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Introduction

Juliet Floyd and Sanford Shieh

In this introduction we present the principal themes of the special issue and highlight the main interpretive theses of the contributions.

Special Issue: Frege in Philosophical-Historical Context
Edited by Sanford Shieh and Juliet Floyd

Introduction

Juliet Floyd and Sanford Shieh

The centrality of Frege's writings in analytical philosophy is now beyond dispute. Long gone are the days when the only words of Frege's that were widely read, or at least referred to and assigned in classes, are those that make up the first few paragraphs of the famous article "Über Sinn und Bedeutung." (1892) While debates over the sense/reference distinction continue in philosophy of language, and continue on the basis of examining Frege's own views, we are now able to appreciate far better the depth and fecundity of Frege's ideas in philosophy of mathematics and philosophy of logic. Some of the most interesting work on Frege in the past quarter century has resulted from a combination of careful attention to philosophical historical context with sensitivity of philosophical interpretation. The six essays that compose this special issue exemplify this direction in philosophical historical work on Frege, bringing to light less well-known aspects of logic, philosophy of logic, and philosophy of mathematics of Frege's time, and posing new solutions to long-standing interpretive puzzles.

We briefly summarize the contents of this special issue on Frege, which showcases recent creative work by up and coming scholars of his work.

Richard Lawrence provides an account of the philosophy of arithmetic of Hermann Hankel, which Frege criticizes at length in *Die Grundlagen der Arithmetik* (1884) as a formalist account of arithmetic. Lawrence shows that although formalism is a reasonable characterization of Hankel's views, Hankel is also a logicist critic of Kant's philosophy of arithmetic. Indeed, Lawrence shows that Hankel's objections to Kant are sufficiently similar to those espoused by Frege in *Grundlagen* that it is plausible to sup-

pose that they influenced Frege's opposition to Kant. Lawrence then provides a re-examination of Frege's critique of Hankel's formalism. The central basis of the critique is the mathematical practice of provision of existence proofs, which Frege uses to oppose Hankel's view that consistency of a freely created concept is enough to guarantee mathematical existence. Lawrence connects this line of objection to Frege's concept/object distinction in *Grundlagen*: a concept's consistency should not be confused with, and does not imply, any object's falling under that concept.

Moving from mathematical to logical context, Elkind's and Rohr's essays discuss the philosophical significance of features of Frege's logic in relation to the algebra of logic tradition and to the Kantianism in Frege's philosophical surroundings.

Landon Elkind focuses on the striking two-dimensionality of Frege's *Begriffsschrift*,¹ and claims that its importance lies in providing or enforcing a separation of logical structure from content: logical form lies in the horizontal construction using logical signs of a *Begriffsschrift* formula, while the content of such a formula is expressed in the rightmost vertically aligned array of signs. This separation, Elkind argues, enables *Begriffsschrift* to provide a more perspicuous expression of content than the one-dimensional formulas of the algebraists of logic: Boole, Peano, and Schröder. Even more importantly according to Elkind, the separation of logical form from content has an anti-Kantian significance: the expression of inference in *Begriffsschrift* formula displaying such a separation eliminates even the appearance of relying on sensible intuition.

Tabea Rohr advances a re-assessment of the significant differences between the logic of Frege and that of Schröder. She accepts Volker Peckhaus's objection to Jean van Heijenoort's view that Frege's logic expresses quantification while Schröder's does not,

¹We use "Begriffsschrift," unitalicized, to mention the (closely related) languages defined and used in the books *Begriffsschrift* (1879) and *Grundgesetze der Arithmetik* (1893; 1903). The language(s) are sometimes called "concept-script" in English.

since quantification is expressible in both formulations of logic. But she shows that there remains a critical logical/philosophical difference between Frege and Schröder stemming from their conceptions of concept formation. For Frege, concept formation consists in discerning functions in the conceptual contents expressed by statements. Many commentators have noted that in *Begriffsschrift* (1879), Frege conceives of discerning functional structures of conceptual contents through the removal of expressions from statements and replacement of the removed expressions with other expressions. The patterns that remain in place through such replacements are functions, and the places in the functional patterns where the removals and replacements are made are argument-places. Rohr emphasizes another factor in Frege's procedure of concept formation: the subsequent filling in of those argument-places with logically complex expressions. In this way, Frege's *Begriffsschrift* accommodates far greater possibilities of concept-formation than Schröder's algebraic notation. In particular, Rohr shows how Frege's two-factor procedure of concept formation allows the expression of purely logical judgeable contents, none of whose constituents comes from outside of logic, a conclusion which stands in some tension with Elkind's account of the *Begriffsschrift*.

