normatively constrained by the desiderata freely chosen by the explicators, the functional properties of concepts belonging to different conceptual frameworks. Finally, we saw how, in the light of this characterization of explication as a philosophical transfer principle, the evolution of two main philosophical ideals of Carnap, i.e., his structuralism and the exceptionality of logic in his thought, can be better understood.

More generally, the present work shows how the geometrical roots of the explication procedure allowed Carnap to fully embrace his pluralist ideal of prescribing to the would-be philosopher a constant engineering quest to search and construct better conceptual tools for advancing philosophical issues. In this conceptual metrology, functionally-similar concepts belonging to different frameworks are connected by the method of explication depending on the task at issue and on the values of the explicators. If philosophy, according to Carnap, is a science of conceptual possibilities (see Mormann 2000), the method of explication is its kaleidoscope.

Acknowledgements

I am indebted to Hannes Leitgeb for many discussions about Carnap and explication. Georg Schiemer helped me to navigate the scattered literature on transfer principles. I am also indebted to two anonymous reviewers and to audiences in Tilburg and Còrdoba for very helpful comments that greatly improved this paper.

Matteo De Benedetto
Ruhr-Universität Bochum
matteo.debenedetto@rub.de

References

Awodey, Steve and André Carus, 2003. "Carnap versus Gödel on Syntax and Tolerance." In *Logical Empiricism: Historical and Contemporary Perspectives*, edited by P. Parrini, W. C. Salmon and M. H. Salmon, pp. 57–64. Pittsburgh: University of Pittsburgh Press.

———, 2007. "Carnap's Dream: Gödel, Wittgenstein, and Logical Syntax." *Synthese* 159(1): 23–45.

———, 2009. "Wittgenstein's Prison to the Boundless Ocean: Carnap's Dream of Logical Syntax." In Wagner (2012), pp. 79–108.

Beaney, Michael, 2013. "Analytic Philosophy and History of Philosophy: The Development of the Idea of Rational Reconstruction." In Reck (2013), pp. 231–60.

Biagioli, Francesca, 2020. "Structuralism and Mathematical Practice in Felix Klein's Work on Non-Euclidean Geometry." *Philosophia Mathematica* 28(3): 360–84.

Brun, Georg, 2016. "Explication as a Method of Conceptual Re-Engineering." *Erkenntnis* 81(6): 1211–41.

———, 2020. "Conceptual Re-Engineering: From Explication to Reflective Equilibrium." *Synthese* 197(3): 925–54.

Cappelen, Herman, David Plunkett and Alexis Burgess, eds., 2020. *Conceptual Engineering and Conceptual Ethics*. New York: Oxford University Press.

Carnap, Rudolf, 1922. *Der Raum: Ein Beitrag zur Wissenschaftslehre*. Berlin: Kant-Studien, Ergänzungshefte.

———, 1928a. *Der logische Aufbau der Welt*. Berlin: Weltkreis.

———, 1928b. *Scheinprobleme in der Philosophie*. Berlin: Weltkreis.

———, 1934. *Logische Syntax der Sprache*. Vienna: Springer.

———, 1947. *Meaning and Necessity*. Chicago: University of Chicago Press.

———, 1950a. "Empiricism, Semantics, and Ontology." *Revue Internationale de Philosophie* 4(11): 20–40.

———, 1950b. *Logical Foundations of Probability*. Chicago: University of Chicago Press.

———, 1963a. "Carnap's Intellectual Autobiography." In *Sch63*, pp. 3–84.

———, 1963b. "The Philosopher Replies." In *Sch63*, pp. 859–1013.

———, 1966. *Philosophical Foundations of Science*. New York: Basic Books. Reprinted as *An Introduction to the Philosophy of Science*, 1972.

Carsten, Klein and Steve Awodey, eds., 2004. *Carnap Brought Home—The View from Jena*. La Salle, IL: Open Court.

Carus, André, 2007. *Carnap and Twentieth-Century Thought: Explication as Enlightenment*. Cambridge: Cambridge University Press.

———, 2012a. “Engineers and Drifters; The Ideal of Explication and Its Critics.” In Wagner (2012), pp. 225–39.

———, 2012b. From Analysis to Explication. Unpublished Manuscript.

———, 2017. “Carnapian Rationality.” *Synthese* 194: 163–84.

Chang, Hasok, 2004. *Inventing Temperature: Measurement and Scientific Progress*. New York: Oxford University Press.

Coffa, Alberto, 1986. “From Geometry to Tolerance: sources of conventionalism in nineteenth-century geometry.” In *From Quarks to Quasars: Philosophical Problems of Modern Physics*, edited by R. G. Colodny, pp. 3–70. Pittsburgh: University of Pittsburgh Press.

———, 1991. *The Semantic Tradition From Kant to Carnap: To the Vienna Station*. Cambridge: Cambridge University Press.

