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What did Russell Learn from Leibniz?

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Russell's rejection in 1898 of the doctrine of internal relations—the view that all relations are grounded in the intrinsic properties of the terms related—was a decisive part of his break with Hegelianism and opened the way for his turn to analytic philosophy. Before rejecting it, Russell had given the doctrine little thought, though it played an essential role in the most intractable of the problems facing his attempt to construct a Hegelian dialectic of the sciences. I argue that it was Russell's early reading of Leibniz, in preparation for his lectures on Leibniz given at Cambridge in 1899, that most probably alerted him to the role the doctrine was playing in his own philosophy. Leibniz defended a similar doctrine and extricated it from difficulties like those faced by Russell by means of devices that were not open to Russell. Russell would have come across these views of Leibniz in writings by Leibniz that he read in the summer of 1898, just before he rejected the doctrine of internal relations.

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The lectures on Leibniz which Russell gave at Cambridge during Lent Term 1899 and published the following year as *A Critical Exposition of the Philosophy of Leibniz* had a remarkable influence on Leibniz scholarship, where for better or worse they set much of the agenda for the next fifty years and beyond. But what influence did Russell's study of Leibniz have on the development of Russell's own philosophy?

In *My Philosophical Development* Russell said that it was as a result of working on Leibniz that he 'first realized the importance of the question of relations' (Russell 1959, p. 61). But what exactly did he mean by 'the question of relations'? He could not have meant that he discovered the philosophical importance of relations from studying Leibniz, for Russell's first philosophical book, *An Essay on the Foundations of Geometry* (1897), propounds a relational theory of space, and his recognition of the importance of relations in mathematics had been growing steadily since then. By mid-1898, which was when he started preparatory reading for his lectures on Leibniz, he had come to hold that relations were central to the whole of mathematics.

In fact, 1898 was a very important year in Russell's study of relations. Having recognized their importance, Russell began to study their formal properties. In *An Analysis of Mathematical Reasoning*, a work written between April and July 1898 (Russell 1989a, p. 155), Russell for the first time distinguished between symmetrical, transitive, and reflexive relations, although he does not use these terms in quite their modern senses (Russell 1989c, p. 191).[1] But the classification of relations was only part of the advance Russell made in his understanding of relations in 1898. To appreciate the role relations played in the development of Russell's philosophy, we need to look more broadly at Russell's philosophical position at the time he started work on Leibniz.

Since 1895 Russell's main philosophical endeavour had been to construct a neo-Hegelian encyclopedia of the sciences, starting with the more abstract of the special sciences and moving, via a series of dialectical supersessions, to an all-encompassing metaphysical science of the Absolute. Each special science, Russell held, attempted to create as full a picture of the world as possible from the limited set of concepts in its repertoire. But this attempt, he thought, ended inevitably in contradictions which could only be removed by adding concepts to those which constituted the original science, thereby transiting to a new science less abstract than the one which preceded it. The task of the philosopher was to establish the basic concepts and principles involved in each science, identify the contradictions to which it gave rise, and to show how they could be eliminated by the addition of new concepts which resulted in the next science in the dialectic.

The contradictions which Russell found in the special sciences were of various types, but one, which by 1898 he had come to call the contradiction of relativity (Russell 1989c, p. 166), proved to be both especially prevalent and especially troubling. The contradiction of relativity appeared first in geometry as the 'antinomy of the point', namely that, while every point is distinct from every other, all are exactly alike (Russell 1956, p. 188). By the following year, the contradiction had turned up in kinematics, dynamics, and 'almost, if not quite, universally' in pure mathematics (Russell 1989c, p. 166). In all forms it arose, where, as Russell put it in 1897 with a nice Hegelian flourish, we have 'a conception of difference without a difference of conception' (Russell 1989b, p. 81). What he means is that we conceive of there being different points or other types of mathematical entities, such as quantities, (i.e., we have a conception of their being different), though all of them fall under exactly the same concepts (there is no difference of conception to individuate them), since all individual differences between them have been abstracted out in order to create the abstract science in question. In so far as pure mathematics deals with quantity, it is with abstract quantity, not with this or that concrete quantity.

