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## Russell on Generality 1910 to 1918

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### ABSTRACT

In *Principia Mathematica*, Russell thought that there are irreducibly general judgements with their own mode of truth. They are true in virtue of what the elementary judgements they collect together correspond to. In *The Philosophy of Logical Atomism*, Russell thought that they are true in virtue of general facts. In 1910, general facts are not even considered in order to reject them. In 1918, Russell announces that it cannot be doubted that there are general facts. This raises an intriguing question. What was it that led Russell to drop his 1910 view and drove him to the conclusion that there are general facts? I propose an answer that relies on a core aspect of Russell's views on which he stayed firm throughout this period and on a preoccupation of his that began well before 1910: theories of truth and the problem of how false judgements are possible. I argue that Russell's 1910 view failed to account adequately for the falsity of general judgements, while the correspondence theory of truth pushed him to accepting general facts. To round things off I also consider three independent arguments for general facts.

I do not profess to know what the right analysis of general facts is. It is an exceedingly difficult question, and one which I should very much like to see studied.

— Russell

## 1. Introduction

In virtue of what are general judgements such as 'All humans are mortal' true? One option is that they are true in virtue their instances, in the example that Diotima is human and mortal, Socrates is human and mortal, Plato is human and mortal, Aristotle is human and mortal, etc.. Another view is that they are true in virtue of relations between

properties, for instance that the property *human* is subsumed under the property *mortal*. Yet a third view is that, just as there are facts concerning particulars in virtue of which judgements concerning them are true, there are general facts in virtue of which general judgements are true.

The present paper is an investigation of Russell's views on this question in the period from 1910 to 1918. As so often during his philosophical development, Russell changed his mind, and so he did on this topic. Russell held a version of the view that general judgements are true in virtue of their instances in 1910, which saw the publication of the first volume of *Principia Mathematica* (*PM*), and came to hold that they are true in virtue of general facts by 'the first months of 1918' (1956, 177), the time of the lectures *The Philosophy of Logical Atomism* (*PLA*).<sup>1</sup> These provide the cornerstones of the present investigation. They contain comprehensive and definitive expositions of Russell's views at these times, even where those views are expressed with caution. There will also be occasions for glances back to his engagement with the monistic theory of truth and a year or so forward. A number of Russell's most important writings from between 1910 to 1918 will be consulted, too, in particular *Theory of Knowledge*, *The 1913 Manuscript* (*TK*) and *Our Knowledge of the External World* (*OKEW*). I shall quote Russell extensively, in the case of works collected in Marsh's volume *Logic and Knowledge* (*LK*) with reference to it and to the original source.

The development from *PM* to *PLA* with respect to general judgements is striking and deserves explanation. What might have been Russell's reasons for this change of mind? Surprisingly little has been written on this topic, including by Russell himself.<sup>2</sup>

A major aim of this paper is to draw attention to this question, and I shall propose an explanation for this change in Russell's thought. Russell abandoned the *PM* account as a response to the problem of how false judgement is possible, which occupied him in his engagement with theories of truth, applied to general judgements. *PM* does not provide a satisfactory account of the falsehood of general judgements. The problem is solved by an appeal to general facts. Additionally, the

<sup>1</sup> Reprinted in Russell (1986).

<sup>2</sup> A number of philosophers have argued from more or less Russellian perspectives that there are general facts: Armstrong (1997, 197ff), Armstrong (2004, 72ff), Elkind (2021), Grossmann (1992, 71ff), Hochberg (1969), Hochberg (1981), Hossack (2007, sec. 2.7), Kürbis (2022), Tegtmeier (1992, I. §4). But none of them address the question raised here.

correspondence theory of truth, which Russell adopted as the result of his engagement with theories of truth, pushed him to the conclusion that there must be general facts. I shall also give three independent arguments for general facts.

Being closely tied to Russell's writings, my approach is historical. There are, however, systematic aspects to the present investigation. Philosophers tend to focus on truth, with falsehood often neglected. The present paper thus draws attention to the importance of falsehood and gives an example where it pays off to consider the opposite of truth.<sup>3</sup> Secondly, independently of Russell's thought the lines of argument traced in this paper indicate that a certain version of the correspondence theory of truth supports the conclusion that there are general facts.

During the period from 1910 to 1918 Russell changed his mind on a number of topics central to his philosophy. In 1910, Russell holds the multiple relation theory of judgement (*PM*, 46). In 1918, he notes mistakes in its previous formulations (*PLA III & IV*, 58ff; *LK* 224ff), and while he still holds on to it there, already in 'On Propositions: What they are and how they mean' (*OP*), 'composed a year after the lectures on logical atomism' (Russell 1956, 283), he does not hold it anymore (*OP* 25ff; *LK* 305ff).<sup>4</sup> In 1910, Russell thinks that propositions, as that to which the mind supposedly relates in judgement or the contents of judgements, are incomplete symbols, which only have meaning in the context of sentences in which they occur—namely those of the form 'S judged that *p*'—and which are to be explained by the multiple relation theory of judgement (*PM* 46f); thus propositions do not form part of the inventory of the world, just as the present King of France does not. In 1919, Russell rehabilitates content (*OP* 27; *LK* 307). In 1910, there is a subject that does the judging (*PM* 46), in 1919, there is not (or there is no commitment that there is one) (*OP* 25; *LK* 305).<sup>5</sup> Finally, there is the nature of universals, whether they can be terms in a relation and named or not and thus can only be referred to by verbs or predicates. Russell held the latter view in 1918 (*PLA III & IV* 59, *V & VI* 192ff, *VII & VIII*

<sup>3</sup> It illustrates a point made by Denyer (1991).

<sup>4</sup> *OP* is reprinted in Russell (1986).

<sup>5</sup> His reasons are worth a footnote in the present context: 'The advantages [of his current view] are those derived from the rehabilitation of the content, making it possible to admit propositions as actual complex occurrences, and doing away with the difficulty of answering the question: what do we believe when we believe falsely?' (*OP* 27; *LK* 307).

350f, 362; *LK* 225, 230ff, 258f, 267), but gravitated towards the former before 1910.

Although they complicate matters, these changes of mind do not affect the issue addressed in the present paper. Russell concludes that there must be general facts independently thereof.

The key to my approach is that throughout the period under consideration, there are two things about which Russell did *not* change his mind: there are general judgements and truth consists in correspondence to facts. Explaining these two aspects of Russell's philosophy is the topic of Section 3. Together, they result in a view according to which true general judgements correspond to general facts. In 1910 the correspondence theory of truth is directly applied only to the simplest kind of judgements, those Russell calls 'elementary'. General judgements require a different mode of truth. Only after developing his logical atomism did he accept that there are general facts as that to which true general judgements correspond. In *PM*, Russell notes the necessity to admit irreducibly general judgements, soon after, in *The Problems of Philosophy* (*PP*), he speaks of irreducibly general knowledge. General facts enter in *PLA*. Russell's thinking moves from accepting that there are irreducibly general judgements to accepting that there are general facts.

I shall avoid questions regarding the nature of general facts as much as possible. However, a few words on this topic are necessary and sketches of three options are in Section 2.

In Section 4 I argue that the *PM* account of the truth of general judgements is unsatisfactory. It is not an argument that Russell gives himself, but its theme is at the heart of the development of Russell's understanding of what a theory of truth must achieve: a theory of truth must also provide a satisfactory account of falsehood. The *PM* account of the truth of general judgements, I shall argue, fails to do so. This is also the place where Russell's earlier engagement with the monistic theory of truth comes in: Russell rejected it on the grounds that it fails to explain how error or false judgement is possible. Applying his own standards to the account of the truth of general judgements of *PM*, he should have noticed that it fails in this respect.

If the account of the truth of general judgements of *PM* must be rejected, the view that general judgements correspond to general facts suggests itself. But are there independent arguments for this view? This

question is addressed in Section 5. I give three answers. The first builds on a remark Russell makes in *OP* (6; *LK* 289) that similar considerations that led him to accept negative facts also support his acceptance of general facts. I shall argue that in both cases, an attempt to avoid them in one place merely sees them emerge in another. The second is an argument not given explicitly by Russell, but it is suggested by the multiple relation theory of judgement and the correspondence theory of truth. In *PM*, general and elementary judgements have different modes of truth and the question arises what the constituents of general judgements are. The answer that suggests itself is that they are a mind, universals and a certain form in which they are combined. But that is a form of general facts. Furthermore, if general facts are admitted, truth for both kinds of judgements consists in straightforward correspondence to a fact. The difference lies only in the nature of the facts and the constituents of the judgements. The third is an argument Russell gives explicitly. Universally quantified propositions cannot be deduced from particular premises. This requires some premises that are universally quantified, and these state general facts.<sup>6</sup>

Throughout this paper, I shall only be concerned with generality of first order. The issues with which Russell grapples arise already there, and considering higher-order generality is a needless complication. But one may reasonably surmise that equal considerations as those voiced here apply to the case of higher-order generality, too.

To close this introduction, a few words on terminology are necessary.

Russell names the *genus* after the *species*: the *genus* 'general fact' comprises the facts that all things are  $\phi$  and that some things are  $\psi$ , where the former are also called 'general facts' and the latter 'existence-facts' (*PLA* V & VI 198; *LK* 234f). I consider only universal facts, that all things are  $\phi$ , but there are also arguments for existence-facts.