Creath, Richard, 1992. “Carnap’s Conventionalism.” *Synthese* 93: 141–65.

———, 2012. “Before Explication.” In Wagner (2012), pp. 161–74.

De Benedetto, Matteo, 2021. “Explication as a Three-Step Procedure: the case of the Church-Turing Thesis.” *European Journal for Philosophy of Science* 11(21).

———, 2022. “Explicating ’Explication via Conceptual Spaces.” *Erkenntnis* 87: 853–89.

Dutilh Novaes, Catarina and Erich Reck, 2017. “Carnapian Explication, Formalisms as Cognitive Tools, and the Paradox of Adequate Formalization.” *Synthese* 194(1): 195–215.

Eder, Günther, 2019. “Frege and the Origins of Model Theory in Nineteenth Century Geometry.” *Synthese* 198(6): 5547–75.

———, 2021. “Projective Duality and the Rise of Modern Logic.” *Bulletin of Symbolic Logic* 27(4): 351–84.

Friedman, Michael, 1999. *Reconsidering Logical Positivism*. New York: Cambridge University Press.

———, 2000. *A Parting of the Ways: Carnap, Cassirer, and Heidegger*. New York: Open Court.

———, 2011. “Carnap on Theoretical Terms: Structuralism without metaphysics.” *Synthese* 180(2): 249–63.

———, 2012. “Rational Reconstruction, Explication, and the Rejection of Metaphysics.” In Wagner (2012), pp. 190–204.

Friedman, Michael and Richard Creath, 2008. *The Cambridge Companion to Carnap*. Cambridge: Cambridge University Press.

Gärdenfors, Peter, 2000. *Conceptual Spaces: The Geometry of Thought*. Cambridge, MA: MIT Press.

Gray, Jeremy, 2007. *Worlds Out of Nothing: A Course in the History of Geometry in the 19th Century*. New York: Springer.

———, 2008. *Plato’s Ghost: The Modernist Transformation of Mathematics*. Princeton, NJ: Princeton University Press.

Hanna, Joseph F., 1967. “An Explication of ’Explication’.” *Philosophy of Science* 35: 28–44.

Hawkins, Thomas, 1988. “Hesse’s Principle of Transfer and the Representation of Lie Algebras.” *Archive for History of Exact Sciences* 39: 41–73.

Hesse, Ludwig Otto, 1866a. “Vier Vorlesungen aus der analytischen Geometrie.” *Zeitschrift für Mathematik und Physik* 11: 369–425.

———, 1866b. "Ein Uebertragungsprincip." *Journal für die reine und angewandte Mathematik* 66: 15–21.

Horsten, Leon, 2011. *The Tarskian Turn. Deflationism and Axiomatic Truth*. Cambridge, MA: MIT Press.

Jeffrey, Richard Carl, 1994. "Carnap's Voluntarism." In *Logic, Methodology and Philosophy of Science IX*, Studies in Logic and the Foundations of Mathematics 134, edited by D. Prawitz, B Skyrms and D. Westerstáhl, pp. 847–66. Amsterdam: Elsevier.

Justus, James, 2012. "Carnap on Concept Determination: Methodology for Philosophy of Science." *European Journal for Philosophy of Science* 2: 161–79.

Klein, Felix, 1872. *Vergleichende Betrachtungen über neuere geometrische Forschungen*. Erlangen: Deichert.

Lavers, Gregory, 2016. "Carnap on Abstract and Theoretical Entities." In *Ontology After Carnap*, edited by S. Blatti and S. Lapointe, pp. 200–19. Oxford: Oxford University Press.

Leitgeb, Hannes and André Carus, 2020. Rudolf Carnap. Stanford Encyclopedia of Philosophy (Summer 2020 Edition), `https://plato.stanford.edu/archives/sum2020/entries/carnap/`.

Menger, Karl, 1943. "What is Dimension?" *The American Mathematical Monthly* 50(1): 2–7.

———, 1979a. *Selected Papers in Logic, and Foundations, Didactics, Economics*, Vienna Circle Collection Vol. 10. Dordrecht: Springer.

———, 1979b. "On Definitions, Especially of Dimension." In Menger (1979a), pp. 207–17. This includes reprinted sections of Menger (1943).

Mormann, Thomas, 2000. "Carnaps Philosophie als Möglichkeitwissenschaft." *Zeitschrift für philosophische Forschung* 55(1): 79–100.

———, 2008. "Geometrical Leitmotifs in Carnap's Early Philosophy." In Friedman and Creath (2008), pp. 43–64.

Psillos, Stathis, 1999. *Scientific Realism: How Science Tracks Truth*. London.

———, 2000. "Rudolf Carnap's 'Theoretical Concepts in Science'." *Studies in History and Philosophy of Science* 31: 151–72.