As a neo-Hegelian, Russell took the contradiction of relativity to show the perils of abstraction, rather than the failings of the dialectic. The solution for each version of the contradiction was to build a new science on the basis of the old by adding new concepts which would enable the diverse but indiscernible elements of the original science to be distinguished in the new. The procedure was always to provide the missing difference of conception by means of a dialectical supersession to a new, less abstract science. Thus geometry was to be superseded by a Boscovichian kinematics in which different spatial points were to be distinguished by the material point-atoms which occupied them. But kinematic point-atoms merely reproduced the problems of geometrical points: each, in itself, was exactly like all the others, a bare centre of converging or diverging motion. Accordingly, in the next step of concretization, Russell endowed them with forces, thereby transiting to dynamics. But this also was no more than a stop-gap, for force itself was a relative concept, identifiable only by the relative motion of matter, and thus its addition had still not produced the ultimate differences of conception that Russell's system required. As a neo-Hegelian, of course, Russell was an idealist and thus did not expect that a purely material world would pass muster metaphysically. The final stage of the dialectic was to be a transition to psychology, in which dynamical point-atoms, were to be replaced by monads. This left him with what would seem to be the considerable difficulty of deriving the laws of physics from the psychology of monads. Of how this was to be accomplished Russell left few clues, since he abandoned the dialectic before the transition to psychology was seriously tackled.

The situation I've described was essentially what Russell had arrived at by the time he started his work on Leibniz, with one important addition: in Russell (1989c, pp. 225–6) he provided a general argument, derived from his new classification of relations, to show that the contradiction of relativity would appear wherever asymmetrical relations were involved. Since he held that such relations 'pervade almost the whole of mathematics', he concluded, that the 'fundamental importance of this contradiction to Mathematics is thus at once proved and accounted for' (Russell 1989c, p. 226).

Except, we might protest, that it is not clear why a conception of difference without a difference of conception is a contradiction at all, for surely two items may be exactly alike as far as their intrinsic properties are concerned and yet be easily distinguished by their differing relations. Russell, however, was precluded from taking this line by his hitherto unquestioned neo-Hegelian view that all relations were internal, that is, they were all in some way grounded in the intrinsic properties of their terms. The doctrine of internal relations makes it clear why it was asymmetrical relations that gave rise to the contradiction of relativity. If all relations are grounded in the intrinsic properties of their terms and R is an asymmetrical relation holding between a and b, then a and b must have different intrinsic properties. For whatever it is about the nature of a that grounds its relation R to b, b must have a different nature for b neither has the relation R to a nor to itself. According to the doctrine of internal relations, asymmetrical relations are impossible for objects which do not differ in intrinsic properties. And yet for the mathematical sciences, as Russell had increasingly been discovering, they are essential. It is the doctrine of internal relations which makes the contradiction of relativity a genuine contradiction.

Up to this point, Russell had not only not discussed the doctrine of internal relations, he had neither stated nor even identified it. It is simply taken for granted, an unrecognized assumption at the centre of his philosophy. Indeed, the doctrine is stated only after it is rejected. It is explicitly stated[2] in 'The Classification of Relations' (Russell 1989a, pp. 138–46), a paper which was read to the Cambridge Moral Sciences Club on 27 January 1899, just after his lectures on Leibniz had started. There, Russell does not provide an argument against it, but his grounds for rejecting it can be found in parts of the typescript of the *Analysis of Mathematical Reasoning* (AMR) which he incorporated into the 1899–1900 draft of *The Principles of Mathematics*. They are, rather surprisingly, precisely that, in the case of asymmetrical relations, the doctrine of internal relations leads to the contradiction of relativity. In fact, he takes the very two pages from the AMR typescript on which he had presented the argument which showed that the contradiction was endemic in mathematics and simply changed the conclusion. Instead of a *modus ponens* argument from the doctrine of internal relations as unstated premiss to the contradiction of relativity as conclusion, he gives a *modus tollens* argument refuting the doctrine of internal relations because it entails the contradiction of relativity.[3] It is hard to over-estimate the importance of this move, for it constituted an important, indeed perhaps the decisive, part of Russell's break from neo-Hegelianism, the one true revolution, he said, in his philosophical development (Russell 1959, p. 11).[4]