In the passages quoted in this paper, Russell uses four different words for that to which truth and falsehood apply: 'judgement', 'belief', 'statement' and 'proposition'. There is some terminological confusion regarding the word 'proposition'. It is not to be understood in the sense of what are sometimes called 'Russellian propositions',<sup>7</sup> which, if true,

<sup>6</sup>Frege makes a related point in §3 of *Grundlagen der Arithmetik*. See below, footnote 50.

<sup>7</sup>As explained by Gaskin: 'Propositions, on the Russellian account, are purely extensional entities: simple propositions may be conceived as ordered  $n$ -tuples of objects and properties or relations' (Gaskin 1997, 140). This terminology goes back to Salmon and

are virtually indistinguishable from facts.<sup>8</sup> Rather, its use is in line with that of his *Introduction to Mathematical Philosophy*. Here Russell admits that ‘the word [“proposition”] cannot be formally defined’, but in order to say ‘something as to its meaning’ he explains: ‘We mean by a “proposition” primarily a form of words which expresses what is either true or false.’ (Russell 1919a, 155) It is also that of OKEW: ‘A form of words which must be either true or false I shall call a *proposition*. Thus a proposition is the same as what may be significantly asserted or denied.’ (52) A proposition in this sense is neither a Russellian proposition nor a mere string of words. It is a meaningful sentence, more precisely, ‘a proposition, one may say, is a sentence in the indicative, a sentence asserting something, not questioning or commanding or wishing.’ (PLA I & II 504; LK 185).<sup>9</sup> Part of this terminological confusion results from the need for a word for something that represents a fact independently of any judgement made. A commonly used term for this is ‘proposition’, and Russell follows suit.

Russell often uses “‘belief” and “judgement” as synonyms’ (1910a, 172 fn 1), and I will follow him.<sup>10</sup> Furthermore ‘the truth or falsehood of statements can be defined in terms of the truth or falsehood of beliefs. A statement is true when a person who believes it believes truly, and false when a person who believes it believes falsely.’ (Russell 1910a, 172) This indicates that Russell uses ‘statement’ as close in meaning to ‘proposition.’ I favour ‘judgement’ as the term for truth bearers, except where the discussion demands otherwise, as, all things considered, it is the term preferred by Russell.

Soames and their direct reference theory of meaning (e.g., Soames (1987) and Salmon (1986)). See also the SEP entry on structured propositions (King 2019) and Kaplan’s classic (1975). Not even Russell held on to them long and abandoned them around 1910. See Linsky (1993) and Linsky (1999, sec. 3.1).

<sup>8</sup>For an attempt to distinguish them, which, however, had to fail on Russell’s account, see Linsky (1999, sec. 3.2).

<sup>9</sup>This coheres with the use of ‘proposition’ recommended by Geach, to mean ‘a form of words in which something is propounded, put forward for consideration’ (Geach 1972, 255). Problems would arise with indexicals. Yet another explanation is given in OP, where Russell declares that ‘a proposition may be defined as: *What we believe when believe truly or falsely*’ (OP 1; LK 285), and then defines ‘a “proposition” as the content of a belief’ (OP 28; LK 308). According to this definition, a proposition need not be something linguistic, if what is believed is not always tied to language.

<sup>10</sup>Russell (1910a) is reprinted in Russell (1992a).

## 2. The Facts in Virtue of Which General Judgements are True

I shall try to avoid as much as possible questions regarding the nature of general facts and what Russell’s views on this issue may have been. This intricate topic deserves a separate investigation.<sup>11</sup> But a few words are necessary to clarify what it might be that we are speaking of in general judgments.

For present concerns it suffices to consider three options for the facts in virtue of which, e.g., ‘All humans are mortal’ may be true:<sup>12</sup>

- a. all the elementary facts of individual humans being mortal;
- b. the fact that the universal *human* is subsumed under the universal *mortal*;
- c. the fact that all humans are mortal.

I’m not claiming that Russell held (b), although it is not an unreasonable option, and there is a suggestion of it in *PP*. It is mentioned to distinguish it from (c). Between *PM* and *PLA* Russell moved from (a) to (c). My question is: why?

In case (a), the facts in virtue of which ‘All humans are mortal’ is true contain, in addition to the universals *human* and *mortal*, the particular humans, Diotima, Socrates, Plato, Aristotle, etc.. It is true in virtue of particulars that instantiate the first universal always also instantiating the second. In cases (b) and (c) the facts contain no particulars, but only universals. But only in (c) is there a genuine general fact.

I shall say that in both (b) and (c) the facts concern relations between universals. In case (c) the relation is of a very special kind: it is a logical relation. It is the relation called *formal implication* in *PM*.

From *Principles of Mathematics* (*PoM*) onwards, Russell was occupied with the question whether universals can be terms of relations or only relating relations, that is, whether they can appear as subjects (also referred to by verbal nouns) and as what is referred to by verbs, or only as what is referred to by verbs (§§48, 49, 54). The difference may

<sup>11</sup>To be carried out in Kürbis (n.d.).

<sup>12</sup>A fourth option might be Armstrong’s ‘totality facts’ (1997, 197ff; 2004, 72ff).



be exemplified, to use Russell's example, by the difference between 'is different from' and 'difference'.

I won't speak of verbs but only of predicates, and I won't be concerned with the question, as Russell was, whether there are any properties or qualities, i.e., one-place universals, or whether every universal is a relation. 'Relations and predicates together will be called "universals"'. (TK 81) I'll use 'predicate' and 'relation' to cover the cases of one as well as many places.

In *PoM*, Russell attributes to universals a curious twofold nature (45) that permits them to be terms in relations as well as relating relations. The nature of universals is not a topic of *PM*, and the theory of types only applied to individuals and propositional functions and to everything else derivatively thereof (*PM* 40, 51ff, 138). Linsky argues that nonetheless universals that cannot be terms, and indeed universals stratified into levels of logical types, are implicit in the metaphysics of *PM*, but concludes that the language of *PM* does not suit adequate reference to them (Linsky 1988; 1999, sec. 2.3). It 'does not serve well as a logic of universals' (Linsky 1988, 459), as it only has expressions for propositional functions and its higher-order quantifiers range over them, but universals are not propositional functions.

The view that universals can only be relating relations is explicitly Russell's in *PLA*. *PLA* contains considerations about the differences in the meanings of different expressions: names mean particulars, predicates mean universals, sentences mean facts. But the three relations referred to by the same word 'means' are essentially different. (*PLA I & II* 506f, *VII & VIII* 363f; *LK* 186f, 268f) Russell is adamant that only particulars can be named and only facts can be asserted or denied. Particulars cannot be asserted or denied, as presumably everyone agrees, and, perhaps more controversially, facts cannot be named. Consequently, predicates cannot name universals, but neither can names. Predicates (of first order) can only be asserted or denied of particulars. 'A predicate can never occur except as a predicate' (*PLA* 34; *LK* 205), and 'a relation can never occur except as a relation, never as a subject' (*PLA* 35; *LK* 206).<sup>13</sup> Thus universals can only be referred to by predicates, that is, by expressions Frege would call 'unsaturated' or that show, in Linsky's words, their 'predicative nature'. Universals belong to a different logical

<sup>13</sup>I thank a referee for reminding me of these passages.

type from particulars, and thus cannot be particulars (*PLA V & VI* 195ff, *VI & VIII* 350f; *LK* 232ff, 258f). To use colours as examples of universals, as does Russell, the universal *cyan* can then really only be referred to by the predicate 'is cyan' and 'cyan is a colour' is not really of subject-predicate form.<sup>14</sup> The universal 'cyan' cannot take the place of a particular, and *colour*, being a universal applying to universals, cannot apply to particulars. Then 'Fred is a colour', where 'Fred' names a particular, is meaningless, and so is 'Cyan is magenta', 'is magenta' being understood so that 'Fred is magenta' is meaningful.

It is sometimes said that if universals cannot be terms in a relation, then 'magenta is a colour' must be analysed as 'for any  $x$ , if  $x$  is magenta, then  $x$  is coloured'.<sup>15</sup> I do not see why this should be so. The formal analysis of 'magenta is a colour' would be that 'magenta' is a first-order predicate, while 'colour' is an expression that takes a first-order predicate and forms a sentence out of it, i.e., a second-order predicate. 'Magenta is a colour' is, therefore, contrary to appearance, a sentence formed from two predicates, one of first order, one of second order. There may not be a natural way of rendering the formal analysis of 'magenta is a colour' in English, but that is a problem of English. It merely shows the need for a formal language such as Russell's in which the distinction can be drawn and its importance, as it permits to clarify the issue more satisfactorily than English. 'Continuous' is another example: it takes, e.g., the first-order function  $1/x$  and forms the sentence ' $1/x$  is continuous'. The binding of the variable  $x$  is left implicit. This could be made explicit, as Frege does (1893, sec. 25). Mimicking *PM*'s notation for the universal quantifier,  $(x) \cdot \phi x$ , we could write '(continuous  $x$ )  $\cdot 1/x$ ' and '(colour  $x$ )  $\cdot x$  is magenta'. First-order logic is distinguished from second-order logic in that the former admits of only one kind of expressions that form sentences from first-order predicates—the

<sup>14</sup>Russell treats colours as universals at various points, e.g., (*PLA VII & VIII* 351; *LK* 258). Another example is: 'We are aware of the universal *yellow*; this universal is the subject in such judgements as "yellow differs from blue" or "yellow resembles blue less than green does". And the universal yellow is the predicate in such judgements as "this is yellow", where "this" is a particular sense datum' (Russell 1911a, 11; 1917, 212). A referee points out that if universals can't be terms, this makes acquaintance with universals problematic, as acquaintance is a relation between a mind and a term. This is not the place to attempt to resolve this conundrum.