We move next to an essay centering on philosophical context. Jacob Rump focuses on the notion of *Vorstellung*, variously translated as "presentation" or "representation," as it appears in Frege, Kant, and Husserl's works. The principal issue he addresses is whether a role is played by a conception of a subject in the notion of *Vorstellung*. Rump argues that although in *Begriffsschrift* Frege appears to allow some room for a subject's "combining" *Vorstellungen* into judgeable contents, his conception of logic and the anti-psychologistic stance this conception supports drive him to the view that conceptual content, and, later, thought, are not in any way constituted by cognitive subjects. Frege conceives of these subjects as empirical individuals, and their cognitive activities as psychological processes that are

of no concern to logic. In contrast, both Kant and Husserl hold that subjectivity plays an ineliminable logically significant role in the constitution of *Vorstellungen* and in judgment. Rump shows that for Kant and Husserl, the logically significant subject is not an empirical subject, and so their conceptions of *Vorstellungen* and judgment are not psychologistic. Rump's analyses lead into a number of interesting issues. What is the non-psychologistic subject? Should it be understood as something like a cognitive act type that particular empirical thinkers may instantiate? (On this suggestion, compare the approaches in Moltmann 2017 and Soames 2015.) According to one of us (Shieh 2019, chapter 3), Frege's conception of judgment centers on recognition of what is the case in the realm of reference. If so, then Frege should perhaps accept Husserl's characterization of judgment in terms of "fulfillment," and thereby accept also the involvement of a non-psychologistic subject in judgment.

We now move to essays that center more on philosophical interpretation. Rachel Boddy offers a resolution of a well-known puzzle about Frege's conception of definition. Frege requires definitions to be "fruitful" (*fruchtbar*), seemingly in the sense of being indispensable for the construction of gap-free proofs; but, he also holds that definitions, strictly speaking, are mere abbreviations, so that in every proof in which it occurs the *definiendum* of a legitimate definition may be replaced by the *definiens*. Boddy's solution turns on ascribing to Frege a notion of "analytic definition" as a definition that explains a central concept of a scientific discipline. Frege's definition of the concept of number in *Grundlagen* is analytic in this sense, providing an explanation of the central concept of the science of arithmetic. Such a definition is indispensable in the sense that without it, the proofs in *Grundgesetze der Arithmetik* would not be recognizable as proofs of the basic laws of *arithmetic*. They would, rather, remain merely proofs of statements of a purely logical theory of *value-ranges*. At the same time, the replaceability of the *definienda* by the *definienda* is required in order for the *proofs* of the basic laws of arith-

metic to be purely logical. Boddy's solution appears to suggest a stance on a long-standing issue on whether Frege took the thoughts expressed by the conclusions of his proofs of the basic laws of arithmetic to be exactly the thoughts, if any, expressed by arithmetical language prior to these proofs. Boddy's view would seem to support the contention that Frege did not envision any requirement for preservation of "pre-existing" senses of arithmetical language, but rather only the requirement of a mapping of the linguistic structure of "pre-existing" sentences to Begriffsschrift theorems. This mapping subserves an aim of theoretical unification common in the mathematical sciences of Frege's time. But this theoretical unification may also provide us with insight into our ordinary, unreflective involvement with arithmetic, as manifest in our use of informal arithmetical language. Such a stance, we note, coheres with Lawrence's view of Frege's logicism in *Grundlagen*. For both Boddy and Lawrence, Frege's aims are neither purely mathematical nor purely philosophical; the interest and value of Fregean logicism lies in the specific ways in which it displays the interpenetration of mathematical practice and philosophical understanding.

Alexander Yates takes up an equally puzzling question about Frege's conception of the basic laws of logic: how do we come to judge these laws as true, and to recognize them as *logically* true? This question is bound up with another: what exactly is the status of the seeming arguments Frege seems to advance in *Grundgesetze* for the truth of the Basic Laws and the correctness of the modes of inference he adopts? Recently readers of Frege have tended to two types of answers. On the one hand, some take these arguments to be justifications of the truth of the Basic Laws or of their status as logical truths. On the other hand, some take Frege's arguments to be teaching tools, elucidations that provide his readers with an understanding of how to operate his Begriffsschrift formal system, or instill a recognition of the generality and undeniability of the his Basic Laws. Yates steers a path between these types of readings towards an interpretation

that avoids difficulties confronting each. He takes up a suggestion one of us made (Shieh 2019, chapter 5), holding that these apparent arguments are intended to exhibit the exercise of a logical capacity. These exhibitions have the aim of bringing Frege's readers into a state in which they are warranted in acknowledging his Basic Laws as true, and in judging that they are logically true. Yates provides a detailed account of the inferential capacities at play in such exhibitions, and explains how the uptake of the exhibitions lead to non-inferential warrant for the generality, undeniability, and logicity of the Basic Laws. Since exhibitions are not inferential justifications of logicity, they are not open to problems of circularity. But it is in virtue of engaging inferential capacities that these exhibitions provide non-inferential warrant for judgments of logicity. Hence Yates is in a position to explain why accepting the logicity of a Basic Law by taking in an exhibition, as opposed to suffering a blow to the head, provide one with warrant for that acceptance.

In conclusion, we would like to emphasize that the essays of this special issue do not merely advance our understanding of Frege's thought in historical context. They also indicate the relevance of this understanding to contemporary philosophical issues such as: the nature of definition and theoretical unification, the possibilities of concept formation in logic, the differences between consistency and existence proofs, the ground of our knowledge of logic, and the limits of empirical psychological conceptions of belief. They thereby exemplify the way in which philosophical history is simply a mode of philosophy.

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