We do not know precisely when this change took place. Russell subsequently said that he broke with neo-Hegelianism in 1898[5] and this, I think, must include his seeing that rejecting internal relations would eliminate the contradiction of relativity — though not necessarily, of course, to his rewriting the passage from the AMR typescript. So the change came sometime between his finishing AMR in July 1898 and the end of the year. What I suggest is that it may well have come about as a result of his study of Leibniz. In previously writing on this topic (Griffin 1991 pp. 341–6) I had assumed that the chronology ruled this out. But this now seems far less likely.[6]

It seems likely that Russell accepted the invitation to lecture on Leibniz in the early summer of 1898, for the reading list he kept during the 1890s (Russell 1983, Appdx. 1) records him reading Langley's recent translation of the *Nouveaux essais* (Leibniz 1896) in June 1898, a work which otherwise he would have had no reason to read at that time. Thereafter, Leibniz works begin to appear regularly but not frequently in his reading list for the next nine months. The next works to appear are two recent English translations of selections of Leibniz's writings: Duncan's *The Philosophical Works* (Leibniz 1890) and Latta's *Monadology and Other Writings* (Leibniz 1898), in August and October respectively. Russell's main source, however, was Gerhardt's seven-volume *Philosophischen Schriften* (Leibniz 1875–90), at that time the most extensive collection of Leibniz material available in print, a copy of which Russell acquired in December 1898. It appears in the reading list for February 1899, which was presumably when Russell finished reading it. Russell's copy of Gerhardt's *Philosophische Schriften* survives in his library[7] and is quite extensively marked up, indicating that Russell read the entire work (or very nearly). A study of his marginalia strongly suggests that he read Gerhardt's volumes systematically, starting with volume 1.

In the preface to his book on Leibniz Russell said that, prior to giving his lectures, he had felt that Leibniz's *Monadology* was 'a kind of fantastic fairy tale, coherent perhaps, but wholly arbitrary', and that it was only when he read the correspondence with Arnauld and the *Discourse on Metaphysics* that 'a flood of light was thrown on all the inmost recesses of Leibniz's philosophical edifice' (Russell 1975, pp. xiii–xiv). The two crucial works Russell mentions are to be found in volumes 2 and 4 of Gerhardt respectively. And since he had only started reading Gerhardt in December, it seemed unlikely (though not impossible) that he would have got all the way through volume 4 by the end of the year, but took another two months to finish the remaining three volumes. But this assumes that he could not have derived an important insight from Leibniz — an insight concerning Russell's own philosophy rather than Leibniz's — until after he had read the *Discourse* and the Arnauld correspondence. As a result of paying much closer attention to the Leibniz texts Russell read, and the order in which he read them, it now seems to me that he could have derived his crucial insight into the role that the doctrine of internal relations was playing in his own philosophy before he read either the Arnauld correspondence or the *Discourse on Metaphysics*. The insight he subsequently gained from reading those works concerned the role of the doctrine of internal relations in *Leibniz*'s philosophy, especially in underwriting Leibniz's so-called 'containment principle' as an account of truth, a theme which Russell very much emphasized in his book. The two points were closely related. It was because he maintained that all relations were internal — that, as he put it 'There is no denomination so extrinsic as not to have an intrinsic one for its foundation' (Leibniz 1875–90, ii, p. 240; Russell 1975, p. 205)[8] — that Leibniz was able to maintain his 'great principle' that the concept of an individual substance contains 'all its events and all its denominations, even those which are commonly called extrinsic' (Leibniz 1875–90, ii, p. 56 = Leibniz 1956, i, p. 517). But Russell would certainly have been able to identify the doctrine of internal relations in Leibniz before he realized the use to which Leibniz intended to put it. Moreover, he would have found the doctrine in Leibniz, and in a role very close to that which it played in his own philosophy, in work by Leibniz which we know he read in the summer of 1898.