<sup>15</sup>The point appears to stem from Ramsey (1978, 19). See Linsky (1988, 455), Linsky (1999, Ch. 1).

quantifiers—while the latter admits of any number of them. If Linsky is right about the language of *PM*, the approach proposed here would not be available to Russell in *PM*, but this may be down to the nature of the topic of *PM*, namely, nothing in particular, rather than a limitation intrinsic to its language. The language of *PM* has no individual constants nor any constants of any other type—apart from the logical constants, but they are in a rather different category. If one were to apply it to a particular subject matter, one would introduce names for particulars and predicate constants for their properties, and there would be no reason not to introduce also constants for properties of those properties. To illustrate with an example of Russell's (*PLA VII & VIII* 351; *LK* 258), 'colours exists in the spectrum between blue and yellow' means that the propositional function ' $x$  is a colour between blue and yellow' can be true, but, colours being universals, the  $x$  does not stand for a particular. Expressions referring to universals being first-order, 'is in the spectrum between ... and ...' is a three-place relation of second order, which we could represent by ' $(\text{SpecBetween } x) \cdot (\phi x, \psi x, \chi x)$ '. Letting 'blue  $x$ ' and 'yellow  $x$ ' represent the predicates 'is blue' and 'is red', Russell's example would then be analysed as ' $(\exists \phi) : (\text{SpecBetween } x) \cdot (\phi x, \text{blue } x, \text{yellow } x)$ '.

In attributing to Russell the view that there are higher-order universals I'm following Linsky, who observes that, although there is little explicit discussion of this in Russell's writings, it is strongly suggested in many passages (Linsky 1999, 34). To round of this discussion, here is one: in *PLA* the world is divided into simples and facts, where simples comprise particulars and universals and 'there are particulars and qualities and relations of various orders, a whole hierarchy of different kinds of simples' (*PLA* 365; *LK* 270). The view is also plausible independently, once a Russellian ontology of universals is accepted: universals are what things have in common, and universals have things in common, which are higher-order universals.<sup>16</sup>

I shall permit myself to refer to universals by nouns and noun phrases even if their nature requires predicates, and by predicates (generally omitting the 'is' of predication) even if their nature would permit a noun

<sup>16</sup>The view that there are no particulars, only universals, also suggests that there are higher-order universals: what holds together the universals that make up, say, Socrates, is not the particular, Socrates, but a higher-order universal.

or noun phrase. There is no way around this, because of the difference between (b) and (c).

In (b), the universals *human* and *mortal* may be terms or relating relations, depending on whether *subsumed under* is understood as a first- or as a second-order relation. In (c), because the fact mirrors the structure of 'All humans are mortal', which really has the form 'For all  $x$ , if  $x$  is human,  $x$  is mortal', in which the universals are referred to by predicates, the universals are relating relations, not terms.

Another difference between (b) and (c) is that while (b) is atomic, (c) is not. The judgement 'All humans are mortal' is general. In case (a), it is true in virtue of many atomic facts. In case (b) it is true in virtue of a single atomic fact. Only in case (c) it is true in virtue of a single non-atomic fact.<sup>17</sup>

Russell stays largely silent on the nature of general facts. Next to nothing on this topic is said in *PLA*, except that their technically convenient treatment is by propositional functions, but that this is 'not the whole of the right analysis', and that Russell does not profess to know what the right analysis of general facts is (*PLA V & VI* 200f; *LK* 236f). General judgements being non-atomic (*TK* 99), they may have been amongst the topics to be covered in later parts of *TK*, which—alas—apparently never got written, as Russell gave up the views developed in Parts I and II in reaction to Wittgenstein's criticism.<sup>18</sup> Nonetheless Russell's work

<sup>17</sup>In *TK*, molecular propositions are defined as those that have other propositions as parts, atomic ones as those that do not (*TK* 106). An atomic complex is expressed by the latter, a molecular one by the former, and an atomic complex contains only one relating relation (*TK* 80). According to these definitions, general propositions and facts are not molecular. On view (b) with *subsumed* understood as first-order, they are atomic. Facts of kind (b) with *subsumed* understood as second-order would be atomic according to (*TK* 106) and the first half of (*TK* 80), but, containing more than one relating relation, not atomic according to the second half of (*TK* 80). It should be noted that Russell says the definition of (*TK* 80) is 'merely provisional'. On both understandings of *subsumed*, facts of kind (b) should count as atomic—in the first as of first order, in the second of second order—because of their lack of any structure correlated with a logical constant. Because of their logical aspect, which they have in common with molecular facts, facts of kind (c) are not atomic, and indeed, Russell calls general judgements 'non-atomic' (*TK* 99). There is a suggestion of a tripartite division of propositions and facts into atomic, molecular and general, for which see also (*OKEW* 53ff).

<sup>18</sup>See Eames (1992, ix, xvi, xixf). *TK* (116) remarks that the content of the projected Part III would have been molecular propositions.

contains two suggestions of what the nature of general facts might be like that are relevant to the present discussion.

In *PP*, Russell at one point mentions abstract logical universals (*PP* 171).<sup>19</sup> We may surmise that these are the referents of logical words such as ‘or’, ‘not’, ‘all’ and ‘some’ and, like other universals, are constituents of facts.<sup>20</sup> In *TK*, the objects of logic are formal and occupy a third category alongside universal and particular (*TK* 99). Logical constants are ‘concerned with pure *form*’ (*TK* 98) and not constituents of facts.<sup>21</sup> The two options of understanding the nature of general facts then are:

- i. General facts contain a logical universal referred to by ‘all’, a second-order universal that requires first-order universals to form facts.<sup>22</sup>
- ii. General facts are facts of a certain form: they combine first-order universals in general fashion.

In case (i) we could say that there is a universal referred to by formal implication, in (ii) formal implication expresses a form of facts.<sup>23</sup>

There is merely a hint of (i) as an option in Russell’s work, and although Russell is not explicit that he held (ii), this is suggested by his introduction of logical forms in *TK* and a brief discussion of negative facts in *OP*. Presumably, then, Russell also gravitated towards the view

<sup>19</sup>See also Russell’s assertion that  $2 + 2 = 4$  deals exclusively with universals in the quotation from *PP* (163f) below: for Russell the logicist these must be logical universals.

<sup>20</sup>If this approach is adopted, we could, *pace* Linsky (1988, 459), say that *PM* does in fact have expressions that stand for universals, namely the connectives and quantifiers. The view goes against the *Grundgedanke* of Wittgenstein’s *Tractatus*: logical constants do not refer to, stand for or represent anything, a view also expressed in *PLA*. Hossack (2007, Ch. 2) develops such a view. Hossack’s work is thoroughly Russellian and repays close study. For an overview, see Hossack (2022) and Kürbis, Assadian, and Nassim (2022), and for critical assessment of Hossack’s account of facts and knowledge, see Weiss (2022).

<sup>21</sup>In *PP*, abstract logical universals are said to be knowable by acquaintance (171) (and see 163f). In *TK* Russell says that ‘general logical facts...are often such as can be perceived’ (*TK* 47). Logical intuition, acquaintance with logical form, is said to be ‘required to account for our understanding such words as “or”, “not”, “all” and “some”’ (*TK* 101). See also the remarks on seeing clearly and being acquainted with the entities concerned in the discussion of indefinables in the Preface to *PoM*.

<sup>22</sup>Recall that I am restricting consideration to generality expressible in first-order logic.

<sup>23</sup>Even if there are logical universals, and formal implication expresses such a universal, the difference between (b) with *subsumed under* understood as second-order and (c) remains: (b) is an atomic fact and lacks the logical force of (c). See Section 5.3.

that being general is a form of facts by the time of *PLA*. In *OP* (4; *LK* 287), Russell says that positive and negative are forms of facts: negative facts contain the same constituents as positive facts, only combined in a different, negative way. The negative fact that Socrates is not alive combines Socrates and the universal *alive* in the form of negative facts, just as the fact that Socrates is mortal combines Socrates and the universal *mortal* in the form of positive facts. Russell then draws a parallel between negative and general facts (*OP* 6; *LK* 289), on which more later: the conclusion that general facts must exist is supported by considerations similar to those that establish the existence of negative facts.

### 3. Truth, Correspondence and General Judgements

Despite Russell’s changes of mind, I shall take two things as standing firm throughout the period under investigation:

1. Truth consists in correspondences with facts.
2. There are general judgements.

Let’s look at each one in turn.

#### 3.1. Truth and Correspondence

It is well known that Russell adopted a correspondence theory of truth. Indeed, he became its foremost proponent. Arthur Prior observes that ‘the term “correspondence theory of truth” has circulated among modern philosophical writers largely through the influence of Bertrand Russell, who sets the view (which he himself adopts) that “truth consists in some form of correspondence between belief and fact” against the theory of the absolute idealists that “truth consists in coherence,” that is, that the more our beliefs hang together in a system, the truer they are.’ (Prior 2006, 539) Some quotations from Russell should suffice for illustration.<sup>24</sup>

When we judge truly, some entity “corresponding” in some way to our judgement is to be found outside our judgement, while when we judge falsely there is no such “corresponding” entity...The truth or falsity of our judgement depends upon the presence or absence of a “corresponding” entity of some sort. (Russell 1910a, 176)

<sup>24</sup>For an overview of the correspondence theory of truth, see Kürbis (2024b).