Quinon, Paula, 2019. "Can Church's Thesis Be Viewed as a Carnapian Explication?" *Synthese* 198(Suppl. 5): 1047–74.

Reck, Erich, 2012. "Carnapian Explication: a Case Study and Critique." In Wagner (2012), pp. 96–116.

———, ed., 2013. *The Historical Turn in Analytic Philosophy*. London: Palgrave Macmillan.

Reck, Erich and Michael Price, 2000. "Structures and Structuralism in Contemporary Philosophy of Mathematics." *Synthese* 125(3): 341–83.

Reck, Erich and Georg Schiemer, eds., 2020. *The Pre-History of Mathematical Structuralism*. Oxford: Oxford University Press.

Richardson, Alan, 1997. *Carnap's Construction of the World: The Aufbau and the Emergence of Logical Empiricism*. Cambridge: Cambridge University Press.

———, 2003. "The Geometry of Knowledge: Lewis, Becker, Carnap and the formalization of philosophy." *Studies in the History and Philosophy of Science Part A* 34(1): 165–82.

———, 2013. "Taking the Measure of Carnap's Philosophical Engineering: Metalogic as Metrology." In Reck (2013), pp. 60–77.

Schiemer, Georg, 2020a. "Transfer Principles, Klein's Erlanger Program, and Methodological Structuralism." In Reck and Schiemer (2020), pp. 106–41.

———, 2020b. "Carnap's Structuralist Thesis." In Reck and Schiemer (2020), pp. 383–420.

———, 2021. "Non-Standard Logicism." In *The Routledge Handbook of Logical Empiricism*, edited by T. Uebel and C. Limbeck-Lilienau, pp. 211–19. New York: Routledge.

———, 2022. "Logicism in logical empiricism." In *Origins and Varieties of Logicism*, edited by F. Boccuni and A. Sereni, pp. 243–66. New York: Routledge.

Schilpp, P. A., ed., 1963. *The Philosophy of Rudolf Carnap*. La Salle, IL: Open Court.

Sieg, Wilfried, 2002. "Calculations by man and machine: Conceptual analysis." In *Reflections on the Foundations of Mathematics. Essays in honor of Solomon Feferman*, Lecture Notes in Logic 115, Assoc. for Symbolic Logic, edited by W. Sieg, R. Sommer and C. Talcott, pp. 390–409. Natick, MA: AK Peters.

Stein, Howard, 1992. "Was Carnap Entirely Wrong After All?" *Synthese* 93: 275–95.

Steinberger, Florian, 2016. "How Tolerant Can You Be? Carnap on Rationality." *Philosophy and Phenomenological Research* 92(3): 645–68.

Strawson, Peter Frederick, 1963. "Carnap's Views on Constructed Systems versus Natural Languages in Analytic Philosophy." In Schilpp (1963), pp. 503–18.

Sznajder, Marta, 2018. "Inductive Logic as Explication: The Evolution of Carnap's Notion of Logical Probability." *The Monist* 101(4): 417–40.

Tappenden, James, 2005. "Metatheory and Mathematical Practice in Frege." In *Gottlob Frege: Critical Assessments of Leading Philosophers*, Vol. II, edited by M. Beaney and E. Reck, pp. 190–228. New York: Routledge.

———, 2006. "The Riemannian Background to Frege's Philosophy." In *The Architecture of Modern Mathematics: Essays in History and Philosophy*, edited by J. Ferreirós and J. Gray, pp. 107–50. Oxford: Oxford University Press.

Tarski, Alfred, 1933. "The concept of truth in formalized languages." In *Logic, Semantics: Metamathematics*, pp. 152–278. Oxford: Oxford University Press. English translation (1956).

Turing, Alan Mathison, 1936. "On Computable Numbers, with an Application to the Entscheidungsproblem." *Proc. London Math. Soc.* 2(42): 230–65.

———, 1937. "A correction." *Proc. London Math. Soc.* 2(43): 544–46.

Uebel, Thomas, 2011. "Carnap and Kuhn: On the Relation between the Logic of Science and the History of Science." *Journal for General Philosophy of Science* 42: 129–40.

———, 2012. "The Bipartite Conception of Metatheory and the Dialectical Conception of Explication." In Wagner (2012), pp. 117–30.

———, 2018. "Carnap's Transformation of Epistemology and the Development of His Metaphilosophy." *The Monist* 101(4): 367–87.

Wagner, Pierre, ed., 2012. *Carnap's Ideal of Explication and Naturalism*. London: Palgrave Macmillan.

Wilson, Mark, 1992. "Frege: The Royal Road from Geometry." *Noûs* 26(2): 149–80.

Wussing, Hans, 1984. *The Genesis of the Abstract Group Concept: A Contribution to the History of the Origin of Abstract Group Theory*. Cambridge, MA: MIT Press.