One important passage can be found in Duncan's collection, where Leibniz criticizes Johann Christoph Sturm's view that motion consists simply in 'the successive existence of a thing in different places' (Leibniz 1890, p. 129) — a position, incidentally, which Russell himself defends in *The Principles of Mathematics*. Contrary to Sturm, Leibniz held that a moving body has, in addition to its successive positions, 'a tendency or effort to change its place' (Leibniz 1890, p. 129). Were this not the case, Leibniz argues, there would be no difference between the body in motion and the same body at rest (Leibniz 1890, p. 129). But then there would be no difference in bodies at all. Leibniz held that matter formed a plenum, with bodies occupying every portion of space, thus moving a body, *m*, from *a* to *b* would involve replacing the body, *m*′, congruent with *m*, which previously occupied *b*. But then the body replaced must be exactly like the body replacing it, for on Sturm's view the two are not distinguished by their tendencies to move, and 'there is no basis for a distinction [between bodies] in a plenum of mass uniform in itself other than that which concerns motion.'[9] But if *m* and *m*′ were exactly alike then God would have no reason for replacing one by the other and *m*'s move would violate the principle of sufficient reason (Leibniz 1890, pp. 129–30).

It is hard to miss the parallel between this problem which Leibniz finds in Sturm and Russell's contradiction of relativity. In both cases, the problem arises because of the need to postulate a plurality of objects which have no internal differences between them. If Sturm's view is to work, we need a conception of difference, m must be different from *m*′ otherwise no motion would occur, without a difference of conception, since *m* and *m*′ must be alike. Leibniz holds that this is impossible because of the principle of the identity of indiscernibles, which he derives from the principle of sufficient reason. There is no motion under these circumstances, for, given the identity of indiscernibles, to replace a body with one exactly like it is to replace it with itself. Russell's argument, as we've seen, was different, for it did not depend upon either the identity of indiscernibles or the principle of sufficient reason, but required rather that if m was to occupy b after *m*′, *m* and

m' must have different intrinsic properties on which their relation to one another could be grounded.

Just as Russell sought to distinguish kinematic atoms from one another by endowing them with differential forces, Leibniz sought to avoid the problem of motion by maintaining that a moving body has, besides its successive positions, 'a tendency or effort to change its place' (Leibniz 1890, p. 129). These tendencies or efforts (appetitions) served to distinguish one body from another, and thus make motion possible, not withstanding the identity of indiscernibles. As Leibniz notes elsewhere, each 'substantial thing ... must always differ from every other in respect of *intrinsic denominations*' (Leibniz 1981, II, i, 2).[10] So the appetitions of bodies must be genuinely intrinsic denominations, for otherwise the moving substances would not be appropriately distinguished. Indeed, the account would be circular, for the appetitions would distinguish the bodies by reference to distinct positions (an extrinsic denomination), positions which, in a relational theory of space, could only be distinguished by reference to the different bodies which occupied them. This was, in fact, the fate of Russell's forces, since he could only distinguish the differing forces by means of the relative motions of bodies. But the best analogues in Russell's system for Leibniz's appetitions of bodies are not the forces Russell ascribes to atoms, but the psychological states he ascribes to monads. These are supposed to provide the basic distinction between entities on which the whole system depends.

Although Russell had little to say about this final step in his dialectic, he probably originally expected to arrive at something like the conclusion Leibniz did, namely that each monad can be distinguished from all others by its 'internal qualities and actions, which can be nothing else than its *perceptions* ... and its *appetitions* (that is, its tendencies to pass from one perception to another).'[11] By 1898, however, his hopes of reaching this conclusion must have begun to wear thin. The price of dealing with the contradiction of relativity was one Leibniz was happy to pay, namely to have each monad mirror the entire universe. But this was not a cost Russell was willing to bear. For one thing, it equipped the monads with an infinity of unconscious perceptions, an extravagance at which any scientific psychology might blanche. Moreover, if an infinity of monads each mirrored the Absolute in its entirety, how was this different from having an infinity of Absolutes?[12] It would seem difficult to distinguish the Absolute from its infinitely many mirror-images. Ontological efficiency would suggest that one such object was enough, and, indeed, as Russell came to the end of his neo-Hegelian career he seemed to turn away from the monadism which had underlain his earlier work and towards a Bradleian monism. Moreover, as Bradley (1893 , p. 152) had famously said: the Absolute does not make eyes at itself in the mirror.