*PM* gives details about the nature of these entities and contains an explicit definition of truth in terms of correspondence:

We may define *truth*...as consisting in the fact that there is a complex *corresponding* to the discursive thought which is the judgment. That is, when we judge "*a* has the relation *R* to *b*" our judgment is said to be *true* when there is a complex "*a*-in-the-relation-*R*-to-*b*," and is said to be *false* when this is not the case. (*PM* 46)

This is the notion of truth applicable to the simplest judgements, those composed from names and predicates that apply to names. Russell calls these 'elementary judgements'. *PM* defines truth differently for general judgements, on which more soon.

More can be said about the nature of correspondence, but for present purposes it suffices that truth consists in correspondence to a fact, as Russell calls the complexes of *PM* later: 'A belief is true when there is a corresponding fact, and is false when there is no corresponding fact' (*PP* 202).<sup>25</sup> Let's move on to general judgements.

### 3.2. General Judgements and their Truth in *PM*

#### 3.2.1. General Judgements in *PM*

According to Whitehead and Russell, there are irreducibly general judgements:

But take now such a proposition as "all men are mortal." Here the judgement does not correspond to *one* complex, but to many, namely "Socrates is mortal," "Plato is mortal," "Aristotle is mortal," etc... Our judgment that all men are mortal collects together a number of elementary judgments. It is not, however, composed of these, since (*e.g.*) the fact that Socrates is mortal is no part of what we assert, as may be seen by considering the fact that our assertion can be understood by a person who has never heard of Socrates. In order to understand the judgment "all men are mortal," it is not necessary to know what men there are. We must admit, therefore, as a radically new kind of judgment, such general assertions as "all men are mortal." We assert that, given that *x* is human, *x* is always mortal. That is, we assert "*x* is mortal" of *every x* which is human. Thus we are able to judge (whether truly or falsely) that *all* the objects which have some assigned property also have some other assigned property. That is, given any propositional functions  $\phi\hat{x}$

<sup>25</sup>In *PP* Russell uses both expressions, 'fact' and 'complex'.

and  $\psi\hat{x}$ , there is a judgment asserting  $\psi x$  with every *x* for which we have  $\phi x$ . Such judgments we will call *general judgments*. (*PM* 47)

The example of a general judgement belongs to the kind called 'universally affirmative' in Aristotelean logic: two propositional functions are combined into a proposition of the form 'All *A* are *B*'. But the account applies equally to general propositions formed from only one or more than two propositional functions. Many of the former are false, such as 'Everything is human', but some of them may be true, such as, borrowing from Quine, 'Everything exists' or "Everything is identical to something".<sup>26</sup>

One part of the passage omitted from the quotation comments on the nature of elementary judgements and the difficulty of finding any. The examples given are not, in fact, elementary, at least not for those not acquainted with Socrates, Plato and Aristotle. 'Truly elementary judgements are not very easily found' (*PM* 47). This need not concern us here, and with Russell, we may treat the examples as elementary for heuristic purposes.

The other part reads: 'We do not mean to deny that there may be some relation of the concept *man* to the concept *mortal* which may be *equivalent* to "all men are mortal," but in any case this relation is not the same thing as what we affirm when we say that all men are mortal.' As far as I can see, this is the only use of 'concept' in *PM*. In *PP*, 'a universal of which we are aware is called a *concept*' (81); in *TK* concepts are 'universals with which we are acquainted' (101). Presumably, then, in *PM* concepts are universals, too, and the judgement that is different from but equivalent to "all men are mortal" is 'the concept *man* is subsumed under the concept *mortal*', or something to that effect. If the judgement that all men are mortal is true, so is the judgement that the universal *man* is subsumed under the universal *mortal*, and conversely. Thus in *PM* there is a suggestion of a distinction on the side of judgements that is

<sup>26</sup>The question has been put to me whether Whitehead and Russell understand quantification objectually or substitutionally. I take it that the intention is that it is objectual, but nothing much seems to hang on this question for this paper. A referee points out that in the *Tractatus* Wittgenstein appears to accept substitutional quantification and to reject general facts and asks whether the two are connected, and whether it is Russell's rejection of substitutional quantification that is connected to his eventual acceptance of general facts. This is of great interest. I won't attempt to address these questions and mention them only for the readers' considerations.



analogous to the distinction between (b) and (c) of Section 2 on the side of facts. The judgement that all men are mortal is a judgement about all particulars, that is, things of a lower type than the universals *man* and *mortal*, something expressed by formal implication. The judgement that *man* is subsumed under *mortal* is not a judgement about particulars, but a judgement that is only about those universals and the universal *subsumed under*.

### 3.3. The Truth of General Judgements in *PM*

In *PM* general judgements have their own mode of truth:

the definition of *truth* is different in the case of general judgements from what it is in the case of elementary judgements. Let us call the meaning of *truth* which we gave for elementary judgements "elementary truth." Then when we assert that it is true that all men are mortal, we shall mean that all judgements of the form "*x* is mortal," where *x* is a man, have elementary truth. We may define this as "truth of second order" or "second-order truth."...

We use the symbol " $(x) \cdot \phi x$ " to express the general judgement which asserts all judgements of the form " $\phi x$ ." Then the judgement "all men are mortal" is equivalent to " $(x) \cdot 'x$  is a man' implies ' $x$  is mortal' "

...the meaning of *truth* which is applicable to this proposition is not the same as the meaning of *truth* which is applicable to "*x* is a man" or to "*x* is mortal." And generally, in any judgment  $(x) \cdot \phi x$ , the sense in which this judgment is or may be true is not the same as that in which  $\phi x$  is or may be true. If  $\phi x$  is an elementary judgment, it is true when it *points to* a corresponding complex. But  $(x) \cdot \phi x$  does not point to a single corresponding complex: the corresponding complexes are as numerous as the possible values of *x*. (*PM* 47f)

Thus second-order truth consists in correspondence to numerous complexes. One way of putting this would be that an elementary judgement corresponds to one complex, a general judgement to many.

According to *PM*, then, general judgements:

- a. collect together the elementary judgements that are their instances;
- b. correspond to the complexes to which their instances correspond, if true.

Following Russell, I will also say that general judgements point to the complexes that correspond to the elementary judgements they collect together.<sup>27</sup>

In Section 4 I shall argue that this account of the truth of general judgements is unsatisfactory. This is likely to provoke the following criticism.<sup>28</sup> Three pages before the passage just quoted, and a page after the symbol ' $(x) \cdot \phi x$ ' is introduced as denoting 'the proposition which asserts *all* the values for  $\phi \hat{x}$ ' (*PM* 43), there is also an explanation of the truth of such propositions. The theory of types requires that  $(x) \cdot \phi x$  has a different sort of truth than its instances. Call the latter 'first truth'. If  $(x) \cdot \phi x$  'has truth of the sort appropriate to it, that will mean that every value  $\phi x$  has "first truth." Thus if we call the sort of truth appropriate to  $(x) \cdot \phi x$  "second truth," we may define " $\{(x) \cdot \phi x\}$  has second truth" as meaning "every value for  $\phi \hat{x}$  has first truth," *i.e.* " $(x) \cdot (\phi x$  has first truth)." (*PM* 44). With hindsight this looks like a fair stab at how Tarski would define its truth: ' $(x) \cdot \phi x$ ' is true iff for all values of ' $x$ ', ' $\phi x$ ' is true.<sup>29</sup> And so one might be tempted to say that Russell got the explanation of the truth of general judgements right in *PM*.

The explanation given at this point, however, can only have been heuristic: truth has not been defined yet. This is also shown by the fact that Russell speaks about first and second truth, not first- and second-order truth, which is the official terminology from the theory of types introduced later. Elementary judgements are explained on the following pages and their truth is defined in terms of correspondence. The latter is named 'elementary truth' (48), which also contains the definition of second-order truth, the notion of truth applicable to general judgements. Here, too, we find Russell reverting to correspondence. The earlier passage is phrased entirely in terms of propositions. Truth, however, first and foremost applies to judgements, as explained at (45f). Indeed, propositions are incomplete symbols (46), and hence require explanation in terms of judgements. Ultimately, then, truth as applied to propositions is to be explained in terms of truth applied to judgements.

<sup>27</sup>The *PM* view has some similarities to the one expressed by Wittgenstein in the *Tractatus*. See Anscombe (1971, Ch. 11). Wittgenstein, however, does not speak of judgements, nor does he distinguish modes of truth.

<sup>28</sup>This discussion was added at the request of a referee.

<sup>29</sup>Evidently this glosses over what 'for all values of " $x$ "' means in the clause for the universal quantifier of Tarski's definition of truth.

And this is what happens on p. 47f. I am therefore taking this later account to be the official one.

Even if the earlier explanation was part of Russell's official account of the truth of general judgements, he must have been dissatisfied with it: it is no longer in *PLA* and instead the truth of such propositions is explained in terms of general facts. The focus on the definition of truth in terms of correspondence, and consequently its shortcomings, would explain this. Section 5.1 contains some more thoughts on this matter.

### 3.4. General Judgements in *PP*

*PP* contains an account of what is involved when the mind judges generally. Russell focusses on general knowledge, but his remarks apply to judgements, too.<sup>30</sup> Relations between universals permit *a priori* knowledge: 'All *a priori* knowledge deals exclusively with the relations of universals' (*PP* 162). Russell makes a similar point to the one made in *PM*, that to understand a general proposition, one need not know all its instances, but is now explicit that it is the universals involved, and only them, that need to be known:

One way of discovering what a proposition deals with is to ask ourselves what words we must understand—in other words, what objects we must be acquainted with—in order to see what the proposition means...Many propositions which might seem to be concerned with particulars are really concerned only with universals. In the special case of 'two and two are four,' even when we interpret it as meaning "any collection formed of two twos is a collection of four," it is plain that we can *understand* the proposition, *i.e.* we can see what it is that it asserts, as soon as we know what is meant by "collection" and "two" and "four." It is quite unnecessary to know all the couples in the world: if it were necessary, obviously we could never understand the proposition, since the couples are infinitely numerous and therefore cannot all be known to us. Thus although our general statement *implies* statements about particular couples, *as soon as we know that there are such particular couples*, yet it does not itself assert or imply that there are such particular couples, and thus fails to make any statement whatever about actual particular couples. The statement made is about 'couple,' the universal, and not about this or that couple.

<sup>30</sup>See chapter XIII, which concerns the question under which conditions a judgement or belief constitutes knowledge.

Thus the statement "two and two are four" deals exclusively with universals, and therefore may be known by anybody who is acquainted with the universals concerned and can perceive the relation between them which the statement asserts. (*PP* 163f)

Not only do we not need to know all its instances in order to understand an *a priori* proposition. We need not know any instances at all. The instances the propositions of logic and arithmetic, the most general ones there are, may have are irrelevant to their truth, and acquaintance with any of them is not necessary for the *a priori* reflection through which we come know them.

In *PP* Russell draws attention to the distinction between general judgements, or what is required of someone to understand them, and the evidence for their truth. General judgements concern only universals, but evidence for their truth is sometimes found in their instances. General knowledge need not be *a priori*:

It will serve to make the point clearer if we contrast our genuine *a priori* judgement ["two and two are four"] with an empirical generalization, such as "all men are mortals." Here as before, we can *understand* what the proposition means as soon as we understand the universals involved, namely *man* and *mortal*. It is obviously unnecessary to have an individual acquaintance with the whole human race in order to understand what our proposition means. Thus the difference between an *a priori* general proposition and an empirical generalization does not come in the meaning of the proposition; it comes in the nature of the evidence for it. (*PP* 166)

There is no *prima facie* reason to suppose that we cannot come to know empirical generalisations.<sup>31</sup> But even if the only general knowledge was

<sup>31</sup>In *PLA*, Russell insists that 'if there is, as there seems to be, knowledge of general propositions, then there must be *primitive* knowledge of general propositions (...which is not obtained by inference)' (*PLA* V & VI 199; *LK* 235). Inference here means deductive inference. This does not exclude the possibility that induction from instances may lead to knowledge. Earlier, and in relation to a point to be discussed later in this paper, Russell wrote: 'General truths cannot be inferred from particular truths alone, but must, if they are to be known, be either self-evident, or inferred from premises of which at least one is a general truth. But all empirical evidence is of *particular* truths. Hence, if there is any knowledge of general truths at all, there must be *some* knowledge of general truths which is independent of empirical evidence, *i.e.* does not depend upon the data of sense.' (*OKEW* 56) Russell gives as an example the principle of induction. In the footnote to §3 of *Grundlagen* (Frege 1884, 4), Frege also adduces induction to illustrate his view. See footnote 50.

*a priori* knowledge, there would still be irreducibly general knowledge. Be that as it may, Russell's point applies equally to well supported empirical belief. The difference between empirical and logical general judgements lies in the evidence for their truth. The evidence for the truth of 'all men are mortals' is particular men's mortality. These are the complexes to which *PM* says the general judgement corresponds and the elementary judgements it collects together. The evidence does not consist in our perception of a relation between the universals *man* and *mortal* (*PP* 166). Nonetheless, it is a judgement that concerns a relation between these universals, and it may be understood by anyone who has the concepts involved. In the case of general judgements that are *a priori*, it suffices to be acquainted with the universals: evidence for their truth or falsehood is found through reflection on them.

If all general knowledge is either *a priori* or empirical, then all general knowledge is concerned with the relation of universals. But not all judgements that deal exclusively with universals are general: 'magenta is a colour' or 'yellow is brighter than orange' are not.

### 3.5. The Contrast between *PM* and *PP*

In *PP*, Russell no longer says that a general judgement collects together all judgements that are its instances nor that it corresponds to the numerous elementary complexes they correspond to. These are the evidence, while the judgement concerns universals. The distinction between general judgements and equivalent judgements about universals is also no longer drawn.

In *PP* Russell appears to deny something *PM* appears to assert: 'our general statement...fails to make any statement whatever about actual particular couples.' (*PP* 163) *PM*, by contrast, says that a general judgement asserts all its instances (43, 48), and hence, as these instances are judgements about particular complexes, one should expect that the general judgement does the same.

*PP* contains no comparable discussion of the truth of general judgements at all.<sup>32</sup> As in *PP* general judgements concern only universals, this may suggest that Russell now accepted view (b) of Section 2: general judgements are true in virtue of facts concerning the subsumption of

<sup>32</sup>Neither does *TK*, as already mentioned, nor *OKEW*.

universals. If 'proposition' is read as 'Russellian proposition', then a comment made a year before supports such a conclusion, too: 'La proposition : "Tous les hommes sont mortels", est composée exclusivement de concepts : les hommes actuels n'en sont pas des constituants. Cela est évident, puisqu'on peut comprendre la proposition sans connaître tous les hommes.' (Russell 1911b, 9) But Russell is not explicit that (b) was his view, and it is not vital for the following that it was.

I am taking the contrast between *PM* and *PP* to be a hint at a change of mind. It may be that Russell's concerns in *PP* were different from those in *PM*, namely with the epistemology of general judgements rather than with the metaphysics that explains their truth.<sup>33</sup> It may also be that in a popular text like this one, he wished to avoid the complexities of the latter, which would have required a discussion of the theory of types. But Russell had no qualms of at least touching upon the complexities of the theory of types in *PLA*, which also had a generalist audience in mind. It is remarkable that this aspect of the *PM* account is ignored altogether in *PP*. Doubts about its correctness would explain the omission.

## 4. A Problem with Falsehood in the *PM* Account of the Truth of General Judgements

*PP* contains a famous announcement of 'three requisites which any theory [of truth] must fulfil': (a) 'Our theory of truth must be such as to admit its opposite, falsehood'; (b) 'truth and falsehood are properties of beliefs and statements'; (c) 'the truth or falsehood of a belief always depends on something outside the belief itself.' (*PP* 188f) I shall argue that the *PM* account of the truth of general judgements violates the first of these principles.

According to *PM*, an elementary judgement is true if there is a complex to which it corresponds or to which it points, false if there is no such complex. The truth of general judgements is explained in terms of correspondence or pointing to many complexes. This gives a plausible account of the truth of general judgements. 'All men are mortal' corresponds or points to all the complexes of particular men

<sup>33</sup>As suggested by a referee.

being mortal. In general, if  $(x) \cdot \phi x$  is true, it corresponds or points to the numerous complexes  $\phi a, \phi b, \phi c \dots$

But now consider a general judgement that is false. There are only those facts that there are and not also those that there are not. So a general judgement that is false also corresponds or points to all the facts it can possibly correspond or point to, namely all the relevant ones that there are.

If  $\phi x$  is not true of anything, then there are no elementary complexes to correspond or point to, and we can say  $(x) \cdot \phi x$  is false. But suppose  $\phi x$  is true of some things and false of others. In that case, as  $(x) \cdot \phi x$  collects together all the judgements that are its instances, it corresponds to those complexes that correspond to the true instances of  $\phi x$ . There are no complexes corresponding to the false instances of  $\phi x$ : there is nothing to correspond or point to regarding those cases. But then  $(x) \cdot \phi x$  corresponds or points to everything it can possibly correspond or point to, and so it looks as if it should be true.  $(x) \cdot \phi x$  corresponds to complexes as numerous as its instances, but there are only the complexes corresponding to its true instances. No others exist. So a judgement  $(x) \cdot \phi x$  that is true for some  $x$ , false for others, corresponds to exactly the same amount of facts as a proposition  $(x) \cdot \psi x$  that is true of all  $x$ : namely all those that there are, which is, all of them. So  $(x) \cdot \phi x$  should be just as true as  $(x) \cdot \psi x$ .

To illustrate, suppose we restrict quantification to birds living in the Tower of London. There are exactly six ravens amongst them: Jubilee, Harris, Poppy, Georgie, Edgar and Branwen.<sup>34</sup> They are all black. Then 'All ravens are black' collects together the judgements 'Jubilee is a black raven', 'Harris is a black raven', 'Poppy is a black raven', 'Georgie is a black raven', 'Edgar is a black raven' and 'Branwen is a black raven'. 'All ravens are black' is true in virtue of the complexes corresponding to those six instances: Jubilee being a black raven, Harris being a black raven, Poppy being a black raven, Georgie being a black raven, Edgar being a black raven and Branwen being a black raven. Suppose there were also two black crows, Muninn and Gripp, and one white crow, Grog, amongst the birds living in the Tower of London.<sup>35</sup> Then 'All

<sup>34</sup>As of 31 December 2024: <https://www.hrp.org.uk/tower-of-london/whats-on/the-ravens/#gs.iwnhoy>.