Since every monad mirrors the entire world, it follows that every monad has infinitely many intrinsic properties. Indeed, it is an important feature of Leibniz's philosophy that for him every individual substance has infinitely many intrinsic properties. It does not follow from this, however, that no substance is simple. Simple substances lack parts, they do not lack properties. '[T]he simplicity of a substance', Leibniz writes, 'does not prevent the plurality of modifications which must be found together in the same simple substance'. And he offers a memorable analogy: in a single point, 'though it is perfectly simple', 'there may be found' 'an infinity of angles formed by the lines which meet in it' (Leibniz 1956, ii, p. 1034, re-arranged = Leibniz 1890, p. 299). Thus in Leibniz's system substances, whether simple or not, never lack for intrinsic denominations (modifications, or properties) by means of which they may be distinguished in accordance with the identity of indiscernibles. The idea that two substances should differ 'only by means of external denominations with no internal foundation ... is contrary to the greatest principles of reason' (Leibniz 1981, II, xxvii, 3).

But from this it does not follow that there are no items in Leibniz's system which are distinguished by their external denominations only. Like Russell, Leibniz acknowledges that places and times can only be distinguished by their external denominations: 'places and times are in themselves exactly alike' (Leibniz 1981, II, xxvii, 2). But this shows only that 'they are not substances or complete realities' (*ibid*.). The identity of indiscernibles applies only to substances, not to such abstract items as places and times. So it seems that Leibniz, unlike Russell, is well-able to defang the contradiction of relativity: in the case of substances, which have to be individuated by their intrinsic denominations, there is no problem because they are always provided with an abundantly rich set of them; in the case of abstract items which have a thinner set of intrinsic denominations, there is no problem either because they can be individuated by means of their external denominations.

Russell has no such easy way out because (i) in the case of concrete things (or substances), he is not prepared to countenance Leibniz's ontological and psychological extravagance and (ii) in the case of abstract objects, Russell holds that even they must be distinguished internally if they are to have relations to one another. It seems clear, therefore, that this very early reading of Leibniz in the summer of 1898 would have alerted Russell to the role that relations were playing in producing in his own philosophy a problem very similar to the one that Leibniz was able to overcome in his.

But another text which Russell read in the summer of 1898 brings him directly to the problem that concerned him. This is the passage which Russell frequently quotes from §47 of the 5th letter to Clarke. It is of 'capital importance' Russell says (Russell 1975, p. 13):

> The ratio or proposition between two lines *L* and *M* may be conceived in three several ways; as a ratio of the greater *L* to the lesser *M*; as a ratio of the lesser *M* to the greater *L*; and lastly, as something abstracted from both, that is, as the ratio between *L* and *M*, without considering which is the antecedent, or which the consequent; which the subject and which the object.… In the first way of considering them, *L* the greater is the subject, in the second *M* the lesser is the subject of that accident which philosophers call relation or ratio. But which of them will be the subject, in the third way of considering them? It cannot be said that both of them, *L* and *M* together, are the subject of such an accident; for if so, we should have an accident in two subjects, with one leg in one, and the other in the other; which is contrary to the notion of accidents.[13]

It is the third view of the relation between *L* and *M* that would naturally suggest itself to anyone who held that relations were external. But Leibniz flatly rules out this option as 'contrary to the notion of accidents'. It is plain that for Leibniz there are no polyadic accidents.