<sup>35</sup>A referee points out that the formal system of *PM* requires that at least one particular exists see *PM* (20f, 239), and so logical truths have at least one instance. However, I take

crows are black' collects together the judgements 'Muninn is a black crow', 'Gripp is a black crow' and 'Grog is a black crow'. But when it comes to the complexes corresponding to these judgements, there are only Muninn's being a black crow and Gripp's being a black crow. There is no complex of Grog's being a black crow, as he is white. The complex of Grog's being a black crow does not exist. There is nothing there for 'All crows are black' to point to when it comes to Grog. And so all that 'All crows are black' could correspond or point to are the two complexes of Muninn and Gripp being black crows. But then 'All crows are black' should be true, as it corresponds to all the relevant complexes there are.<sup>36</sup>

The problem with the *PM* account of the truth of general judgements, then, is the following. Truth is explained in terms of the existence of facts. Judgements are true if facts exist to which they correspond, false if no such facts exist. Now consider a judgement  $(x) \cdot \phi x$  such that  $\phi x$  is true of some  $x$ . Then there is something to which  $(x) \cdot \phi x$  corresponds, namely those facts. So  $(x) \cdot \phi x$  should be true. Moreover it corresponds to everything that exists and is relevant to its truth. However, in some such cases  $(x) \cdot \phi x$  should be false. The attempt to explain the truth of  $(x) \cdot \phi x$  in terms of correspondence fails, as it does not yield an adequate account of the falsehood of general judgements.

I have said that on the *PM* account a general judgement that has no instances may be declared false as then there is nothing to which it corresponds. The problem, then, may be even worse. The propositions of logic are supposed to remain true even if they have no instances: they are true no matter what exists or does not.<sup>37</sup> We cannot say that a logical truth is 'true by default': true if nothing speaks against it, as it

this to be a feature of *PM* that is not wholly satisfactory. Indeed, Russell came to regard it as a defect (1919a, 203 fn 1), as there is no logical necessity that at least one particular must exist. Elkind argues for the existence of general facts from a thoroughly Russellian perspective from the assumption that it is logically or metaphysically possible that no particular exists (2021, sec. 6).

<sup>36</sup>'All crows are black' should not point to the complex of Grog's being white, as then, once more, it would point to an existing complex and thus to something in virtue of which judgements are true.

<sup>37</sup>A referee points out that the formal system of *PM* requires that at least one particular exists see *PM* (20f, 239), and so logical truths have at least one instance. However, I take this to be a feature of *PM* that is not wholly satisfactory. Indeed, Russell came to regard it as a defect (1919a, 203 fn 1), as there is no logical necessity that at least one particular must exist. Elkind argues for the existence of general facts from a thoroughly Russellian



were. General judgements that are contradictory have the same property: nothing speaks against them, as they, too, have no instances. Thus the *PM* account may fail to draw an adequate distinction between the logical truth and the logical falsehood of general judgements. I won't follow up on this: one problem suffices to show the need for an alternative.

The argument given in this section is not taken directly from Russell's work. That some such line of thought may have driven him to give up on the *PM* account of the truth of general judgements is, however, not mere speculation. During the first decade of the 20th century Russell was intensively occupied by the nature of truth, which at one point he pronounced to be the most fundamental question of philosophy (Russell 1910d, 130).<sup>38</sup> Russell regularly adduces failure to account adequately for falsehood as a decisive argument against a theory of truth. But it is only in *PP* that it is announced as requisite of a theory of truth that it adequately account for falsehood. To make the case, it will help to go through Russell's development on this issue.

The years 1906 and 7 saw the publication of three articles by Russell on the nature of truth, all of which engage with Harold Joachim's *The Nature of Truth* (1906), and two of which are reviews of this book.<sup>39</sup> In Russell (1906b), Russell does not yet present himself as endorsing the correspondence theory of truth, but rather holds the view 'that truth is primarily a property of *facts*' and that a belief 'is true in a derivative sense, namely the sense that it is a belief in a *fact*'. Russell notices that error is a problem for Joachim's coherence theory of truth, but mostly because he finds the chapter on error of Joachim's book wanting. Although the final four paragraphs of his review are an amusing and insightful discussion of error, there are no signs yet of his later announcement that being able to account for falsity is a requisite any satisfactory theory of truth must fulfil. His review of the same book for *Mind* (Russell 1906a) is more serious in tone. It contains no discussion of error, but instead underlines the difficulty of adjudicating between philosophical views that disagree on fundamental principles. Russell observes that the only way of refuting Joachim's view may be a *reductio ad absurdum* of the

perspective from the assumption that it is logically or metaphysically possible that no particular exists (2021, sec. 6).

<sup>38</sup>This essay was originally published in the *Albany Review* in January 1908 under the title 'Transatlantic "Truth"' (Russell 1910d, 127). Reprinted in Russell (2014).

<sup>39</sup>Both are reprinted in Russell (2014).

coherence theory of truth on internal grounds, that is, on principles Joachim himself accepts. Russell attempts to provide such a *reductio* on the basis of the observation that 'if coherence (in his [Joachim's] sense) is the essence of truth, then it cannot be quite true that coherence (in his sense) is the essence of truth'. This appeals to a notion of falsehood, in 'that whatever entails its own falsehood is itself false', but the necessity of accounting for falsehood is not mentioned as a requisite of a satisfactory theory of truth. In Russell (1907), Joachim's view is labeled 'monism', 'the view that truth is one'.<sup>40</sup> The first section of this paper contains a discussion of error that begins with the observation that accounting for error is a problem for the monistic theory of truth (1907, 32) and ends in the conclusion that 'there is no explanation, on the coherence-theory, of the distinction commonly expressed by the words *true* and *false*, and no evidence that a system of false propositions might not, as in a good novel, be just as coherent as the system which is the whole truth' (1907, 34). But, as before, accounting for falsity is not put forward as a general criterion of a satisfactory theory of truth, but rather as a problem that specifically afflicts Joachim's theory: the focus lies elsewhere and the problem is only adduced with the aim of refuting monism.<sup>41</sup> Section III sketches two alternative theories of truth. One of them is the correspondence theory, according to which truth is a property of beliefs, namely those that are beliefs in facts. Russell argues that the problem of error is also a problem for this theory, as then it would appear that 'error is belief in nothing' (1907, 46). There is a brief suggestion that what will be developed into the multiple relation theory of belief may provide a solution, but Russell is not yet satisfied with it (1907, 46f fn). The other is the theory according to which truth is a property of propositions, understood as non-mental complexes. The true propositions are the facts, the false ones Russell calls 'fictions', where the difference between truth and falsity is ultimate (1907, 48). Russell remains characteristically cautious and endorses neither theory. Both views are driven by considerations of what Russell calls 'objective falsehood' and the aim of accounting for the existence of error. Russell thus acknowledges problems concerning falsehood in relation to two further theories of truth, but, again, he does not yet pronounce that, quite generally, any theory of truth must admit of falsehood.

<sup>40</sup>Russell (1907) is reprinted in Russell (2014).

<sup>41</sup>The section culminates in a summary of the four objections made to Joachim's theory, with Russell pointing out that each of them amounts to a *reductio ad absurdum*.

The last essay reappeared partially in *Philosophical Essays*, the first two sections as essay VI under the title ‘The Monistic Theory of Truth’, while its third section was replaced by essay VII, entitled ‘On the Nature of Truth and Falsehood’ (Russell 1910b vi). The latter, written for this collection, expounds the views Russell held then: the correspondence theory of truth and the multiple relation theory of judgement, which ‘the possibility of false judgement compels us to adopt’ (1910a, 174). It is presented as providing a solution to the problem posed by the possibility of false judgement. Judgement cannot be a relation between a mind and a fact, as if I judge falsely, there is no fact, and hence nothing to relate to (1910a, 177). The multiple relation theory of judgement accounts for this possibility without admitting the ‘incredible’ existence of objective falsehoods (1910a, 176). Thus the focus is on drawing the distinction between what is true or false, i.e., beliefs or judgements, and what makes them true or false, the facts, which exist or fail to exist. The two corresponding requisites for any theory of truth of *PP* at one point even appear on the same page (Russell 1910a, 173). The title of the new essay is significant: it indicates that Russell now accords falsehood an equally important place as truth, as he does in the three requisites of *PP*. Russell is edging ever more closely to the first requisite of any theory of truth of *PP*, but an announcement of it is still lacking.

Russell’s engagement with the pragmatist theory of truth culminates in two essays published in 1908 and 1909 and also reprinted in *Philosophical Essays*. The first one, on William James’s conception of truth, I have already mentioned. It contains a discussion of what the falsehood of a belief might consist in for pragmatism, and problems one may encounter with it, but these play a comparatively minor role in the overall argument. Russell’s focus is on the notion of truth in order to show that ‘the pragmatist theory of truth is to be condemned on the ground that it does not “work”’ (Russell 1910d, 149). In the other essay, ‘Pragmatism’, on the other hand, the problem of falsehood does play an important role.<sup>42</sup> Russell acknowledges that ‘the pragmatic theory of truth takes credit to itself...for a due consideration of error’, which distinguishes it from most other theories of truth, such as Joachim’s, and emphasises ‘that the proper business of a theory of truth is to show how truth and falsehood

<sup>42</sup>Initially published in the *Edinburgh Review*, April 1909 (Russell 1910c, 87). Reprinted in Russell (1992a).

are distinguished’ (Russell 1910c, 98).<sup>43</sup> Despite its awareness of the importance of falsehood, however, Russell concludes that pragmatism succumbs to the problem it sets out to solve and ‘involves a variety of the very assumption which it criticises in others, namely, that all our beliefs are true’ (Russell 1910c, 113). In these two essays, too, then, we find Russell according greater importance to falsehood from one essay to the next.

To sum up, in these articles the problem of falsehood is noted as it affects various theories of truth, including the correspondence theory, and Russell formulates two major solutions to it, one of which he eventually adopts. Throughout the period, Russell holds the realist view that the truth and falsity of a judgement is dependent on what is judged, not on those who judge, which is principle (c) of *PP*. Initially, Russell endorses or at least leans towards the view that truth and falsity are properties of facts, rather than beliefs. This he gives up in 1910 the latest, as he replaces the third section of ‘On the Nature of Truth’ with a new essay arguing the point, and instead adopts principle (b) of *PP*. It is worth remarking that the listing of the three principles of *PP* reverses the order in which Russell came to adopt them. It is not until *PP*, that is, until after the formulation of the account of the truth of general judgements of *PM*, that to provide an adequate account of falsehood is put forward as a prerequisite for a satisfactory theory of truth—indeed, it is the first such prerequisite.

I conjecture that Russell gave himself some of his own medicine: *PM* fails to provide an adequate account of the falsehood of general judgements. His occupation with the problem that the possibility of falsehood poses for theories of truth rival to the correspondence theory, and, indeed, the problems that this theory itself faces, led him to realise that the *PM* account is not an adequate account of the falsehood of general judgements, and must, therefore, be abandoned and an alternative solution sought.

## 5. Three Arguments for General Facts

After a discussion of negative facts in *PLA*, Russell announces a general principle: ‘A thing cannot be false except because of a fact’ (*PLA III & IV*

<sup>43</sup>Reprinted in Russell (1992a).

46; *LK* 214). *PM* failed to take into account any facts in virtue of which general judgements are false. If all we admit are the complexes to which the elementary judgements correspond, then so long as there are some, these are all the complexes that there are, and the general judgement should be true, as it corresponds to all the facts that are admitted.

If  $(x) \cdot \phi x$  is false, then (or so we can grant Russell) it entails some false elementary proposition. But entailment is rather different from collecting together, because propositions entail all kinds of propositions Russell probably wouldn't be happy to count amongst those collected together by a general judgement. A more promising option is the following. If there are negative facts, then some (or all) of the judgements collected together by a false judgement  $(x) \cdot \phi x$  correspond to negative facts. The general judgement  $(x) \cdot \phi x$  points to all elementary facts involving the universal  $\phi$ . If some of them are negative, the judgement is false. But even though in *PLA* Russell accepts that there are negative facts, this is not the route he takes to explain the falsity of general judgements. The reasons, I suggest, are found in the three arguments for general facts to be discussed next. One is explicitly given by Russell (Section 5.3), the others are not, but they are suggested by his writings.

Appealing to general facts gives a better account of the truth of general judgements: a general judgement is true if it corresponds to a general fact. The truth of a general judgement is explained straightforwardly in terms of correspondence. The difference between orders of truth required by the theory of types has then something to do, not with the way the judgements correspond, but with what they correspond to. The problem of the falsehood of general judgements is easily solved by an appeal to general facts. It is a matter of whether or not the universals in question are related in general facts as the judgements judge them to be. Instances do not come into the equation at all. Accepting general facts therefore has clear advantages.

### 5.1. All the Facts

After Tarski, one is presumably inclined to say that the judgement that all men are mortal is true if and only if all men are mortal.<sup>44</sup>

<sup>44</sup>Glossing over rather a lot of subtle detail.

As noted earlier, Russell appears to say something very much like this. However, he also says that general judgements collect together elementary judgements and correspond or point to the corresponding elementary complexes. Russell's and Tarski's accounts are not the same. The essence of Tarski's semantic definition of truth resides in the homophonic clauses that avail themselves of 'all' in the metalanguage. The essence of Russell's account is correspondence. Again, a general judgement can only correspond to what exists, not also to what does not exist, and thus arises the question concerning the falsehood of general judgements.

Even if, possibly with hindsight, we read the Tarskian truth conditions into *PM*, the question remains why Russell does not repeat anything similar later on and why he introduces general facts instead.

I suggest an answer that exploits a comment of Russell's, 'that a not dissimilar set of considerations [as those showing that there must be negative facts] shows the necessity of admitting general facts, i.e., facts about all or some of a collection.' (*OP* 6; *LK* 289) The comment comes after Russell has considered and rejected two attempts to make do without admitting negative facts. One is to attempt to explain away negative facts in terms of a relation of incompatibility as holds between being round and being square. Russell concludes that 'the only reason why we can deny "the table is square" by "the table is round" is that what is round is *not* square. And this has to be a *fact*, though just as negative as the fact that this table is not square. Thus it is plain that incompatibility cannot exist without negative facts.' (*OP* 4; *LK* 288) The other option is to 'substitute for a negative fact the mere absence of a fact. . . But the absence of a fact is itself a negative fact; it is the fact that there is *not* such a fact' (*OP* 4f; *LK* 288).<sup>45</sup> To this we may add, should anyone believe there to be a difference between the absence and the non-existence of a fact, that the non-existence of a fact is also fact and

<sup>45</sup>Russell also discusses Demos's view that negation is to be explained in terms of a primitive relation of opposition between propositions (*Demos* 1917). But this notion is effectively incompatibility applied to propositions. Russell focuses on Demos's epistemology rather than his metaphysics. Needless to say, he finds it wanting. Demos's proposal is discussed in more detail at the end of lecture III of *PLA*. There the conclusion is that it is simpler to admit negative facts, as incompatibility holds between propositions, and so one would need to admit propositions as real in addition to facts (*PLA* III & IV 45f; *LK* 214f). For further discussions of the strengths and weaknesses of the incompatibility view and some of its history, see Kürbis (2019a) and Kürbis (2019b, Ch. 4).

just as negative as those one might wish to avoid by an appeal to the non-existence of facts.

The gist of Russell's argument for negative facts is that any attempt to avoid them merely sees them crop up again elsewhere. And this is exactly what happens when one endorses the explanation that the judgement that all men are mortal is true if and only if all men are mortal and takes this to explain the truth of a general judgement in terms of its instances. That all men are mortal is a general fact. Similarly, keeping closer to Russell's actual wordings, that all elementary judgements collected together by 'all men are mortal' correspond to elementary facts, that all facts of the form ' $x$  is mortal', where  $x$  is a man, exist, or that there is a fact ' $x$  is mortal' for all values of  $x$  that are men, are general facts.

The correspondence theory of truth does not require that for every kind of true propositions, there is a kind of fact to which they correspond, facts the structure of which is somehow reflected in the propositions. It suffices if the truth of any proposition can be accounted for by some facts. It may be the case that propositions require analysis. There is no kind of fact that corresponds to propositions containing definite descriptions, for example, according to Russell's Theory of Definite Descriptions.<sup>46</sup> 'The author of *The Philosophy of Logical Atomism* smoked a pipe' is not true in virtue of a fact that reflects its composition, but of the fact that one and only person authored *The Philosophy of Logical Atomism* and that person smoked a pipe. But the generality of propositions cannot be analysed in terms of something else, and, as argued, attempts to explain the truth of general judgements in terms of something else either failed or merely saw them crop up again elsewhere. So we are left with taking general judgements and their truth at face value.

As argued, Russell appears to have changed his mind about the truth of general judgments already by the time of *PP*. A yet stronger hint is in a brief discussion of general propositions in *OKEW* (53, 55f), which also alludes to the argument spelled out in Section 5.3. Russell observes that in order to deduce a general proposition such as 'all men are mortal' from facts about each particular man's mortality, we would also need to know that 'those are all the men there are' (*OKEW* 56). The issue here is one that concerns inference, but it is also one regarding the truth of 'all

<sup>46</sup>First in Russell (1905), reprinted in Russell (2014). See Kürbis (2024a) for exposition.

men are mortal'. It suggests that even if we tried to define the truth of general judgements in terms of their instances along the lines attempted in *PM*, an additional clause 'these are all instances' would be required, which is missing from *PM*, and which states a general fact.<sup>47</sup>

## 5.2. From Judgements to General Facts

Concerning elementary judgements, Russell says:

When a judgement occurs, there is a certain complex entity, composed of the mind and the various objects of the judgement. When the judgement is *true*,...there is a corresponding complex of the *objects* of the judgement alone. Falseness...consists in the absence of a corresponding complex composed of the objects alone. (*PM* 46)

This is the celebrated multiple relation theory of judgement coupled with truth defined in terms of correspondence.<sup>48</sup>

In *PM*, Russell does not think that this account is applicable when it is judged that  $(x) . \phi x$ . Instead, general judgements and second-order truth as new kinds of judgements and truth are introduced to deal with these cases. The question arises, what are the objects that, together with the mind, compose a general judgement?

*PM* does not address this question, except for noting that it cannot be the constituents of all its instances; hence the need for a 'radically new kind of judgement'. From Russell's metaphysics, however, the

<sup>47</sup>Are general facts positive or negative? In *OKEW* Russell partitions the general propositions into positive and negative ones. The former are those of form 'All  $A$  are  $B$ '. The latter have the form 'Some  $A$  is  $B$ ' and are negative because they are denials of positive general propositions (55). 'The knowledge that there are no other atomic facts is positive, general knowledge; it is the knowledge that "all atomic facts are known to me," or at least "all atomic facts are in this collection"—however the collection may be given.' (*OKEW* 55) If this is followed up to the level of facts, then there are positive and negative general facts, and 'All philosophers are wise' is false because of the negative general fact that some philosophers are not wise. Russell, however, does not retain this distinction very long. In (*PLA* V & VI 190f; *LK* 228f) he recommends not applying 'positive' and 'negative' to general and existential propositions, because the decision which is which is arbitrary. More recent advocates of universal facts are in two minds over whether they are negative or positive. Armstrong thinks they are negative, Hochberg that they are positive. See Kürbis (2022) for a comparison of their views with those of Russell and arguments that the Russell's account is to be preferred.