Leibniz is more explicit in other passages Russell notes but would have found only from his reading of Gerhardt. For example, in his letter of 21 April 1714 to Des Bosses, Leibniz first repeats the point that an accident cannot inhere in two substances, as if the opposite opinion were an obvious absurdity: 'I do not believe that you will admit an accident that is in two subjects at the same time'. He then continues:

> My judgment about relations is that paternity in David is one thing, sonship in Solomon another, but that the relation common to both is a merely mental thing whose basis is the modifications of the individuals. (Leibniz 1875–90, ii, p. 486 = Leibniz 1956, ii, p. 992; Russell 1975, p. 206)[14]

Ignoring the main issue of whether relations are 'mere mental things', what is noteworthy here is the fact that Leibniz seeks to identify the intrinsic denominations on which the relation is based, namely paternity in David and sonship in Solomon. Evidently, these have to be monadic accidents, which form part of the individual concept of David and Solomon respectively.

There can be no doubt, I think, (1) that Leibniz held the doctrine of internal relations; (2) that Russell was aware of this and held it to be of the highest importance; and (3) that Russell had access to at least some of the evidence for this (the frequently-cited letter to Clark) by the late summer of 1898.

It is equally important that, at no point in his discussion, does Leibniz give a general logical argument for the doctrine of internal relations. On two of the most important occasions on which he mentions it, he dismisses polyadic accidents as if they were an evident impossibility. His grounds for doing so come entirely from the authority of the scholastic tradition. Mugnai (1992) does a magnificent job tracing Leibniz's views on relations back through a host of philosophers in the scholastic tradition to the authority of Thomas Aquinas, who held that an accident 'never extends beyond the subject in which it inheres'.[15] He shows that this view was especially widely held by philosophers in the sixteenth and seventeenth centuries and that Leibniz 'conformed ... to the traditional doctrine' (Mugnai 2012, §3). There is no reason to suppose that Russell, at the time he worked on Leibniz, knew much, if anything, about this tradition; and less to suppose that, had he done, he would have accorded it any respect.

Leibniz did not make a case for the doctrine which demanded a reply. But he did state the doctrine with admirable clarity, perhaps the first time that Russell had seen it so stated. And this was one of the cases, which are not infrequent in philosophy, when to state a widely held assumption clearly, even as an obvious truth, is to open it to criticism. Leibniz's various, complicated, and dubious efforts to show it could be maintained in the face of putative counter-evidence, served to show that not only was the doctrine an unsupported assumption, but that it was one that was difficult to defend, at least in its full generality. In this paper, I have made the case as strongly as possible for thinking that it was Russell's reading of Leibniz that led him to realize that the doctrine of internal relations was an assumption which was essential for the contradiction of relativity, but that it was an assumption that could relatively easily be dispensed with. It is impossible to know if this causal story is correct; for it is also possible that Russell came to this conclusion on his own and that his subsequent study of Leibniz did no more than show him that Leibniz provided no grounds for recanting. Nonetheless, Russell did acknowledge subsequently that he learnt from Leibniz the importance of the question of relations, and the dispensability of the doctrine of internal relations was certainly important enough for Russell to count as '*the* question of relations'. If this was what Russell learnt from Leibniz, it was a very important lesson indeed, with massive implications for Russell's subsequent philosophy. Ironically, however, it was not a lesson that Leibniz intended to teach.[16]

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Notes

1 Awkwardly for modern commentators, Russell uses the term 'symmetrical' for relations we would call symmetrical and transitive; 'transitive' for those we would call transitive and asymmetrical; 'reciprocal' for those we would call symmetrical and non-transitive; and 'one-sided' for those which we would regard as neither symmetrical nor transitive.

2 'It is argued that a relation must make a difference to the related terms, and that the difference must be marked by a predicate which the terms would not otherwise possess. This I deny.' (Russell 1989d, p. 143) On the same page, Russell says that his position is that 'all relations are external'. Interestingly, he expresses some uncertainty about the 'internal'/'external' terminology, which he ascribes to Bradley: 'I am not certain whether I understand what he means by this expression'. The statement quoted above is thus what Russell takes the doctrine of internal relations to mean. Later, in dispute with Joachim (one of its chief advocates), Russell described it as the view that every relation is grounded in the natures of its terms. (Russell 1968, p. 139)

3 Compare Russell (1989a), pp. 225–6 with Russell (1993a, p. 93). The final argument appears, somewhat rephrased, in Russell (1964, p. 224).