<sup>48</sup>On which Russell changed his mind. See Weiss (1995) for discussion. Miller (2006) points out further difficulties with it. For Russell's changing views on propositions and judgements, see Ito (2020).



conclusion is inevitable that at least universals must be involved. The general judgement that all men are mortal must consist at least of a mind and the universals *man* and *mortal*.

There must be something involved in the judgement over and above these two universals, as otherwise there would be no difference between the judgements 'All men are mortal', 'All mortals are men' and 'Some men are mortal'. One answer would be view (b) of Section 2: a third universal, *subsumed under*, is involved, and, as for elementary judgements, there is a certain order in which the mind and these three universals form the complex entity that is the judgment. But it is explicitly denied in *PM* that the judgement that 'all men are mortal' is the same as this one.

The by now evident alternative is to apply what is suggested in (*TK* 97ff) also to general judgements: in any judgement, the mind judges the objects of the judgement to be combined according to a certain form. In the elementary judgement 'Socrates precedes Plato' the mind judges Socrates, Plato and the universal *precedes* to be combined by the form  $xRy$ , where Socrates is  $x$  and Plato is  $y$ . Analogously, then, in the general judgement 'All men are mortal' the mind judges the universals *man* and *mortal* to be combined by the form 'All  $A$  are  $B$ ', where *man* is  $A$  and *mortal* is  $B$ . Similarly for 'some men are mortal': here the universals are the same, but the form is different.<sup>49</sup>

On this account, general judgements are not of a radically new kind. What makes them general is that only universals and certain forms are involved. Apart from that, they are just like elementary judgements.

Now the forms occurring in judgements are the forms of facts. Thus the irreducibility of general judgements together with the multiple relation theory of judgement leads naturally to a metaphysics of general facts. Consequently, what makes a truth second-order is the form involved in the judgement, not a different kind of correspondence.

This account of general judgements and their truth gels better with the multiple relation theory of judgement and the correspondence theory of truth than the *PM* account. General judgements are explained in terms of straightforward acquaintance with forms and universals, their truth in terms of straightforward correspondence to facts. Both explanations are the same, no matter whether the judgement is elementary or general.

<sup>49</sup>See Landini (2011, 265f).

### 5.3. Complete Descriptions and Inferences

In *PLA*, Russell gives two arguments for the view that there are general facts, which, however, he sees as closely connected. One argument is based on what a complete description of the world would be like. It is not enough, says Russell, to list all the atomic facts. One also needs to say that these are all of them:

The distinction between particular and general facts is one of the most important. There again it would be a very great mistake to suppose that you could describe the world completely by means of particular facts alone. Suppose that you had succeeded in chronicling every single particular fact throughout the universe, and that there did not exist a single particular fact of any sort anywhere that you had not chronicled, you still would not have got a complete description of the universe unless you also added: "These that I have chronicled are all the particular facts there are." So you cannot hope to describe the world completely without having general facts as well as particular facts. (*PLA* I & II 502f; *LK* 183f)

The second argument is based on the logical point that 'you never can arrive at a general proposition by inference from particular propositions alone. You will always have to have at least one general proposition in your premises.' (*PLA* V & VI 199; *LK* 235)<sup>50</sup> Russell observes that from elementary propositions about particular men's mortality, 'A is a man that is mortal', 'B is a man that is mortal', 'C is man that is mortal' and so on, one cannot infer 'All men are mortal' unless the premise 'All men are amongst those I have enumerated' is also given. (*PLA* V & VI 198f; *LK* 235) This additional premise is an indispensable general proposition. Even if there were only two objects in the universe,  $a$  and  $b$  name them, and each of them has the property  $\phi$ , the inference from premises (1)  $\phi a$  and (2)  $\phi b$  to the conclusion  $(x) \cdot \phi x$  is invalid. What is needed is a further premise to the effect that  $a$  and  $b$  are the only objects there are, i.e., (3)  $(x) \cdot x = a \vee x = b$ .<sup>51</sup> If the domain is infinite, Russell's

<sup>50</sup>In the footnote to §3 of *Grundlagen*, Frege writes that 'if it is at all acknowledged that there are general truths, it must be admitted that there are such *Urgesetze* [general laws that can neither be proved nor stand in need of proof], because nothing follows from individual facts, except because of a law' (1884). According to the definition of §3, propositions that can be proved from only such laws are *a priori*. There is no hint that these are only the propositions of logic and geometry. Intriguingly, laws of nature, in as far as they do not refer to particulars, would be classified as *a priori*, albeit synthetic.

<sup>51</sup>This reconstruction follows Hochberg (1969, 336f). I discuss Hochberg's paper in more detail and exhibit a flaw in it in Kürbis (2022).

point hits home even more forcefully: because then we cannot even try to list all particular premises  $\phi a_1, \phi a_2, \phi a_3 \dots$  nor could we produce a list of the names of all the object there are and write down a premise corresponding to (3). The required additional premises is what was to be proved:  $(x) \cdot \phi x$ .

In *OKEW*, Russell connects both arguments by considering how premises of the kind ‘these are all the facts there are’ may be used in deductions: ‘If we knew all atomic facts, and also knew that there were none except those we knew, we should, theoretically, be able to infer all truths of whatever form.’ (*OKEW* 55) This point also made it into the second edition of *PM*:

Given all true atomic propositions, together with the fact that they are all, every other true proposition can theoretically be deduced by logical methods. That is to say, the apparatus of crude fact required in proofs can all be condensed into the true atomic propositions together with the fact that every true atomic proposition is one of the following: (here the list should follow). If used, this method would presumably involve an infinite enumeration, since it seems natural to suppose that the number of true atomic propositions is infinite, though this should not be regarded as certain. In practice, generality is not obtained by the method of complete enumeration, because this method requires more knowledge than we possess. (*Whitehead and Russell* 1997 xv)

One worry of which Russell couldn’t have been aware is that in an incomplete logic, such as Russell’s higher-order logic, an infinite set of formulas  $\Gamma$  may validly entail a formula  $\sigma$  (i.e., every model of  $\Gamma$  is a model of  $\sigma$ ), while there is no deduction of  $\sigma$  from  $\Gamma$ , i.e., a finite list of formulas derived by a finite number of rules of inference from finitely many axioms or assumptions. But we may shelve this worry.

Both arguments establish the need for irreducibly general propositions. For them to be true, the correspondence theory demands that they be true in virtue of facts, and these are general facts.

There is a slight difference between the two arguments: one adduces general propositions that delimit the facts there are, the other ones that delimit the things there are.<sup>52</sup> It is possible that Russell did not consider

<sup>52</sup>Russell argument was adopted by Armstrong, who agrees that there must be general facts that impose limits (*Armstrong* 1997, 198ff; 2004, 70ff). Kürbis (2022) contains a critical discussion of Armstrong’s account.

this distinction to be of particular importance, as evidenced by an appeal to both in the same breadth:

I do not think one can doubt that there are general facts. It is perfectly clear, I think, that when you have enumerated all the atomic facts in the world, it is a further fact about the world that those are all the atomic facts there are about the world, and that is just as much an objective fact about the world as any of them are. It is clear, I think, that you must admit general facts as distinct from and over and above particular facts. The same thing applies to “All men are mortal.” When you have taken all the particular men that there are, and found each one of them severally to be mortal, it is definitely a new fact that all men are mortal; how new a fact, appears from what I said a moment ago, that it could not be inferred from the mortality of the several men that there are in the world. (*PLA V & VI* 200; *LK* 236)

These arguments may not establish the need for a large variety of general facts. Maybe all cases of generality can be reduced to one or both of the two kinds: either that these are all the particulars or that these are all the facts. Whether this is so is not crucial: in either case there are irreducibly general facts. Russell’s purpose is to point out that there must be general facts, of whatever kind they may be. Once general facts such as those in the two arguments are admitted there is little reason not to accept further kinds of general facts.<sup>53</sup>

Logical inference, incidentally, provides an argument against view (b) of Section 2. ‘Socrates is mortal’ follows logically without the addition of any further premises from ‘Socrates is a man’ and ‘All men are mortal’. But it does not follow logically without any further premises from ‘Socrates is a man’ and ‘the universal *man* is subsumed under the universal *mortal*’. It would require the further premise that whenever something instantiates a universal that is subsumed under another universal, it also instantiates that other universals, which is, of course, a general proposition. A crucial difference between (b) and (c) of Section 2 is that (b) lacks the logical force of (c).

<sup>53</sup>For further analysis of Russell’s argument from *PLA* I refer the reader to Elkind (2021, secs. 4, 5).

## 6. Conclusion

This paper has traced Russell's changing views on generality between 1910 and 1918. I proposed that a problem with the falsehood of general judgements on the account of general truth of *PM* may have led him to abandon it and instead adopt the view that there are general facts in virtue of which general judgements are true. Three independent arguments for the existence of general facts have also been considered.

Russell wanted to see the question of the right analysis of general facts to be studied. My aim in the present paper was to contribute to this study. Much more could be said about the nature of general facts, but this is a matter for another occasion.

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