4 See Griffin (1991, Ch. 8) for further details.

5 See, e.g, 'My Mental Development' (1944), Russell (1997, p. 11); 'Beliefs: Discarded and Retained' (1955), Russell (1997, p. 103).

6 See also Griffin (2012, §7) (but written four years previously) where I consider the possibility in greater detail but, for reasons which will become clear, still failed to give it as much credibility as I now think it has.

7 The other works do not. While he likely had his own copies of Latta, Duncan, and the *Nouveaux essais*, he seems not to have owned a copy of his other great source, Gerhardt's seven-volume *Mathematische Schriften*—he told Moore in June 1900 that he was using the copy from Trinity College Library.

8 This is, in fact, Russell's first statement in print of the doctrine of internal relations.

9 '[I]f there is no difference between any portion of matter and another portion equal and congruent to it (which the illustrious man [Sturm] must admit, since he has destroyed active forces or impetuses and all other qualities and modifications except for existence in this place and successively some future existence or other, all qualities and modifications having been removed), and, furthermore, if the state of this matter at one moment does not differ from its state at another moment except through the transposition of equal and congruent portions of matter which agree in everything, it obviously follows that, because of the perpetual substitution of indistinguishables, the state of the corporeal world can in no way be distinguished at different moments.' Here I follow Loemker's translation (Leibniz 1956, ii, p. 821), since Duncan's (p. 130) does not clearly express what is intended. Importantly, Leibniz goes on to argue that no purely 'extrinsic denomination' of the body (Leibniz 1890, p. 130), that is, no characterisation of it by means of its relations alone, could serve to distinguish it from another. The argument is from 'On Nature in Itself; or On the Force Residing in Created Things, and their Actions' (1698) (Leibniz 1875–90, iv, pp. 504–16). Cf also, 'Monadology' §§8–9 (Leibniz 1890, p. 309)

10 I follow Remnant and Bennett (and also Russell 1975 p. 219) in using ‘denomination’ here: Duncan has ‘characteristic” (Leibniz 1890, p. 204) and Langley has ‘connotation’ (Leibniz 1896, p. 110). (Russell corrects Langley’s translation.) The contrast, important for our purposes, between intrinsic and extrinsic denomination is between those characteristics of a thing which involve only the thing itself and those which relate it to another thing. Cf Remnant and Bennett’s helpful note, Leibniz (1981, pp. xxxvi–xxxvii).

11 ‘The Principles of Nature and Grace’ (1714), Leibniz (1890, p. 299; Leibniz 1875–90, vi, pp. 598–606).

12 Self-referential problems intrude here, for if each monad mirrors the entire Absolute exactly, then it must mirror the fact that it mirrors the Absolute, and also that it mirrors its mirroring of the Absolute, and so on.

13 Leibniz (1875–90, vii, p. 401) = Leibniz (1956, ii, p. 1147) as quoted by Russell (1975, pp. 12–13). Russell quotes the passage again in Russell (1964, p. 222). The passage is to be found in Leibniz (1890, p. 359), so Russell would have read it in August 1898.

14 There are many other passages, assembled by Mugnai (1992), in which Leibniz attempts to capture the content of relational propositions without having to admit polyadic accidents, often using completely different techniques. It is impossible to say which (or which group) of them, if any, constituted his final verdict on the topic. None of them seem in the slightest degree tenable in the face of Russell’s devastating general argument about the irreducibility of asymmetrical relations in Russell (1964), pp. 221–6.

15 *In quator Libros Sententiarum*, II, d. 27, q. 1, ar. 6, quoted in Mugnai (2012, § 2).

16 I am grateful to Richard Arthur, Jolen Galaugher and an anonymous referee for helpful comments.

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